Computer Algebra Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/182-6.6.2-e-x-^m-a+b-csch-c+d-x^n-^p

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Contents

1 Introduction **3**

2 detailed summary tables of results **21**

3 Listing of integrals **45**

4 Appendix **575**

CHAPTER 1

INTRODUCTION

1.1 Listing of CAS systems tested 4
1.2 Results 5
1.3 Time and leaf size Performance 8
1.4 Performance based on number of rules Rubi used 10
1.5 Performance based on number of steps Rubi used 11
1.6 Solved integrals histogram based on leaf size of result 12
1.7 Solved integrals histogram based on CPU time used 13
1.8 Leaf size vs. CPU time used 14
1.9 list of integrals with no known antiderivative 15
1.10 List of integrals solved by CAS but has no known antiderivative 15
1.11 list of integrals solved by CAS but failed verification 15
1.12 Timing 16
1.13 Verification 16
1.14 Important notes about some of the results 16
1.15 Design of the test system 19

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [83]. This is test number [182].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (83)	0.00 (0)
Mathematica	93.98 (78)	6.02 (5)
Fricas	74.70 (62)	25.30 (21)
Maxima	66.27 (55)	33.73 (28)
Maple	59.04 (49)	40.96 (34)
Mupad	56.63 (47)	43.37 (36)
Giac	51.81 (43)	48.19 (40)
Sympy	43.37 (36)	56.63 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

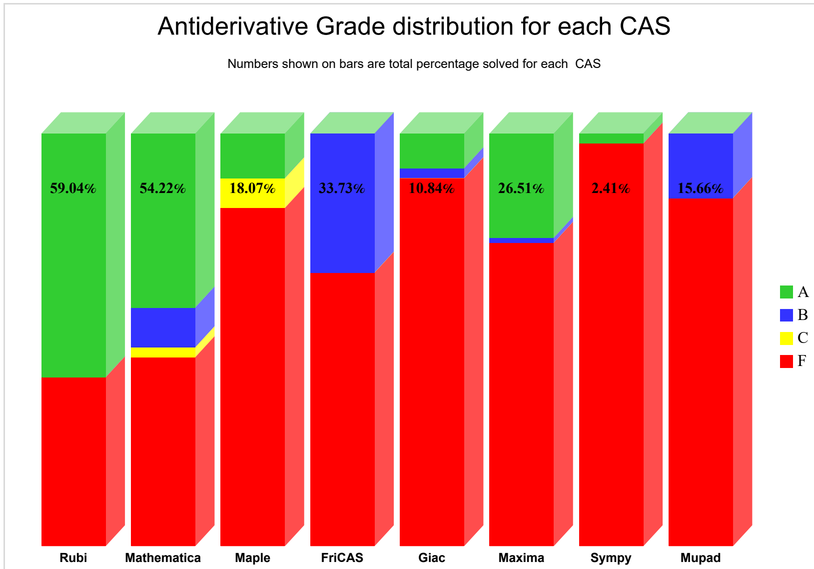
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

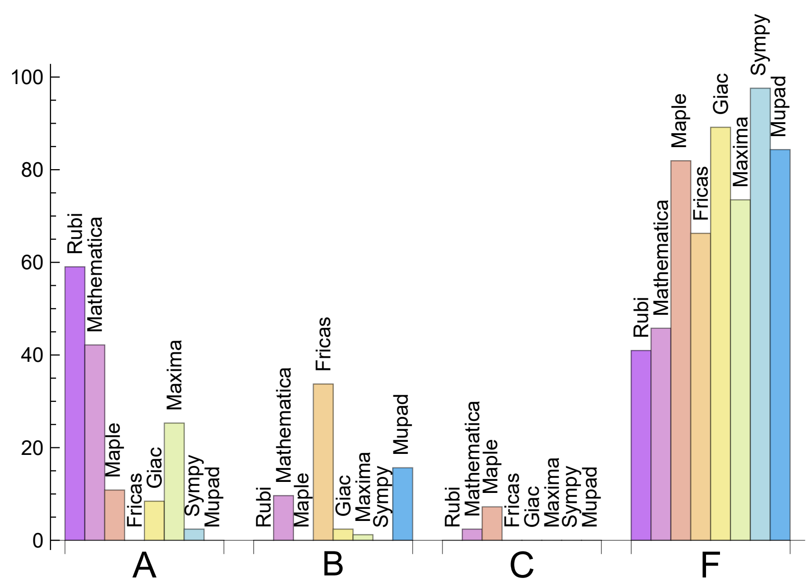
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.036	0.000	0.000	40.964
Mathematica	42.169	9.639	2.410	45.783
Maxima	25.301	1.205	0.000	73.494
Maple	10.843	0.000	7.229	81.928
Giac	8.434	2.410	0.000	89.157
Sympy	2.410	0.000	0.000	97.590
Fricas	0.000	33.735	0.000	66.265
Mupad	0.000	15.663	0.000	84.337

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	5	80.00	20.00	0.00
Fricas	21	95.24	4.76	0.00
Maxima	28	96.43	3.57	0.00
Maple	34	100.00	0.00	0.00
Mupad	36	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Sympy	47	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.28
Rubi	0.30
Maple	0.41
Maxima	0.43
Giac	0.64
Sympy	1.28
Mupad	2.69
Mathematica	16.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	18.58	0.95	17.00	0.94
Giac	36.21	1.11	20.00	1.11
Maple	71.22	1.25	18.00	1.00
Mupad	94.57	1.71	22.00	1.22
Maxima	164.45	4.66	112.00	2.50
Rubi	265.17	1.00	68.00	1.00
Mathematica	351.72	1.38	84.50	1.11
Fricas	629.97	3.77	46.00	2.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

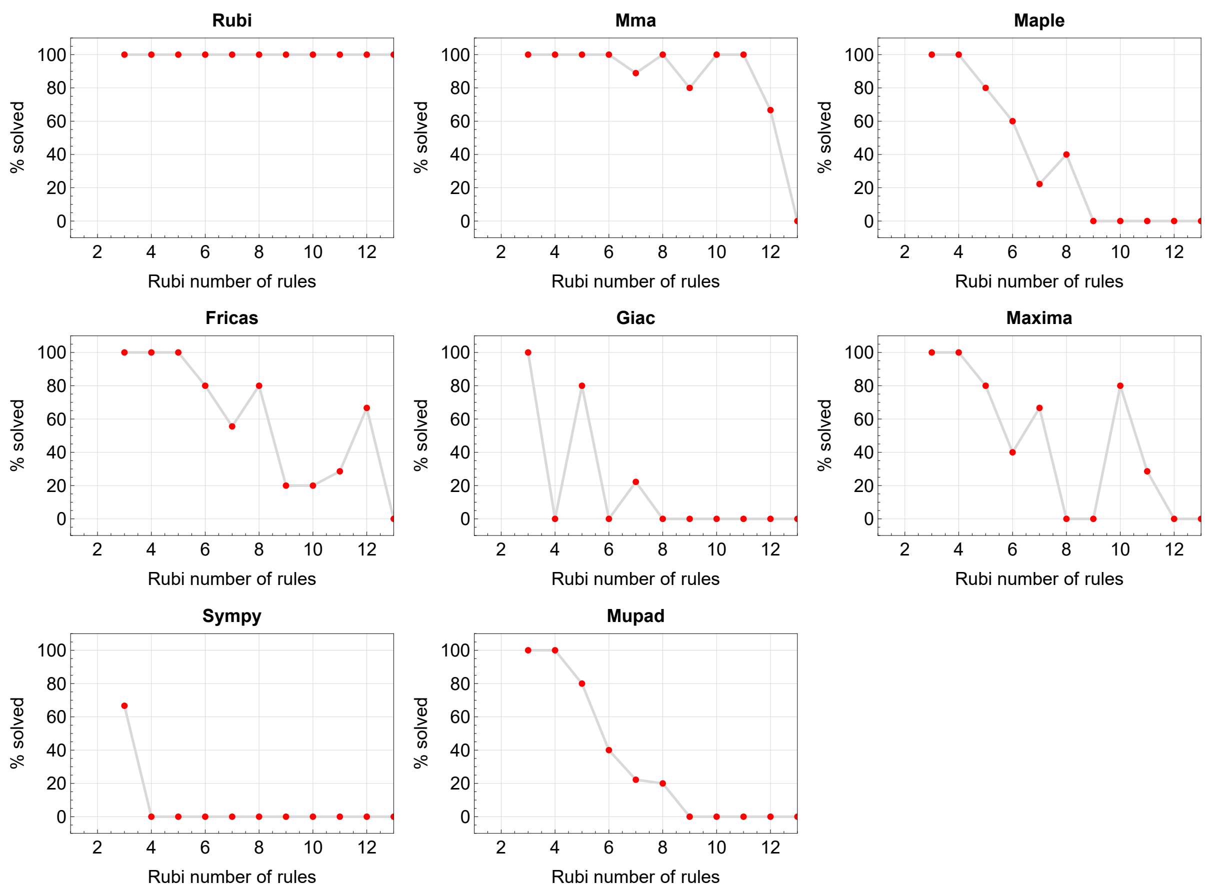


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

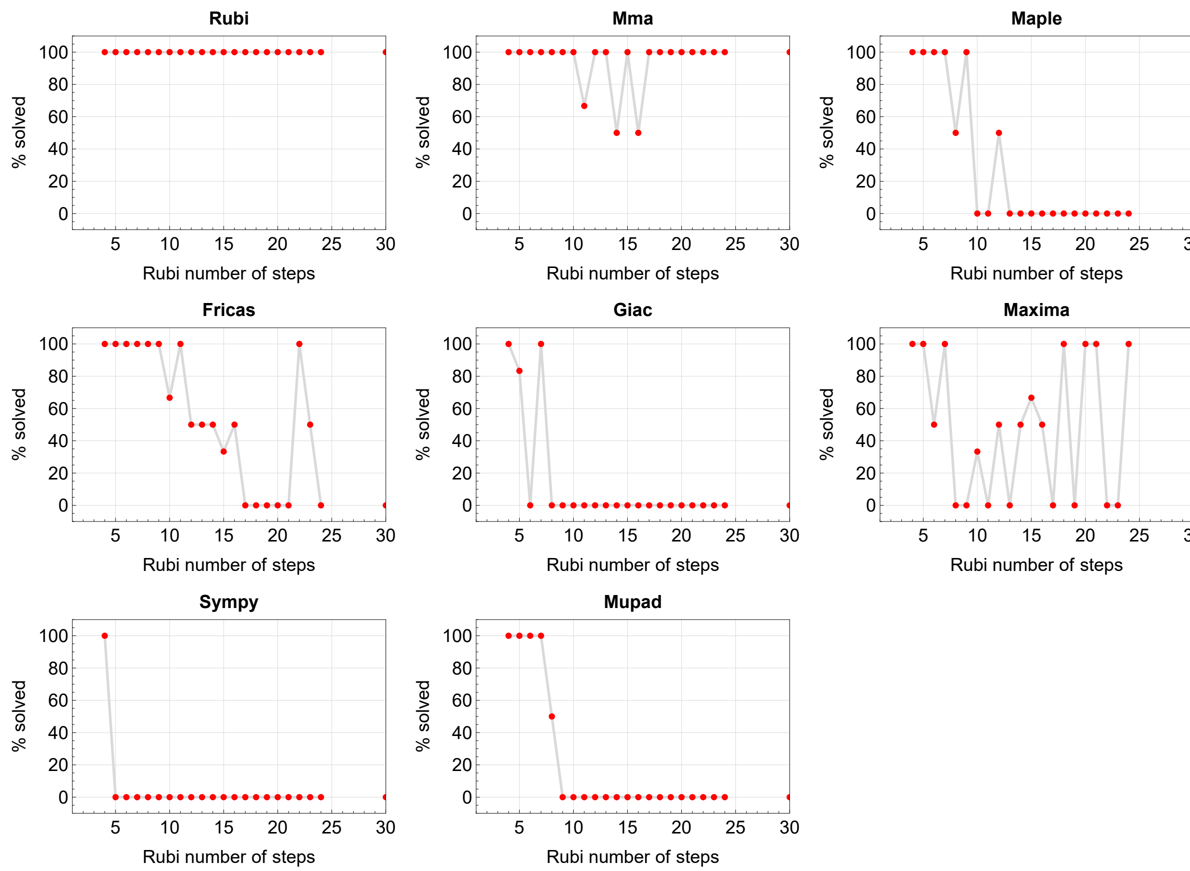


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

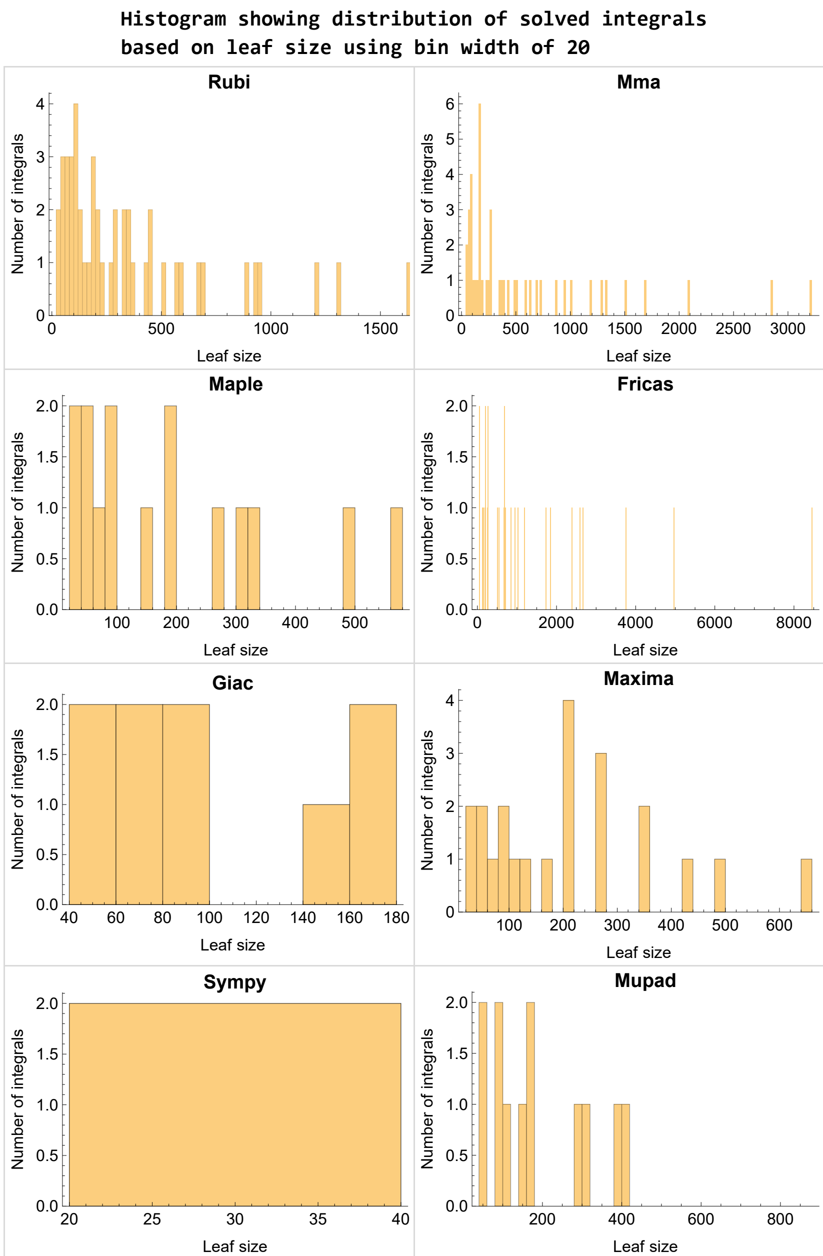


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

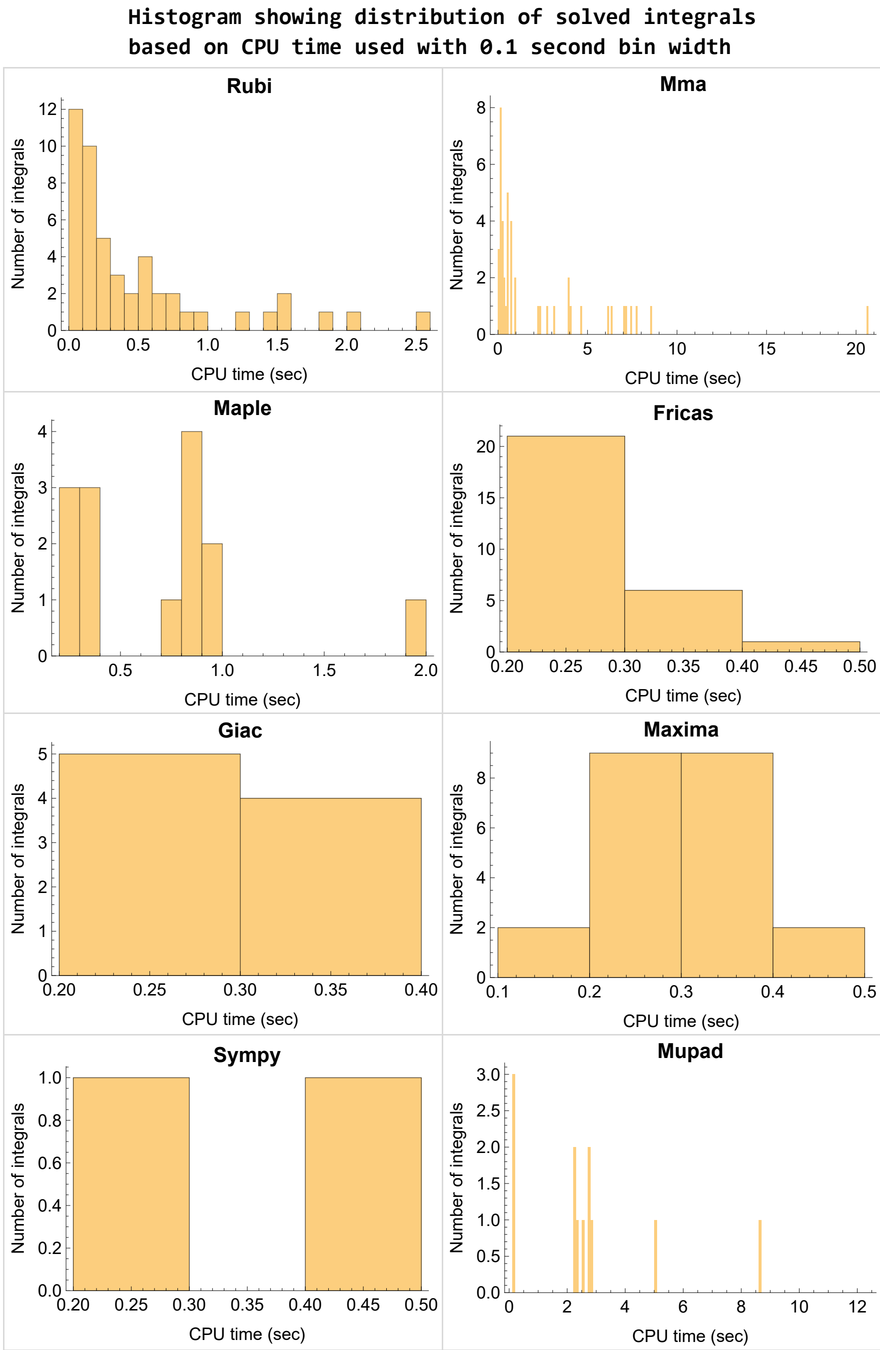


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

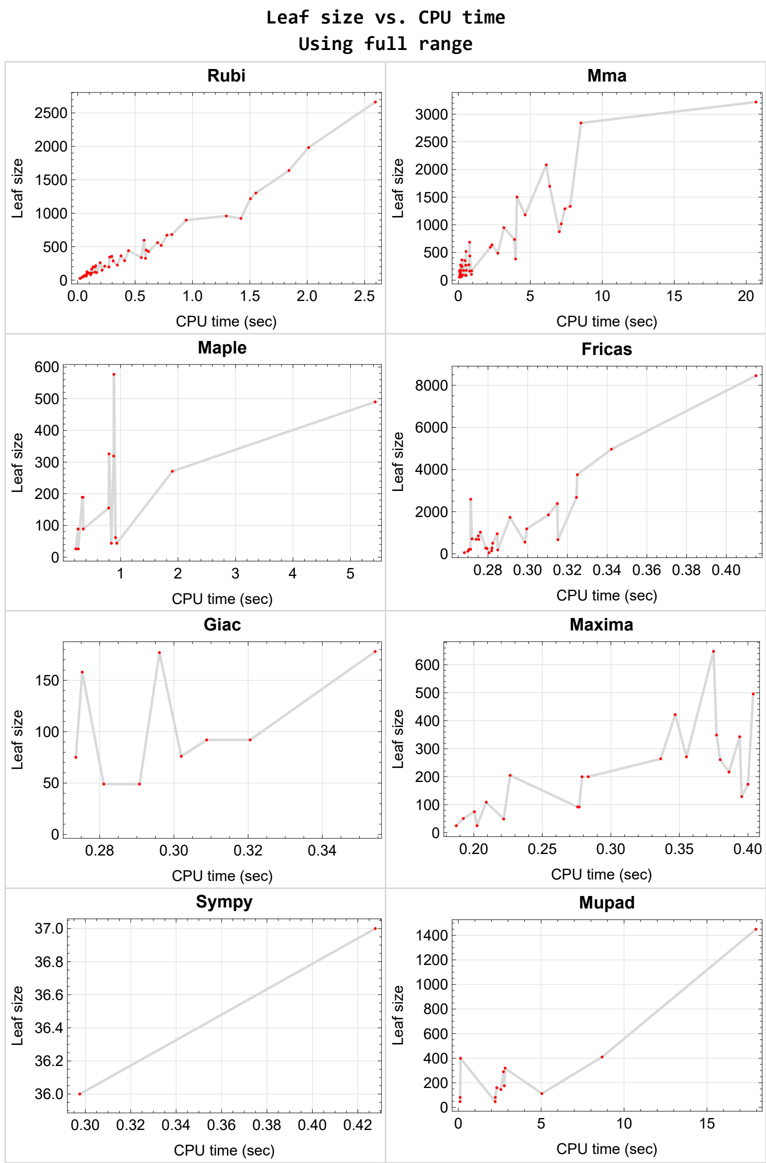


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 69, 70, 71\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {82}

Maple $\{72, 73, 75, 78, 79, 81\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

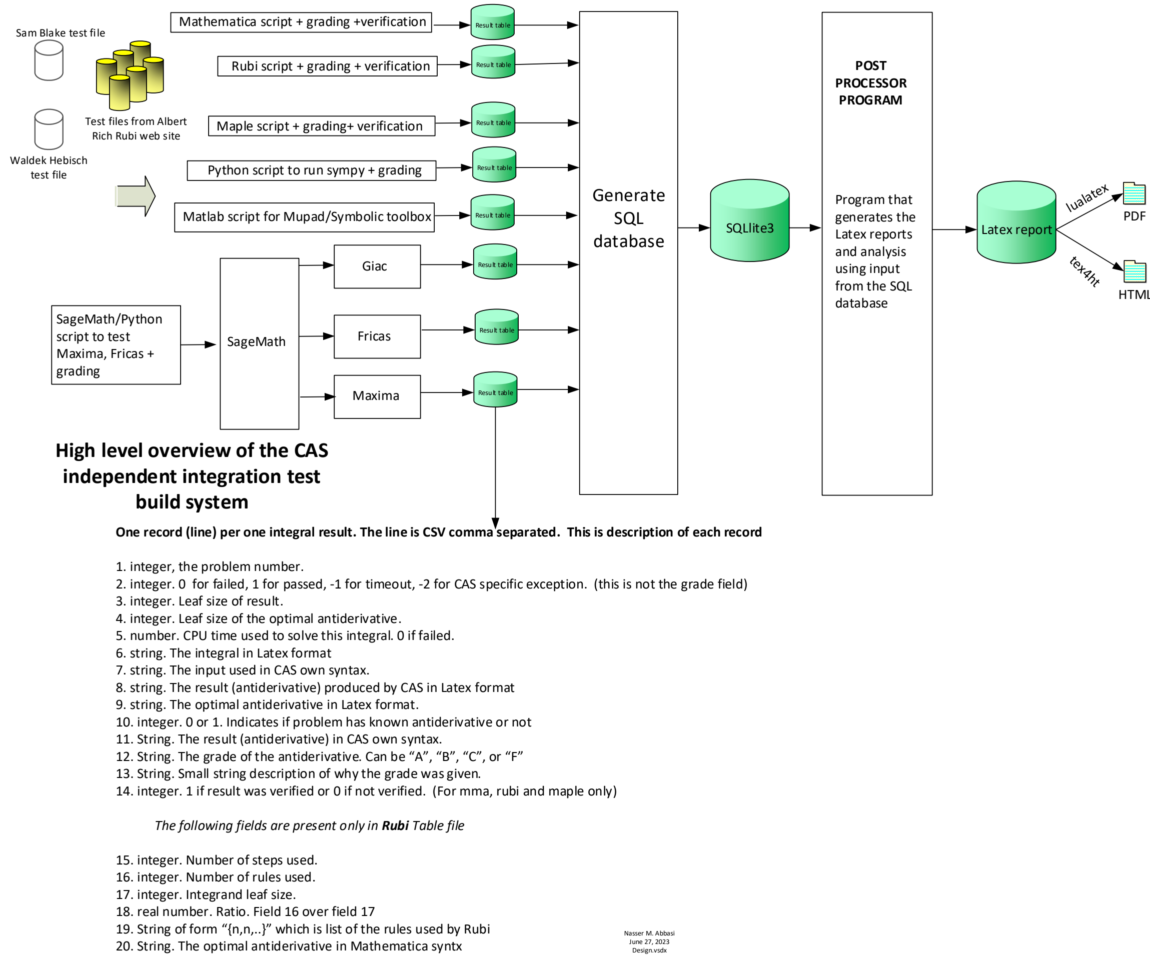
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 42

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 58, 61, 62, 63, 66, 67, 68, 72, 73, 75, 78, 81 }

B grade { 5, 8, 10, 36, 37, 56, 57, 76 }

C grade { 79, 82 }

F normal fail { 74, 77, 80, 83 }

F(-1) timedout fail { 39 }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 53, 58, 63, 68 }

B grade { }

C grade { 72, 73, 75, 78, 79, 81 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 74, 76, 77, 80, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { }

B grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 53, 58, 63, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82 }

C grade { }

F normal fail { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67 }

F(-1) timedout fail { 83 }

F(-2) exception fail { }

Maxima

A grade { 5, 8, 12, 20, 27, 31, 32, 33, 36, 37, 38, 51, 52, 53, 56, 57, 58, 63, 68, 72, 75 }

B grade { 15 }

C grade { }

F normal fail { 1, 3, 10, 16, 18, 23, 25, 41, 42, 43, 46, 47, 48, 61, 62, 66, 67, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { 60 }

F(-2) exception fail { }

Giac

A grade { 12, 15, 20, 27, 58, 63, 68 }

B grade { 5, 53 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 63, 68, 72, 75, 78, 81 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 73, 74, 76, 77, 79, 80, 82, 83 }

F(-2) exception fail { }

Sympy

A grade { 5, 53 }

B grade { }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	133	0	0	209	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.102	0.139	0.000	0.000	0.270	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	40	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.50	1.31	0.94	1.12	1.25
time (sec)	N/A	0.014	10.011	0.088	0.313	0.248	0.383	0.327	2.193

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	97	0	0	144	0	0	0
N.S.	1	1.00	1.43	0.00	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.057	0.124	0.000	0.000	0.282	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	40	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.50	1.31	0.94	1.12	1.25
time (sec)	N/A	0.014	8.350	0.076	0.311	0.240	0.369	0.312	2.229

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	57	26	25	55	37	49	47
N.S.	1	1.00	2.19	1.00	0.96	2.12	1.42	1.88	1.81
time (sec)	N/A	0.021	0.072	0.222	0.202	0.268	0.428	0.291	0.104

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	18	14	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.12	0.88	1.12	1.25
time (sec)	N/A	0.014	7.209	0.066	0.297	0.251	0.853	0.300	2.347

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	40	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.50	1.12	0.94	1.12	1.25
time (sec)	N/A	0.014	8.175	0.071	0.304	0.246	0.346	0.340	2.472

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	595	0	271	1031	0	0	0
N.S.	1	1.00	3.04	0.00	1.38	5.26	0.00	0.00	0.00
time (sec)	N/A	0.272	2.230	0.000	0.355	0.276	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	108	42	17	20	22
N.S.	1	1.00	1.11	1.00	6.00	2.33	0.94	1.11	1.22
time (sec)	N/A	0.019	22.467	0.103	0.418	0.258	0.492	0.811	2.288

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	276	0	0	683	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	6.32	0.00	0.00	0.00
time (sec)	N/A	0.123	0.734	0.000	0.000	0.275	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	98	42	17	20	22
N.S.	1	1.00	1.11	1.00	5.44	2.33	0.94	1.11	1.22
time (sec)	N/A	0.017	20.545	0.103	0.417	0.270	0.473	0.547	2.253

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	85	44	49	271	0	75	81
N.S.	1	1.00	1.89	0.98	1.09	6.02	0.00	1.67	1.80
time (sec)	N/A	0.043	0.505	0.936	0.222	0.279	0.000	0.274	0.114

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	112	36	15	20	22
N.S.	1	1.00	1.11	1.00	6.22	2.00	0.83	1.11	1.22
time (sec)	N/A	0.017	58.644	0.108	0.424	0.270	3.291	0.322	2.389

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	118	36	17	20	22
N.S.	1	1.00	1.11	1.00	6.56	2.00	0.94	1.11	1.22
time (sec)	N/A	0.017	33.333	0.102	0.439	0.254	0.591	0.869	2.387

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	167	62	205	2590	0	158	399
N.S.	1	1.00	1.86	0.69	2.28	28.78	0.00	1.76	4.43
time (sec)	N/A	0.079	0.084	0.914	0.226	0.271	0.000	0.275	0.135

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	325	256	0	0	686	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.591	0.243	0.000	0.000	0.274	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	61	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.39	1.11	0.83	1.11	1.22
time (sec)	N/A	0.020	2.702	0.088	0.290	0.273	0.291	0.313	2.115

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	175	0	0	505	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.345	0.170	0.000	0.000	0.282	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	61	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.39	1.11	0.83	1.11	1.22
time (sec)	N/A	0.020	2.286	0.084	0.293	0.260	0.298	0.295	2.077

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	71	89	92	213	0	92	175
N.S.	1	1.00	1.18	1.48	1.53	3.55	0.00	1.53	2.92
time (sec)	N/A	0.073	0.123	0.261	0.277	0.271	0.000	0.309	2.776

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	59	19	15	20	22
N.S.	1	1.00	1.11	1.00	3.28	1.06	0.83	1.11	1.22
time (sec)	N/A	0.019	2.055	0.078	0.279	0.261	0.783	0.324	2.255

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	40	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.50	1.12	0.94	1.12	1.25
time (sec)	N/A	0.012	0.141	0.014	0.320	0.258	0.354	0.328	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	922	922	1502	0	0	3756	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	1.422	4.079	0.000	0.000	0.325	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	304	38	17	20	22
N.S.	1	1.00	1.11	1.00	16.89	2.11	0.94	1.11	1.22
time (sec)	N/A	0.018	14.953	0.070	0.428	0.256	0.492	0.382	2.282

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	519	519	735	0	0	2383	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	4.59	0.00	0.00	0.00
time (sec)	N/A	0.728	3.906	0.000	0.000	0.315	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	292	38	17	20	22
N.S.	1	1.00	1.11	1.00	16.22	2.11	0.94	1.11	1.22
time (sec)	N/A	0.020	15.012	0.082	0.411	0.258	0.490	0.360	2.280

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	161	189	200	711	0	177	290
N.S.	1	1.00	1.42	1.67	1.77	6.29	0.00	1.57	2.57
time (sec)	N/A	0.165	0.782	0.336	0.279	0.272	0.000	0.296	2.723

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	246	38	17	20	22
N.S.	1	1.00	1.11	1.00	13.67	2.11	0.94	1.11	1.22
time (sec)	N/A	0.018	32.770	0.075	0.408	0.274	1.061	0.941	2.603

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	313	44	19	20	22
N.S.	1	1.00	1.11	1.00	17.39	2.44	1.06	1.11	1.22
time (sec)	N/A	0.018	19.243	0.077	0.443	0.253	0.829	0.356	2.494

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	315	44	19	3	22
N.S.	1	1.00	1.11	1.00	17.50	2.44	1.06	0.17	1.22
time (sec)	N/A	0.018	19.371	0.076	0.435	0.273	0.726	2.849	2.696

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	365	0	349	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.250	0.000	0.377	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	273	0	261	0	0	0	0
N.S.	1	1.00	1.05	0.00	1.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.161	0.000	0.380	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	181	0	173	0	0	0	0
N.S.	1	1.00	1.10	0.00	1.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.131	0.000	0.400	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	43	18	15	18	20
N.S.	1	1.00	1.11	0.89	2.39	1.00	0.83	1.00	1.11
time (sec)	N/A	0.013	34.834	0.178	0.435	0.256	1.591	0.302	2.182

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	53	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.94	1.00	0.94	1.00	1.11
time (sec)	N/A	0.014	32.926	0.164	0.471	0.258	0.540	0.306	2.264

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	597	597	1289	0	648	0	0	0	0
N.S.	1	1.00	2.16	0.00	1.09	0.00	0.00	0.00	0.00
time (sec)	N/A	0.578	7.404	0.000	0.375	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	441	441	1017	0	496	0	0	0	0
N.S.	1	1.00	2.31	0.00	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	7.154	0.000	0.404	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	382	0	343	0	0	0	0
N.S.	1	1.00	1.33	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	3.981	0.000	0.394	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	0	18	117	36	17	20	22
N.S.	1	1.00	0.00	0.90	5.85	1.80	0.85	1.00	1.10
time (sec)	N/A	0.018	0.000	0.329	0.595	0.265	9.162	0.407	2.500

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	143	36	19	20	22
N.S.	1	1.00	1.10	0.90	7.15	1.80	0.95	1.00	1.10
time (sec)	N/A	0.017	127.543	0.324	0.630	0.260	1.530	0.507	2.494

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	897	897	685	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.945	0.784	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	673	673	519	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.777	0.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	353	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.462	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	59	19	17	20	22
N.S.	1	1.00	1.10	0.90	2.95	0.95	0.85	1.00	1.10
time (sec)	N/A	0.019	3.764	0.232	0.527	0.264	1.970	0.336	2.225

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	53	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.94	1.00	0.94	1.00	1.11
time (sec)	N/A	0.012	0.149	0.013	0.444	0.253	0.667	0.308	0.003

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2663	2663	2841	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.594	8.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1983	1983	2085	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.012	6.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1303	1303	1333	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.553	7.777	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	249	38	19	20	22
N.S.	1	1.00	1.10	0.90	12.45	1.90	0.95	1.00	1.10
time (sec)	N/A	0.017	103.453	0.226	0.987	0.274	2.957	1.705	2.564

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	318	44	20	3	22
N.S.	1	1.00	1.10	0.90	15.90	2.20	1.00	0.15	1.10
time (sec)	N/A	0.018	69.896	0.233	1.150	0.278	2.333	2.696	2.657

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	238	0	217	0	0	0	0
N.S.	1	1.00	1.11	0.00	1.01	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.239	0.000	0.386	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	142	0	129	0	0	0	0
N.S.	1	1.00	1.18	0.00	1.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.138	0.000	0.395	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	26	25	55	36	49	47
N.S.	1	1.00	2.00	1.00	0.96	2.12	1.38	1.88	1.81
time (sec)	N/A	0.021	0.096	0.265	0.187	0.281	0.298	0.281	2.224

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	53	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.65	1.25	0.95	0.90	1.00
time (sec)	N/A	0.012	55.211	0.160	0.449	0.267	0.505	0.282	2.443

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	53	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.65	1.25	0.95	0.90	1.00
time (sec)	N/A	0.012	55.876	0.188	0.449	0.261	0.573	0.302	2.433

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	363	363	875	0	422	0	0	0	0
N.S.	1	1.00	2.41	0.00	1.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	7.019	0.000	0.347	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	637	0	264	0	0	0	0
N.S.	1	1.00	3.05	0.00	1.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	2.341	0.000	0.336	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	93	44	51	271	0	76	81
N.S.	1	1.00	1.98	0.94	1.09	5.77	0.00	1.62	1.72
time (sec)	N/A	0.045	0.307	0.841	0.192	0.282	0.000	0.302	2.235

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	138	46	20	20	22
N.S.	1	1.00	1.09	0.82	6.27	2.09	0.91	0.91	1.00
time (sec)	N/A	0.017	126.542	0.329	0.626	0.267	1.464	0.388	2.531

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	1.00
time (sec)	N/A	0.017	125.544	0.321	0.000	0.272	1.640	0.471	2.586

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	561	561	436	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	0.799	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	337	270	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	0.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	73	89	92	123	0	92	145
N.S.	1	1.00	1.16	1.41	1.46	1.95	0.00	1.46	2.30
time (sec)	N/A	0.071	0.158	0.352	0.276	0.270	0.000	0.321	2.571

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	67	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.05	1.23	0.91	0.91	1.00
time (sec)	N/A	0.018	5.551	0.230	0.491	0.260	1.599	0.360	2.230

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	67	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.05	1.23	0.91	0.91	1.00
time (sec)	N/A	0.019	5.759	0.240	0.582	0.268	1.595	0.441	2.236

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1639	1639	1696	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.841	6.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	959	959	948	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.294	3.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	175	189	200	670	0	178	319
N.S.	1	1.00	1.48	1.60	1.69	5.68	0.00	1.51	2.70
time (sec)	N/A	0.149	0.363	0.347	0.284	0.315	0.000	0.354	2.823

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	311	48	22	20	22
N.S.	1	1.00	1.09	0.82	14.14	2.18	1.00	0.91	1.00
time (sec)	N/A	0.018	59.997	0.218	1.037	0.276	2.369	2.207	2.907

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	318	48	22	3	22
N.S.	1	1.00	1.09	0.82	14.45	2.18	1.00	0.14	1.00
time (sec)	N/A	0.018	61.459	0.224	1.364	0.279	2.304	3.037	2.702

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20
time (sec)	N/A	0.037	14.634	0.172	0.376	0.273	0.308	0.761	2.218

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	61	155	75	180	0	0	112
N.S.	1	1.00	1.36	3.44	1.67	4.00	0.00	0.00	2.49
time (sec)	N/A	0.039	0.227	0.794	0.200	0.285	0.000	0.000	5.040

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	175	326	0	555	0	0	0
N.S.	1	1.00	1.41	2.63	0.00	4.48	0.00	0.00	0.00
time (sec)	N/A	0.080	0.550	0.800	0.000	0.299	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	0	0	0	951	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	4.83	0.00	0.00	0.00
time (sec)	N/A	0.133	0.000	0.000	0.000	0.285	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	103	271	109	854	0	0	160
N.S.	1	1.00	1.29	3.39	1.36	10.68	0.00	0.00	2.00
time (sec)	N/A	0.076	0.917	1.903	0.209	0.275	0.000	0.000	2.321

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	488	0	0	2678	0	0	0
N.S.	1	1.00	2.46	0.00	0.00	13.53	0.00	0.00	0.00
time (sec)	N/A	0.150	2.748	0.000	0.000	0.325	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	4967	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	14.44	0.00	0.00	0.00
time (sec)	N/A	0.278	0.000	0.000	0.000	0.342	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	82	82	84	319	0	248	0	0	410
N.S.	1	1.00	1.02	3.89	0.00	3.02	0.00	0.00	5.00
time (sec)	N/A	0.113	0.544	0.880	0.000	0.280	0.000	0.000	8.675

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	291	291	1181	577	0	1183	0	0	0
N.S.	1	1.00	4.06	1.98	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.409	4.648	0.885	0.000	0.299	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	428	428	0	0	0	1850	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	4.32	0.00	0.00	0.00
time (sec)	N/A	0.617	0.000	0.000	0.000	0.310	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	167	490	0	1729	0	0	1449
N.S.	1	1.00	1.12	3.29	0.00	11.60	0.00	0.00	9.72
time (sec)	N/A	0.212	0.925	5.430	0.000	0.291	0.000	0.000	17.956

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	681	681	3219	0	0	8453	0	0	0
N.S.	1	1.00	4.73	0.00	0.00	12.41	0.00	0.00	0.00
time (sec)	N/A	0.820	20.681	0.000	0.000	0.415	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1218	1218	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.505	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.666699999999999959]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	N/A	0	0	1.00	16	0.000
3	A	8	5	1.00	16	0.312
4	N/A	0	0	1.00	16	0.000
5	A	4	3	1.00	14	0.214
6	N/A	0	0	1.00	16	0.000
7	N/A	0	0	1.00	16	0.000
8	A	15	11	1.00	18	0.611
9	N/A	0	0	1.00	18	0.000
10	A	10	7	1.00	18	0.389
11	N/A	0	0	1.00	18	0.000
12	A	5	5	1.00	16	0.312
13	N/A	0	0	1.00	18	0.000
14	N/A	0	0	1.00	18	0.000
15	A	5	3	1.00	12	0.250
16	A	13	8	1.00	18	0.444
17	N/A	0	0	1.00	18	0.000
18	A	11	7	1.00	18	0.389
19	N/A	0	0	1.00	18	0.000
20	A	5	5	1.00	16	0.312
21	N/A	0	0	1.00	18	0.000
22	N/A	0	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	31	12	1.00	18	0.667
24	N/A	0	0	1.00	18	0.000
25	A	22	10	1.00	18	0.556
26	N/A	0	0	1.00	18	0.000
27	A	7	7	1.00	16	0.438
28	N/A	0	0	1.00	18	0.000
29	N/A	0	0	1.00	18	0.000
30	N/A	0	0	1.00	18	0.000
31	A	20	7	1.00	18	0.389
32	A	16	7	1.00	18	0.389
33	A	12	7	1.00	16	0.438
34	N/A	0	0	1.00	18	0.000
35	N/A	0	0	1.00	18	0.000
36	A	30	10	1.00	20	0.500
37	A	24	10	1.00	20	0.500
38	A	18	10	1.00	18	0.556
39	N/A	0	0	1.00	20	0.000
40	N/A	0	0	1.00	20	0.000
41	A	23	9	1.00	20	0.450
42	A	19	9	1.00	20	0.450
43	A	15	9	1.00	18	0.500
44	N/A	0	0	1.00	20	0.000
45	N/A	0	0	1.00	18	0.000
46	A	61	11	1.00	20	0.550
47	A	49	11	1.00	20	0.550
48	A	37	11	1.00	18	0.611
49	N/A	0	0	1.00	20	0.000
50	N/A	0	0	1.00	20	0.000
51	A	14	7	1.00	20	0.350
52	A	10	6	1.00	20	0.300
53	A	4	3	1.00	20	0.150
54	N/A	0	0	1.00	20	0.000
55	N/A	0	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	21	10	1.00	22	0.454
57	A	15	11	1.00	22	0.500
58	A	5	5	1.00	22	0.227
59	N/A	0	0	1.00	22	0.000
60	N/A	0	0	1.00	22	0.000
61	A	17	9	1.00	22	0.409
62	A	13	8	1.00	22	0.364
63	A	5	5	1.00	22	0.227
64	N/A	0	0	1.00	22	0.000
65	N/A	0	0	1.00	22	0.000
66	A	43	11	1.00	22	0.500
67	A	31	12	1.00	22	0.546
68	A	7	7	1.00	22	0.318
69	N/A	0	0	1.00	22	0.000
70	N/A	0	0	1.00	22	0.000
71	N/A	0	0	1.00	20	0.000
72	A	5	4	1.00	20	0.200
73	A	9	6	1.00	22	0.273
74	A	11	7	1.00	22	0.318
75	A	6	6	1.00	22	0.273
76	A	11	8	1.00	24	0.333
77	A	16	12	1.00	24	0.500
78	A	6	6	1.00	22	0.273
79	A	12	8	1.00	24	0.333
80	A	14	9	1.00	24	0.375
81	A	8	8	1.00	22	0.364
82	A	23	11	1.00	24	0.458
83	A	32	13	1.00	24	0.542

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a + b\text{csch}(c + dx^2))\ dx$ 49
3.2 $\int x^4(a + b\text{csch}(c + dx^2))\ dx$ 54
3.3 $\int x^3(a + b\text{csch}(c + dx^2))\ dx$ 57
3.4 $\int x^2(a + b\text{csch}(c + dx^2))\ dx$ 61
3.5 $\int x(a + b\text{csch}(c + dx^2))\ dx$ 64
3.6 $\int \frac{a+b\text{csch}(c+dx^2)}{x}\ dx$ 68
3.7 $\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\ dx$ 71
3.8 $\int x^5(a + b\text{csch}(c + dx^2))^2\ dx$ 74
3.9 $\int x^4(a + b\text{csch}(c + dx^2))^2\ dx$ 82
3.10 $\int x^3(a + b\text{csch}(c + dx^2))^2\ dx$ 85
3.11 $\int x^2(a + b\text{csch}(c + dx^2))^2\ dx$ 90
3.12 $\int x(a + b\text{csch}(c + dx^2))^2\ dx$ 93
3.13 $\int \frac{\left(a+b\text{csch}(c+dx^2)\right)^2}{x}\ dx$ 98
3.14 $\int \frac{\left(a+b\text{csch}(c+dx^2)\right)^2}{x^2}\ dx$ 101
3.15 $\int x\text{csch}^7(a + bx^2)\ dx$ 104
3.16 $\int \frac{x^5}{a+b\text{csch}(c+dx^2)}\ dx$ 111
3.17 $\int \frac{x^4}{a+b\text{csch}(c+dx^2)}\ dx$ 118
3.18 $\int \frac{x^3}{a+b\text{csch}(c+dx^2)}\ dx$ 121
3.19 $\int \frac{x^2}{a+b\text{csch}(c+dx^2)}\ dx$ 127
3.20 $\int \frac{x}{a+b\text{csch}(c+dx^2)}\ dx$ 130
3.21 $\int \frac{1}{x\left(a+b\text{csch}(c+dx^2)\right)}\ dx$ 135
3.22 $\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\ dx$ 138

3.23 $\int \frac{x^5}{\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 141
3.24 $\int \frac{x^4}{\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 157
3.25 $\int \frac{x^3}{\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 160
3.26 $\int \frac{x^2}{\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 170
3.27 $\int \frac{x}{\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 173
3.28 $\int \frac{1}{x\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 179
3.29 $\int \frac{1}{x^2\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 182
3.30 $\int \frac{1}{x^3\left(a+b\text{csch}(c+dx^2)\right)^2}\,dx$ . . . 186
3.31 $\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$ . . . 190
3.32 $\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$ . . . 200
3.33 $\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$ . . . 208
3.34 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x}\,dx$ . . . 214
3.35 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x^2}\,dx$ . . . 217
3.36 $\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$ . . . 220
3.37 $\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$ . . . 237
3.38 $\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$ . . . 251
3.39 $\int \frac{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}{x}\,dx$ . . . 260
3.40 $\int \frac{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}{x^2}\,dx$ . . . 263
3.41 $\int \frac{x^3}{a+b\text{csch}(c+d\sqrt{x})}\,dx$ . . . 267
3.42 $\int \frac{x^2}{a+b\text{csch}(c+d\sqrt{x})}\,dx$ . . . 281
3.43 $\int \frac{x}{a+b\text{csch}(c+d\sqrt{x})}\,dx$ . . . 291
3.44 $\int \frac{1}{x\left(a+b\text{csch}(c+d\sqrt{x})\right)}\,dx$ . . . 298
3.45 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x^2}\,dx$ . . . 301
3.46 $\int \frac{x^3}{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ . . . 304
3.47 $\int \frac{x^2}{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ . . . 319
3.48 $\int \frac{x}{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ . . . 333
3.49 $\int \frac{1}{x\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ . . . 346
3.50 $\int \frac{1}{x^2\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ . . . 350
3.51 $\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$ . . . 354
3.52 $\int \sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$ . . . 361

3.53 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{\sqrt{x}}\,dx$ 366
3.54 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx$ 370
3.55 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x^{5/2}}\,dx$ 373
3.56 $\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$ 376
3.57 $\int \sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$ 386
3.58 $\int \frac{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}{\sqrt{x}}\,dx$ 394
3.59 $\int \frac{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}{x^{3/2}}\,dx$ 399
3.60 $\int \frac{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}{x^{5/2}}\,dx$ 403
3.61 $\int \frac{x^{3/2}}{a+b\text{csch}(c+d\sqrt{x})}\,dx$ 406
3.62 $\int \frac{\sqrt{x}}{a+b\text{csch}(c+d\sqrt{x})}\,dx$ 414
3.63 $\int \frac{1}{\sqrt{x}\left(a+b\text{csch}(c+d\sqrt{x})\right)}\,dx$ 421
3.64 $\int \frac{1}{x^{3/2}\left(a+b\text{csch}(c+d\sqrt{x})\right)}\,dx$ 426
3.65 $\int \frac{1}{x^{5/2}\left(a+b\text{csch}(c+d\sqrt{x})\right)}\,dx$ 429
3.66 $\int \frac{x^{3/2}}{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ 432
3.67 $\int \frac{\sqrt{x}}{\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ 445
3.68 $\int \frac{1}{\sqrt{x}\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ 460
3.69 $\int \frac{1}{x^{3/2}\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ 467
3.70 $\int \frac{1}{x^{5/2}\left(a+b\text{csch}(c+d\sqrt{x})\right)^2}\,dx$ 471
3.71 $\int (ex)^m\,(a+b\text{csch}(c+dx^n))^p\,dx$ 475
3.72 $\int (ex)^{-1+n}\,(a+b\text{csch}(c+dx^n))\,dx$ 478
3.73 $\int (ex)^{-1+2n}\,(a+b\text{csch}(c+dx^n))\,dx$ 483
3.74 $\int (ex)^{-1+3n}\,(a+b\text{csch}(c+dx^n))\,dx$ 489
3.75 $\int (ex)^{-1+n}\,(a+b\text{csch}(c+dx^n))^2\,dx$ 495
3.76 $\int (ex)^{-1+2n}\,(a+b\text{csch}(c+dx^n))^2\,dx$ 500
3.77 $\int (ex)^{-1+3n}\,(a+b\text{csch}(c+dx^n))^2\,dx$ 508
3.78 $\int \frac{(ex)^{-1+n}}{a+b\text{csch}(c+dx^n)}\,dx$ 519
3.79 $\int \frac{(ex)^{-1+2n}}{a+b\text{csch}(c+dx^n)}\,dx$ 524
3.80 $\int \frac{(ex)^{-1+3n}}{a+b\text{csch}(c+dx^n)}\,dx$ 532
3.81 $\int \frac{(ex)^{-1+n}}{\left(a+b\text{csch}(c+dx^n)\right)^2}\,dx$ 540
3.82 $\int \frac{(ex)^{-1+2n}}{\left(a+b\text{csch}(c+dx^n)\right)^2}\,dx$ 548

3.83 $\int \frac{(ex)^{-1+3n}}{\left(a+b\text{csch}(c+dx^n)\right)^2}\,dx$ 560

3.1 $\int x^5(a + b\text{csch}(c + dx^2))\, dx$

Optimal result 49
Rubi [A] (verified) 49
Mathematica [A] (verified) 51
Maple [F] 52
Fricas [B] (verification not implemented) 52
Sympy [F] 52
Maxima [F] 53
Giac [F] 53
Mupad [F(-1)] 53

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{bx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{bx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}+\frac{b\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3}-\frac{b\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3}$$

```
[Out] 1/6*a*x^6-b*x^4*arctanh(exp(d*x^2+c))/d-b*x^2*polylog(2,-exp(d*x^2+c))/d^2+
b*x^2*polylog(2,exp(d*x^2+c))/d^2+b*polylog(3,-exp(d*x^2+c))/d^3-b*polylog(
3,exp(d*x^2+c))/d^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 5545, 4267, 2611, 2320, 6724}

$$\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}+\frac{b\,\text{PolyLog}\left(3,-e^{dx^2+c}\right)}{d^3}-\frac{b\,\text{PolyLog}\left(3,e^{dx^2+c}\right)}{d^3}-\frac{bx^2\,\text{PolyLog}\left(2,-e^{dx^2+c}\right)}{d^2}+\frac{bx^2\,\text{PolyLog}\left(2,e^{dx^2+c}\right)}{d^2}$$

```
[In] Int[x^5*(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 - (b*x^4*ArcTanh[E^(c + d*x^2)])/d - (b*x^2*PolyLog[2, -E^(c + d*
x^2)])/d^2 + (b*x^2*PolyLog[2, E^(c + d*x^2)])/d^2 + (b*PolyLog[3, -E^(c +
d*x^2)])/d^3 - (b*PolyLog[3, E^(c + d*x^2)])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^5+bx^5\text{csch}\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^6}{6}+b\int x^5\text{csch}\left(c+dx^2\right)\,dx\\
&= \frac{ax^6}{6}+\frac{1}{2}b\text{Subst}\left(\int x^2\text{csch}(c+dx)\,dx,x,x^2\right)\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b\text{Subst}\left(\int x\log\left(1-e^{c+dx}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{b\text{Subst}\left(\int x\log\left(1+e^{c+dx}\right)\,dx,x,x^2\right)}{d}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{bx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{bx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}\\
&\quad+\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,x^2\right)}{d^2}-\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,x^2\right)}{d^2}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{bx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{bx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{c+dx^2}\right)}{d^3}-\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{c+dx^2}\right)}{d^3}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{bx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}\\
&\quad+\frac{bx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}+\frac{b\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3}-\frac{b\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\begin{aligned}
\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx &= \frac{ax^6}{6}+\frac{bx^4\log\left(1-e^{c+dx^2}\right)}{2d}-\frac{bx^4\log\left(1+e^{c+dx^2}\right)}{2d}\\
&\quad-\frac{bx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{bx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}\\
&\quad+\frac{b\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3}-\frac{b\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3}
\end{aligned}$$

```
[In] Integrate[x^5*(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + (b*x^4*Log[1 - E^(c + d*x^2)])/(2*d) - (b*x^4*Log[1 + E^(c + d*
x^2)])/(2*d) - (b*x^2*PolyLog[2, -E^(c + d*x^2)])/d^2 + (b*x^2*PolyLog[2, E
^(c + d*x^2)])/d^2 + (b*PolyLog[3, -E^(c + d*x^2)])/d^3 - (b*PolyLog[3, E^(
c + d*x^2)])/d^3
```

Maple [F]

$$\int x^{5}\left(a+b\ \mathrm{csch}\left(d\,x^{2}+c\right)\right)dx$$

```
[In] int(x^5*(a+b*csch(d*x^2+c)),x)
```

```
[Out] int(x^5*(a+b*csch(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.01

$$\int x^5(a+b\mathrm{csch}(c+dx^2))\ dx$$
$$=\frac{ad^3x^6-3\,bd^2x^4\log\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)+1\right)+6\,bdx^2\mathrm{Li}_2(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right))-6\,b}{}$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^3*x^6 - 3*b*d^2*x^4*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + 1) + 6
*b*d*x^2*dilog(cosh(d*x^2 + c) + sinh(d*x^2 + c)) - 6*b*d*x^2*dilog(-cosh(d
*x^2 + c) - sinh(d*x^2 + c)) + 3*b*c^2*log(cosh(d*x^2 + c) + sinh(d*x^2 + c
) - 1) + 3*(b*d^2*x^4 - b*c^2)*log(-cosh(d*x^2 + c) - sinh(d*x^2 + c) + 1)
- 6*b*polylog(3, cosh(d*x^2 + c) + sinh(d*x^2 + c)) + 6*b*polylog(3, -cosh(
d*x^2 + c) - sinh(d*x^2 + c)))/d^3
```

Sympy [F]

$$\int x^5(a+b\mathrm{csch}(c+dx^2))\ dx=\int x^{5}\left(a+b\,\mathrm{csch}\left(c+dx^{2}\right)\right)\ dx$$

```
[In] integrate(x**5*(a+b*csch(d*x**2+c)),x)
```

```
[Out] Integral(x**5*(a + b*csch(c + d*x**2)), x)
```

Maxima [F]

$$\int x^5 (a + b\text{csch}(c + dx^2)) \, dx = \int (b \operatorname{csch}(dx^2 + c) + a) x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 2*b*integrate(x^5/(e^(d*x^2 + c) - e^(-d*x^2 - c)), x)
```

Giac [F]

$$\int x^5 (a + b\text{csch}(c + dx^2)) \, dx = \int (b \operatorname{csch}(dx^2 + c) + a) x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^2 + c) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b\text{csch}(c + dx^2)) \, dx = \int x^5 \left(a + \frac{b}{\sinh(d x^2 + c)}\right) \, dx$$

```
[In] int(x^5*(a + b/sinh(c + d*x^2)),x)
```

```
[Out] int(x^5*(a + b/sinh(c + d*x^2)), x)
```

3.2 $\int x^4(a + b\,\mathrm{csch}(c + dx^2))\,dx$

Optimal result 54
Rubi [N/A] 54
Mathematica [N/A] 55
Maple [N/A] (verified) 55
Fricas [N/A] 55
Sympy [N/A] 55
Maxima [N/A] 56
Giac [N/A] 56
Mupad [N/A] 56

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4 \left(a + b\,\mathrm{csch}\left(c + dx^2\right)\right)\,dx = \frac{ax^5}{5} + b\mathrm{Int}\left(x^4\mathrm{csch}\left(c + dx^2\right), x\right)$$

```
[Out] 1/5*a*x^5+b*Unintegrable(x^4*csch(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4 \left(a + b\,\mathrm{csch}\left(c + dx^2\right)\right)\,dx = \int x^4 \left(a + b\,\mathrm{csch}\left(c + dx^2\right)\right)\,dx$$

```
[In] Int[x^4*(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 + b*Defer[Int][x^4*Csch[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ax^4 + bx^4\mathrm{csch}\left(c + dx^2\right)\right)\,dx \\ &= \frac{ax^5}{5} + b\int x^4\mathrm{csch}\left(c + dx^2\right)\,dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 10.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int x^4(a+b\text{csch}(c+dx^2))\ dx$$

```
[In] Integrate[x^4*(a + b*Csch[c + d*x^2]),x]
[Out] Integrate[x^4*(a + b*Csch[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\ \text{csch}\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^4*(a+b*csch(d*x^2+c)),x)
[Out] int(x^4*(a+b*csch(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int (b\,\text{csch}\left(dx^2+c\right)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^4*csch(d*x^2 + c) + a*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int x^4\left(a+b\,\text{csch}\left(c+dx^2\right)\right)\ dx$$

```
[In] integrate(x**4*(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**4*(a + b*csch(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int \left(b\,\text{csch}\left(dx^2+c\right)+a\right)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/5*a*x^5 + 2*b*integrate(x^4/(e^(d*x^2 + c) - e^(-d*x^2 - c)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int \left(b\,\text{csch}\left(dx^2+c\right)+a\right)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4(a+b\text{csch}(c+dx^2))\ dx=\int x^4\left(a+\frac{b}{\sinh\left(d\,x^2+c\right)}\right)\,dx$$

```
[In] int(x^4*(a + b/sinh(c + d*x^2)),x)
[Out] int(x^4*(a + b/sinh(c + d*x^2)), x)
```

3.3 $\int x^3(a+b\text{csch}(c+dx^2))\,dx$

Optimal result 57
Rubi [A] (verified) 57
Mathematica [A] (verified) 59
Maple [F] 59
Fricas [B] (verification not implemented) 59
Sympy [F] 60
Maxima [F] 60
Giac [F] 60
Mupad [F(-1)] 60

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int x^3(a+b\text{csch}(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{bx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{2d^2}+\frac{b\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{2d^2}$$

```
[Out] 1/4*a*x^4-b*x^2*arctanh(exp(d*x^2+c))/d-1/2*b*polylog(2,-exp(d*x^2+c))/d^2+
1/2*b*polylog(2,exp(d*x^2+c))/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {14, 5545, 4267, 2317, 2438}

$$\int x^3(a+b\text{csch}(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{bx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b\,\text{PolyLog}\left(2,-e^{dx^2+c}\right)}{2d^2}+\frac{b\,\text{PolyLog}\left(2,e^{dx^2+c}\right)}{2d^2}$$

```
[In] Int[x^3*(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (b*x^2*ArcTanh[E^(c + d*x^2)])/d - (b*PolyLog[2, -E^(c + d*x^2)
])/(2*d^2) + (b*PolyLog[2, E^(c + d*x^2)])/(2*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3\operatorname{csch}\left(c + dx^2\right)\right) dx \\
&= \frac{ax^4}{4} + b\int x^3\operatorname{csch}\left(c + dx^2\right) dx \\
&= \frac{ax^4}{4} + \frac{1}{2}b\text{Subst}\left(\int x\operatorname{csch}(c + dx)\,dx, x, x^2\right) \\
&= \frac{ax^4}{4} - \frac{bx^2\operatorname{arctanh}\left(e^{c+dx^2}\right)}{d} - \frac{b\text{Subst}\left(\int \log\left(1 - e^{c+dx}\right)\,dx, x, x^2\right)}{2d} \\
&\quad + \frac{b\text{Subst}\left(\int \log\left(1 + e^{c+dx}\right)\,dx, x, x^2\right)}{2d}
\end{aligned}$$

$$= \frac{ax^4}{4} - \frac{bx^2 \operatorname{arctanh}\left(e^{c+dx^2}\right)}{d} - \frac{b\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{c+dx^2}\right)}{2d^2} + \frac{b\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{c+dx^2}\right)}{2d^2}$$

$$= \frac{ax^4}{4} - \frac{bx^2 \operatorname{arctanh}\left(e^{c+dx^2}\right)}{d} - \frac{b\operatorname{PolyLog}\left(2, -e^{c+dx^2}\right)}{2d^2} + \frac{b\operatorname{PolyLog}\left(2, e^{c+dx^2}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int x^3 \left(a + b\operatorname{csch}\left(c + dx^2\right)\right)\,dx = \frac{ax^4}{4} + \frac{bx^2 \log\left(1 - e^{c+dx^2}\right)}{2d} - \frac{bx^2 \log\left(1 + e^{c+dx^2}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, -e^{c+dx^2}\right)}{2d^2} + \frac{b\operatorname{PolyLog}\left(2, e^{c+dx^2}\right)}{2d^2}$$

```
[In] Integrate[x^3*(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 + (b*x^2*Log[1 - E^(c + d*x^2)])/(2*d) - (b*x^2*Log[1 + E^(c + d*
x^2)])/(2*d) - (b*PolyLog[2, -E^(c + d*x^2)])/(2*d^2) + (b*PolyLog[2, E^(c
+ d*x^2)])/(2*d^2)
```

Maple [F]

$$\int x^3 \left(a + b \operatorname{csch}\left(d\,x^2 + c\right)\right)dx$$

```
[In] int(x^3*(a+b*csch(d*x^2+c)),x)
```

```
[Out] int(x^3*(a+b*csch(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.12

$$\int x^3 \left(a + b\operatorname{csch}\left(c + dx^2\right)\right)\,dx$$
$$= \frac{ad^2x^4 - 2\,bdx^2 \log\left(\cosh\left(dx^2 + c\right) + \sinh\left(dx^2 + c\right) + 1\right) - 2\,bc \log\left(\cosh\left(dx^2 + c\right) + \sinh\left(dx^2 + c\right) - 1\right) +}{}$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - 2*b*d*x^2*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + 1) - 2*b
*c*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - 1) + 2*b*dilog(cosh(d*x^2 + c) +
 sinh(d*x^2 + c)) - 2*b*dilog(-cosh(d*x^2 + c) - sinh(d*x^2 + c)) + 2*(b*d*
x^2 + b*c)*log(-cosh(d*x^2 + c) - sinh(d*x^2 + c) + 1))/d^2
```

Sympy [F]

$$\int x^3(a + b\text{csch}(c + dx^2))\ dx = \int x^3(a + b\operatorname{csch}(c + dx^2))\ dx$$

```
[In] integrate(x**3*(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**3*(a + b*csch(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3(a + b\text{csch}(c + dx^2))\ dx = \int (b\operatorname{csch}(dx^2 + c) + a)x^3\, dx$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 2*b*integrate(x^3/(e^(d*x^2 + c) - e^(-d*x^2 - c)), x)
```

Giac [F]

$$\int x^3(a + b\text{csch}(c + dx^2))\ dx = \int (b\operatorname{csch}(dx^2 + c) + a)x^3\, dx$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a + b\text{csch}(c + dx^2))\ dx = \int x^3 \left(a + \frac{b}{\sinh(d\,x^2 + c)}\right)\, dx$$

```
[In] int(x^3*(a + b/sinh(c + d*x^2)),x)
[Out] int(x^3*(a + b/sinh(c + d*x^2)), x)
```

3.4 $\int x^2(a+b\text{csch}(c+dx^2))\,dx$

Optimal result . . . 61
Rubi [N/A] . . . 61
Mathematica [N/A] . . . 62
Maple [N/A] (verified) . . . 62
Fricas [N/A] . . . 62
Sympy [N/A] . . . 62
Maxima [N/A] . . . 63
Giac [N/A] . . . 63
Mupad [N/A] . . . 63

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+b\text{Int}\left(x^2\text{csch}\left(c+dx^2\right),x\right)$$

```
[Out] 1/3*a*x^3+b*Unintegrable(x^2*csch(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx$$

```
[In] Int[x^2*(a + b*Csch[c + d*x^2]),x]
[Out] (a*x^3)/3 + b*Defer[Int][x^2*Csch[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+bx^2\text{csch}\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^3}{3}+b\int x^2\text{csch}\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx$$

```
[In] Integrate[x^2*(a + b*Csch[c + d*x^2]),x]
[Out] Integrate[x^2*(a + b*Csch[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{2} \left(a +b \,\operatorname{csch}\left(d \,x^{2}+c \right)\right)dx$$

```
[In] int(x^2*(a+b*csch(d*x^2+c)),x)
[Out] int(x^2*(a+b*csch(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\int \left(b\operatorname{csch}\left(dx^2+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^2*csch(d*x^2 + c) + a*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**2*(a + b*csch(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int x^2(a + b\text{csch}(c + dx^2)) \, dx = \int (b \, \text{csch}\,(dx^2 + c) + a)x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 2*b*integrate(x^2/(e^(d*x^2 + c) - e^(-d*x^2 - c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b\text{csch}(c + dx^2)) \, dx = \int (b \, \text{csch}\,(dx^2 + c) + a)x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2(a + b\text{csch}(c + dx^2)) \, dx = \int x^2 \left(a + \frac{b}{\sinh\,(d\,x^2 + c)}\right) \, dx$$

```
[In] int(x^2*(a + b/sinh(c + d*x^2)),x)
[Out] int(x^2*(a + b/sinh(c + d*x^2)), x)
```

3.5 $\int x(a+b\,\mathbf{csch}(c+dx^2))\,dx$

Optimal result 64
Rubi [A] (verified) 64
Mathematica [B] (verified) 65
Maple [A] (verified) 65
Fricas [B] (verification not implemented) 66
Sympy [A] (verification not implemented) 66
Maxima [A] (verification not implemented) 67
Giac [B] (verification not implemented) 67
Mupad [B] (verification not implemented) 67

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\operatorname{arctanh}(\cosh\left(c+dx^2\right))}{2d}$$

```
[Out] 1/2*a*x^2-1/2*b*arctanh(cosh(d*x^2+c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 5545, 3855}

$$\int x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\operatorname{arctanh}(\cosh\left(c+dx^2\right))}{2d}$$

```
[In] Int[x*(a + b*Csch[c + d*x^2]),x]
[Out] (a*x^2)/2 - (b*ArcTanh[Cosh[c + d*x^2]])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax + bx\text{csch}\left(c+dx^2\right)\right)\,dx\\ &= \frac{ax^2}{2} + b\int x\text{csch}\left(c+dx^2\right)\,dx\\ &= \frac{ax^2}{2} + \frac{1}{2}b\text{Subst}\left(\int \text{csch}(c+dx)\,dx, x, x^2\right)\\ &= \frac{ax^2}{2} - \frac{b\text{arctanh}(\cosh\left(c+dx^2\right))}{2d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int x\left(a + b\text{csch}\left(c+dx^2\right)\right)\,dx = \frac{ax^2}{2} - \frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2d} + \frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2d}$$

```
[In] Integrate[x*(a + b*Csch[c + d*x^2]),x]
[Out] (a*x^2)/2 - (b*Log[Cosh[c/2 + (d*x^2)/2]])/(2*d) + (b*Log[Sinh[c/2 + (d*x^2
)/2]])/(2*d)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	26
parallelrisch	$\frac{a d\,x^{2}+b\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	27
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a+b\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	30
default	$\frac{\left(d\,x^{2}+c\right)a+b\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	30
risch	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\mathrm{e}^{d\,x^{2}+c}-1\right)}{2d}-\frac{b\ln\left(\mathrm{e}^{d\,x^{2}+c}+1\right)}{2d}$	42

```
[In] int(x*(a+b*csch(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*b/d*ln(tanh(1/2*d*x^2+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int x\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)\,dx = \frac{adx^2 - b\log\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)+1\right)+b\log\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)-1\right)}{2\,d}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x^2 - b*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + 1) + b*log(cosh(d*
x^2 + c) + sinh(d*x^2 + c) - 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int x\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)\,dx = \begin{cases} \frac{a\left(c+dx^2\right)+b\log\left(\tanh\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2d} & \text{for } d\neq 0 \\ \frac{x^2\left(a+b\,\mathrm{csch}\,(c)\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*csch(d*x**2+c)),x)
[Out] Piecewise(((a*(c + d*x**2) + b*log(tanh(c/2 + d*x**2/2)))/(2*d), Ne(d, 0)),
 (x**2*(a + b*csch(c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x\left(a+b\operatorname{csch}\left(c+dx^{2}\right)\right)\,dx=\frac{1}{2}\,ax^{2}+\frac{b\log\left(\tanh\left(\frac{1}{2}\,dx^{2}+\frac{1}{2}\,c\right)\right)}{2\,d}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*b*log(tanh(1/2*d*x^2 + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x\left(a+b\operatorname{csch}\left(c+dx^{2}\right)\right)\,dx=\frac{(dx^{2}+c)a}{2\,d}-\frac{b\log\left(e^{(dx^{2}+c)}+1\right)}{2\,d}+\frac{b\log\left(\left|e^{(dx^{2}+c)}-1\right|\right)}{2\,d}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] 1/2*(d*x^2 + c)*a/d - 1/2*b*log(e^(d*x^2 + c) + 1)/d + 1/2*b*log(abs(e^(d*x
^2 + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int x\left(a+b\operatorname{csch}\left(c+dx^{2}\right)\right)\,dx=\frac{a\,x^{2}}{2}-\frac{\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x^{2}}\,\mathrm{e}^{c}\,\sqrt{-d^{2}}}{d\,\sqrt{b^{2}}}\right)\,\sqrt{b^{2}}}{\sqrt{-d^{2}}}$$

```
[In] int(x*(a + b/sinh(c + d*x^2)),x)
[Out] (a*x^2)/2 - (atan((b*exp(d*x^2)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)
^(1/2))/(-d^2)^(1/2)
```

3.6 $\int \frac{a+b\operatorname{csch}(c+dx^2)}{x} \, dx$

Optimal result 68
Rubi [N/A] 68
Mathematica [N/A] 69
Maple [N/A] (verified) 69
Fricas [N/A] 69
Sympy [N/A] 69
Maxima [N/A] 70
Giac [N/A] 70
Mupad [N/A] 70

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\operatorname{csch}(c+dx^2)}{x} \, dx = a\log(x) + b\text{Int}\left(\frac{\operatorname{csch}(c+dx^2)}{x}, x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(csch(d*x^2+c)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}(c+dx^2)}{x} \, dx = \int \frac{a+b\operatorname{csch}(c+dx^2)}{x} \, dx$$

```
[In] Int[(a + b*Csch[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Csch[c + d*x^2]/x, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x} + \frac{b\operatorname{csch}(c+dx^2)}{x}\right) dx \\ &= a\log(x) + b\int \frac{\operatorname{csch}(c+dx^2)}{x} \, dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 7.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{csch}(c+dx^2)}{x}\,dx=\int \frac{a+b\text{csch}(c+dx^2)}{x}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*x^2])/x,x]
[Out] Integrate[(a + b*Csch[c + d*x^2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\,\operatorname{csch}\left(d\,x^{2}+c\right)}{x}dx$$

```
[In] int((a+b*csch(d*x^2+c))/x,x)
[Out] int((a+b*csch(d*x^2+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{csch}(c+dx^2)}{x}\,dx=\int \frac{b\operatorname{csch}\left(dx^2+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x,x, algorithm="fricas")
[Out] integral((b*csch(d*x^2 + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{csch}(c+dx^2)}{x}\,dx=\int \frac{a+b\operatorname{csch}\left(c+dx^2\right)}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x**2+c))/x,x)
[Out] Integral((a + b*csch(c + d*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{a + b\text{csch}(c + dx^2)}{x} dx = \int \frac{b\,\text{csch}\,(dx^2 + c) + a}{x} dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x*(e^(d*x^2 + c) - e^(-d*x^2 - c))), x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{csch}(c + dx^2)}{x} dx = \int \frac{b\,\text{csch}\,(dx^2 + c) + a}{x} dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x,x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{csch}(c + dx^2)}{x} dx = \int \frac{a + \frac{b}{\sinh(d\,x^2+c)}}{x} dx$$

```
[In] int((a + b/sinh(c + d*x^2))/x,x)
[Out] int((a + b/sinh(c + d*x^2))/x, x)
```

3.7 $\int \frac{a+b\text{csch}(c+dx^2)}{x^2} \, dx$

Optimal result 71
Rubi [N/A] 71
Mathematica [N/A] 72
Maple [N/A] (verified) 72
Fricas [N/A] 72
Sympy [N/A] 72
Maxima [N/A] 73
Giac [N/A] 73
Mupad [N/A] 73

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+b\text{Int}\left(\frac{\text{csch}(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csch(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx = \int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*x^2])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Csch[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x^2}+\frac{b\text{csch}(c+dx^2)}{x^2}\right)dx \\ &= -\frac{a}{x}+b\int \frac{\text{csch}(c+dx^2)}{x^2}\,dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 8.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{csch}(c + dx^2)}{x^2}\, dx = \int \frac{a + b\text{csch}(c + dx^2)}{x^2}\, dx$$

```
[In] Integrate[(a + b*Csch[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b\ \text{csch}\left(d\,x^2 + c\right)}{x^2} dx$$

```
[In] int((a+b*csch(d*x^2+c))/x^2,x)
[Out] int((a+b*csch(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{csch}(c + dx^2)}{x^2}\, dx = \int \frac{b\,\text{csch}\left(dx^2 + c\right) + a}{x^2}\, dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*csch(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b\text{csch}(c + dx^2)}{x^2}\, dx = \int \frac{a + b\,\text{csch}\left(c + dx^2\right)}{x^2}\, dx$$

```
[In] integrate((a+b*csch(d*x**2+c))/x**2,x)
[Out] Integral((a + b*csch(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx = \int \frac{b\,\text{csch}\,(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x^2*(e^(d*x^2 + c) - e^(-d*x^2 - c))), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx = \int \frac{b\,\text{csch}\,(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx = \int \frac{a+\frac{b}{\sinh(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/sinh(c + d*x^2))/x^2,x)
[Out] int((a + b/sinh(c + d*x^2))/x^2, x)
```

3.8 $\int x^5(a+b\text{csch}(c+dx^2))^2\,dx$

Optimal result 74
Rubi [A] (verified) 74
Mathematica [B] (verified) 78
Maple [F] 79
Fricas [B] (verification not implemented) 79
Sympy [F] 80
Maxima [A] (verification not implemented) 80
Giac [F] 81
Mupad [F(-1)] 81

Optimal result

Integrand size = 18, antiderivative size = 196

$$\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=-\frac{b^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^4\coth\left(c+dx^2\right)}{2d}+\frac{b^2x^2\log\left(1-e^{2(c+dx^2)}\right)}{d^2}-\frac{2abx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{2abx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}+\frac{b^2\,\text{PolyLog}\left(2,e^{2(c+dx^2)}\right)}{2d^3}+\frac{2ab\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3}-\frac{2ab\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3}$$

```
[Out] -1/2*b^2*x^4/d+1/6*a^2*x^6-2*a*b*x^4*arctanh(exp(d*x^2+c))/d-1/2*b^2*x^4*co
th(d*x^2+c)/d+b^2*x^2*ln(1-exp(2*d*x^2+2*c))/d^2-2*a*b*x^2*polylog(2,-exp(d
*x^2+c))/d^2+2*a*b*x^2*polylog(2,exp(d*x^2+c))/d^2+1/2*b^2*polylog(2,exp(2*
d*x^2+2*c))/d^3+2*a*b*polylog(3,-exp(d*x^2+c))/d^3-2*a*b*polylog(3,exp(d*x^
2+c))/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules

used = {5545, 4275, 4267, 2611, 2320, 6724, 4269, 3797, 2221, 2317, 2438}

$$\int x^5 \left(a + b\operatorname{csch}\left(c + dx^2\right)\right)^2 \, dx = \frac{a^2x^6}{6} - \frac{2abx^4\operatorname{arctanh}\left(e^{c+dx^2}\right)}{d} + \frac{2ab \operatorname{PolyLog}\left(3, -e^{dx^2+c}\right)}{d^3} - \frac{2ab \operatorname{PolyLog}\left(3, e^{dx^2+c}\right)}{d^3} - \frac{2abx^2 \operatorname{PolyLog}\left(2, -e^{dx^2+c}\right)}{d^2} + \frac{2abx^2 \operatorname{PolyLog}\left(2, e^{dx^2+c}\right)}{d^2} + \frac{b^2 \operatorname{PolyLog}\left(2, e^{2(dx^2+c)}\right)}{2d^3} + \frac{b^2x^2 \log\left(1 - e^{2(c+dx^2)}\right)}{d^2} - \frac{b^2x^4 \coth\left(c + dx^2\right)}{2d} - \frac{b^2x^4}{2d}$$

```
[In] Int[x^5*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] -1/2*(b^2*x^4)/d + (a^2*x^6)/6 - (2*a*b*x^4*ArcTanh[E^(c + d*x^2)])/d - (b^
2*x^4*Coth[c + d*x^2])/(2*d) + (b^2*x^2*Log[1 - E^(2*(c + d*x^2))])/d^2 - (
2*a*b*x^2*PolyLog[2, -E^(c + d*x^2)])/d^2 + (2*a*b*x^2*PolyLog[2, E^(c + d*
x^2)])/d^2 + (b^2*PolyLog[2, E^(2*(c + d*x^2))])/(2*d^3) + (2*a*b*PolyLog[3
, -E^(c + d*x^2)])/d^3 - (2*a*b*PolyLog[3, E^(c + d*x^2)])/d^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(a+b\text{csch}(c+dx))^2\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\text{csch}(c+dx)+b^2x^2\text{csch}^2(c+dx)\right)\,dx, x, x^2\right)$$

$$= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\text{csch}(c+dx)\,dx, x, x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\text{csch}^2(c+dx)\,dx, x, x^2\right)$$

$$= \frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^4\coth\left(c+dx^2\right)}{2d} - \frac{(2ab)\text{Subst}\left(\int x\log\left(1-e^{c+dx}\right)\,dx, x, x^2\right)}{d} + \frac{(2ab)\text{Subst}\left(\int x\log\left(1+e^{c+dx}\right)\,dx, x, x^2\right)}{d}+\frac{b^2\text{Subst}\left(\int x\coth(c+dx)\,dx, x, x^2\right)}{d}$$

$$= -\frac{b^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^4\coth\left(c+dx^2\right)}{2d} - \frac{2abx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{2abx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}+\frac{(2ab)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{c+dx}\right)\right.}{d^2} - \frac{(2ab)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{c+dx}\right)\,dx, x, x^2\right)}{d^2}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x}{1-e^{2(c+dx)}}\,dx, x, x^2\right)}{d}$$

$$= -\frac{b^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^4\coth\left(c+dx^2\right)}{2d} + \frac{b^2x^2\log\left(1-e^{2(c+dx^2)}\right)}{d^2}-\frac{2abx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2} + \frac{2abx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}+\frac{(2ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{c+dx^2}\right)}{d^3} - \frac{(2ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{c+dx^2}\right)}{d^3}-\frac{b^2\text{Subst}\left(\int \log\left(1-e^{2(c+dx)}\right)\,dx, x, x^2\right)}{d^2}$$

$$= -\frac{b^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d} - \frac{b^2x^4\coth(c+dx^2)}{2d} + \frac{b^2x^2\log\left(1-e^{2(c+dx^2)}\right)}{d^2} - \frac{2abx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2} + \frac{2abx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2} + \frac{2ab\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3} - \frac{2ab\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3} - \frac{b^2\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(c+dx^2)}\right)}{2d^3}$$

$$= -\frac{b^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2abx^4\text{arctanh}\left(e^{c+dx^2}\right)}{d} - \frac{b^2x^4\coth(c+dx^2)}{2d} + \frac{b^2x^2\log\left(1-e^{2(c+dx^2)}\right)}{d^2} - \frac{2abx^2\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2} + \frac{2abx^2\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2} + \frac{b^2\,\text{PolyLog}\left(2,e^{2(c+dx^2)}\right)}{2d^3} + \frac{2ab\,\text{PolyLog}\left(3,-e^{c+dx^2}\right)}{d^3} - \frac{2ab\,\text{PolyLog}\left(3,e^{c+dx^2}\right)}{d^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 595 vs. $2(196) = 392$.

Time = 2.23 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.04

$$\int x^5\left(a+b\text{csch}(c+dx^2)\right)^2\,dx = -\frac{12b^2d^2x^4 + 2a^2d^3x^6 - 2a^2d^3e^{2c}x^6 + 12b^2dx^2\log\left(1-e^{-c-dx^2}\right) - 12b^2de^{2c}x^2\log\left(1-e^{-c-dx^2}\right) + 12abd^2x}{}$$

```
[In] Integrate[x^5*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] -1/12*(12*b^2*d^2*x^4 + 2*a^2*d^3*x^6 - 2*a^2*d^3*E^(2*c)*x^6 + 12*b^2*d*x^
2*Log[1 - E^(-c - d*x^2)] - 12*b^2*d*E^(2*c)*x^2*Log[1 - E^(-c - d*x^2)] +
12*a*b*d^2*x^4*Log[1 - E^(-c - d*x^2)] - 12*a*b*d^2*E^(2*c)*x^4*Log[1 - E^(
-c - d*x^2)] + 12*b^2*d*x^2*Log[1 + E^(-c - d*x^2)] - 12*b^2*d*E^(2*c)*x^2*
Log[1 + E^(-c - d*x^2)] - 12*a*b*d^2*x^4*Log[1 + E^(-c - d*x^2)] + 12*a*b*d
^2*E^(2*c)*x^4*Log[1 + E^(-c - d*x^2)] + 12*b*(-1 + E^(2*c))*(b - 2*a*d*x^2
)*PolyLog[2, -E^(-c - d*x^2)] + 12*b*(-1 + E^(2*c))*(b + 2*a*d*x^2)*PolyLog
[2, E^(-c - d*x^2)] + 24*a*b*PolyLog[3, -E^(-c - d*x^2)] - 24*a*b*E^(2*c)*P
olyLog[3, -E^(-c - d*x^2)] - 24*a*b*PolyLog[3, E^(-c - d*x^2)] + 24*a*b*E^(
2*c)*PolyLog[3, E^(-c - d*x^2)] + 3*b^2*d^2*x^4*Csch[c/2]*Csch[(c + d*x^2)/
```

```
2]*Sinh[(d*x^2)/2] - 3*b^2*d^2*E^(2*c)*x^4*Csch[c/2]*Csch[(c + d*x^2)/2]*Si
nh[(d*x^2)/2] - 3*b^2*d^2*x^4*Sech[c/2]*Sech[(c + d*x^2)/2]*Sinh[(d*x^2)/2]
 + 3*b^2*d^2*E^(2*c)*x^4*Sech[c/2]*Sech[(c + d*x^2)/2]*Sinh[(d*x^2)/2])/(d^
3*(-1 + E^(2*c)))
```

Maple [F]

$$\int x^{5}\left(a+b\,\mathrm{csch}\left(d\,x^{2}+c\right)\right)^{2}dx$$

```
[In] int(x^5*(a+b*csch(d*x^2+c))^2,x)
```

```
[Out] int(x^5*(a+b*csch(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(180) = 360$.

Time = 0.28 (sec) , antiderivative size = 1031, normalized size of antiderivative = 5.26

$$\int x^5 (a + b\text{csch}(c + dx^2))^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*d^3*x^6 + 6*b^2*c^2 - (a^2*d^3*x^6 - 6*b^2*d^2*x^4 + 6*b^2*c^2)*c
osh(d*x^2 + c)^2 - 2*(a^2*d^3*x^6 - 6*b^2*d^2*x^4 + 6*b^2*c^2)*cosh(d*x^2 +
 c)*sinh(d*x^2 + c) - (a^2*d^3*x^6 - 6*b^2*d^2*x^4 + 6*b^2*c^2)*sinh(d*x^2
+ c)^2 + 6*(2*a*b*d*x^2 - (2*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)^2 - 2*(2*a*b*
d*x^2 + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) - (2*a*b*d*x^2 + b^2)*sinh(d*x
^2 + c)^2 + b^2)*dilog(cosh(d*x^2 + c) + sinh(d*x^2 + c)) - 6*(2*a*b*d*x^2
- (2*a*b*d*x^2 - b^2)*cosh(d*x^2 + c)^2 - 2*(2*a*b*d*x^2 - b^2)*cosh(d*x^2
+ c)*sinh(d*x^2 + c) - (2*a*b*d*x^2 - b^2)*sinh(d*x^2 + c)^2 - b^2)*dilog(-
cosh(d*x^2 + c) - sinh(d*x^2 + c)) - 6*(a*b*d^2*x^4 - b^2*d*x^2 - (a*b*d^2*
x^4 - b^2*d*x^2)*cosh(d*x^2 + c)^2 - 2*(a*b*d^2*x^4 - b^2*d*x^2)*cosh(d*x^2
 + c)*sinh(d*x^2 + c) - (a*b*d^2*x^4 - b^2*d*x^2)*sinh(d*x^2 + c)^2)*log(co
sh(d*x^2 + c) + sinh(d*x^2 + c) + 1) + 6*(a*b*c^2 - b^2*c - (a*b*c^2 - b^2*
c)*cosh(d*x^2 + c)^2 - 2*(a*b*c^2 - b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c)
- (a*b*c^2 - b^2*c)*sinh(d*x^2 + c)^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c
) - 1) + 6*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c - (a*b*d^2*x^4 + b^2*
d*x^2 - a*b*c^2 + b^2*c)*cosh(d*x^2 + c)^2 - 2*(a*b*d^2*x^4 + b^2*d*x^2 - a
*b*c^2 + b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) - (a*b*d^2*x^4 + b^2*d*x^2
- a*b*c^2 + b^2*c)*sinh(d*x^2 + c)^2)*log(-cosh(d*x^2 + c) - sinh(d*x^2 + c
) + 1) + 12*(a*b*cosh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c)
+ a*b*sinh(d*x^2 + c)^2 - a*b)*polylog(3, cosh(d*x^2 + c) + sinh(d*x^2 + c)
) - 12*(a*b*cosh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b
```

```
*sinh(d*x^2 + c)^2 - a*b)*polylog(3, -cosh(d*x^2 + c) - sinh(d*x^2 + c)))/(
d^3*cosh(d*x^2 + c)^2 + 2*d^3*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d^3*sinh(d*
x^2 + c)^2 - d^3)
```

Sympy [F]

$$\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^5\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**5*(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**5*(a + b*csch(c + d*x**2))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.38

$$\begin{aligned}\int x^5\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{6}\,a^2x^6-\frac{b^2x^4}{de^{(2\,dx^2+2\,c)}-d} \\ &\quad-\frac{\left(d^2x^4\log\left(e^{(dx^2+c)}+1\right)+2\,dx^2\text{Li}_2\left(-e^{(dx^2+c)}\right)-2\,\text{Li}_3(-e^{(dx^2+c)})\right)ab}{d^3} \\ &\quad+\frac{\left(d^2x^4\log\left(-e^{(dx^2+c)}+1\right)+2\,dx^2\text{Li}_2\left(e^{(dx^2+c)}\right)-2\,\text{Li}_3(e^{(dx^2+c)})\right)ab}{d^3} \\ &\quad+\frac{\left(dx^2\log\left(e^{(dx^2+c)}+1\right)+\text{Li}_2\left(-e^{(dx^2+c)}\right)\right)b^2}{d^3} \\ &\quad+\frac{\left(dx^2\log\left(-e^{(dx^2+c)}+1\right)+\text{Li}_2\left(e^{(dx^2+c)}\right)\right)b^2}{d^3} \\ &\quad-\frac{2\,abd^3x^6+3\,b^2d^2x^4}{6\,d^3}+\frac{2\,abd^3x^6-3\,b^2d^2x^4}{6\,d^3}\end{aligned}$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 - b^2*x^4/(d*e^(2*d*x^2 + 2*c) - d) - (d^2*x^4*log(e^(d*x^2 + c
) + 1) + 2*d*x^2*dilog(-e^(d*x^2 + c)) - 2*polylog(3, -e^(d*x^2 + c)))*a*b/
d^3 + (d^2*x^4*log(-e^(d*x^2 + c) + 1) + 2*d*x^2*dilog(e^(d*x^2 + c)) - 2*p
olylog(3, e^(d*x^2 + c)))*a*b/d^3 + (d*x^2*log(e^(d*x^2 + c) + 1) + dilog(-
e^(d*x^2 + c)))*b^2/d^3 + (d*x^2*log(-e^(d*x^2 + c) + 1) + dilog(e^(d*x^2 +
 c)))*b^2/d^3 - 1/6*(2*a*b*d^3*x^6 + 3*b^2*d^2*x^4)/d^3 + 1/6*(2*a*b*d^3*x^
6 - 3*b^2*d^2*x^4)/d^3
```

Giac [F]

$$\int x^5 \left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = \int \left(b \operatorname{csch}\left(dx^2 + c\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = \int x^5 \left(a + \frac{b}{\sinh\left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^5*(a + b/sinh(c + d*x^2))^2,x)
[Out] int(x^5*(a + b/sinh(c + d*x^2))^2, x)
```

3.9 $\int x^4(a+b\text{csch}(c+dx^2))^2\,dx$

Optimal result . . . 82
Rubi [N/A] . . . 82
Mathematica [N/A] . . . 83
Maple [N/A] (verified) . . . 83
Fricas [N/A] . . . 83
Sympy [N/A] . . . 83
Maxima [N/A] . . . 84
Giac [N/A] . . . 84
Mupad [N/A] . . . 84

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\text{Int}\left(x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x^4*(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Int[x^4*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4*(a + b*Csch[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^4*(a + b*Csch[c + d*x^2])^2,x]
[Out] Integrate[x^4*(a + b*Csch[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\ \text{csch}\left(d\,x^2+c\right)\right)^2dx$$

```
[In] int(x^4*(a+b*csch(d*x^2+c))^2,x)
[Out] int(x^4*(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int \left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^4*csch(d*x^2 + c)^2 + 2*a*b*x^4*csch(d*x^2 + c) + a^2*x^4, x
)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**4*(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**4*(a + b*csch(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.00

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - b^2*x^3/(d*e^(2*d*x^2 + 2*c) - d) + integrate(1/2*(4*a*b*d*x^
4 - 3*b^2*x^2)/(d*e^(d*x^2 + c) + d), x) + integrate(1/2*(4*a*b*d*x^4 + 3*b
^2*x^2)/(d*e^(d*x^2 + c) - d), x)
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^2 + c) + a)^2*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+\frac{b}{\sinh\left(d\,x^2+c\right)}\right)^2\,dx$$

```
[In] int(x^4*(a + b/sinh(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b/sinh(c + d*x^2))^2, x)
```

3.10 $\int x^3(a+b\text{csch}(c+dx^2))^2\,dx$

Optimal result 85
Rubi [A] (verified) 85
Mathematica [B] (verified) 87
Maple [F] 88
Fricas [B] (verification not implemented) 88
Sympy [F] 89
Maxima [F] 89
Giac [F] 89
Mupad [F(-1)] 89

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^2\coth\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sinh\left(c+dx^2\right)\right)}{2d^2}-\frac{ab\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{ab\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}$$

```
[Out] 1/4*a^2*x^4-2*a*b*x^2*arctanh(exp(d*x^2+c))/d-1/2*b^2*x^2*coth(d*x^2+c)/d+1
/2*b^2*ln(sinh(d*x^2+c))/d^2-a*b*polylog(2,-exp(d*x^2+c))/d^2+a*b*polylog(2
,exp(d*x^2+c))/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5545, 4275, 4267, 2317, 2438, 4269, 3556}

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{ab\,\text{PolyLog}\left(2,-e^{dx^2+c}\right)}{d^2}+\frac{ab\,\text{PolyLog}\left(2,e^{dx^2+c}\right)}{d^2}+\frac{b^2\log\left(\sinh\left(c+dx^2\right)\right)}{2d^2}-\frac{b^2x^2\coth\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^3*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 - (2*a*b*x^2*ArcTanh[E^(c + d*x^2)])/d - (b^2*x^2*Coth[c + d*x^
2])/(2*d) + (b^2*Log[Sinh[c + d*x^2]])/(2*d^2) - (a*b*PolyLog[2, -E^(c + d*
x^2)])/d^2 + (a*b*PolyLog[2, E^(c + d*x^2)])/d^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\text{csch}(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\text{csch}(c+dx)+b^2x\text{csch}^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\text{csch}(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\text{csch}^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^2\coth\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(ab)\text{Subst}\left(\int\log\left(1-e^{c+dx}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{(ab)\text{Subst}\left(\int\log\left(1+e^{c+dx}\right)\,dx,x,x^2\right)}{d}+\frac{b^2\text{Subst}\left(\int\coth(c+dx)\,dx,x,x^2\right)}{2d}\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^2\coth\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sinh\left(c+dx^2\right)\right)}{2d^2}\\
&\quad-\frac{(ab)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{c+dx^2}\right)}{d^2}+\frac{(ab)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{c+dx^2}\right)}{d^2}\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^2\coth\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sinh\left(c+dx^2\right)\right)}{2d^2}\\
&\quad-\frac{ab\,\text{PolyLog}\left(2,-e^{c+dx^2}\right)}{d^2}+\frac{ab\,\text{PolyLog}\left(2,e^{c+dx^2}\right)}{d^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 276 vs. $2(108) = 216$.

Time = 0.73 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.56

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{\text{csch}\left(\frac{1}{2}(c+dx^2)\right)\text{sech}\left(\frac{1}{2}(c+dx^2)\right)\sinh(c)(\cosh(c)+\sinh(c))\left(-2b^2dx^2\cosh\left(c+dx^2\right)+2b^2dx^2\sinh\left(c+\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (Csch[(c + d*x^2)/2]*Sech[(c + d*x^2)/2]*Sinh[c]*(Cosh[c] + Sinh[c])*(-2*b^
2*d*x^2*Cosh[c + d*x^2] + 2*b^2*d*x^2*Sinh[c + d*x^2] + a^2*d^2*x^4*Sinh[c
+ d*x^2] + 2*b^2*Log[1 - E^(-c - d*x^2)]*Sinh[c + d*x^2] + 4*a*b*d*x^2*Log[
```

```
1 - E^(-c - d*x^2)]*Sinh[c + d*x^2] + 2*b^2*Log[1 + E^(-c - d*x^2)]*Sinh[c
+ d*x^2] - 4*a*b*d*x^2*Log[1 + E^(-c - d*x^2)]*Sinh[c + d*x^2] + 4*a*b*Poly
Log[2, -E^(-c - d*x^2)]*Sinh[c + d*x^2] - 4*a*b*PolyLog[2, E^(-c - d*x^2)]*
Sinh[c + d*x^2]))/(4*d^2*(-1 + E^(2*c)))
```

Maple [F]

$$\int x^3 \left(a+b\ \mathrm{csch}\left(d\,x^2+c\right)\right)^2 dx$$

```
[In] int(x^3*(a+b*csch(d*x^2+c))^2,x)
```

```
[Out] int(x^3*(a+b*csch(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(97) = 194.

Time = 0.28 (sec) , antiderivative size = 683, normalized size of antiderivative = 6.32

$$\int x^3 \left(a+b\mathrm{csch}\left(c+dx^2\right)\right)^2\,dx =$$

$$-\frac{a^2d^2x^4-4\,b^2c-\left(a^2d^2x^4-4\,b^2dx^2-4\,b^2c\right)\cosh\left(dx^2+c\right)^2-2\left(a^2d^2x^4-4\,b^2dx^2-4\,b^2c\right)\cosh\left(dx^2+c\right)}{}$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*d^2*x^4 - 4*b^2*c - (a^2*d^2*x^4 - 4*b^2*d*x^2 - 4*b^2*c)*cosh(d*
x^2 + c)^2 - 2*(a^2*d^2*x^4 - 4*b^2*d*x^2 - 4*b^2*c)*cosh(d*x^2 + c)*sinh(d
*x^2 + c) - (a^2*d^2*x^4 - 4*b^2*d*x^2 - 4*b^2*c)*sinh(d*x^2 + c)^2 - 4*(a*
b*cosh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^
2 + c)^2 - a*b)*dilog(cosh(d*x^2 + c) + sinh(d*x^2 + c)) + 4*(a*b*cosh(d*x^
2 + c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^2 + c)^2 -
a*b)*dilog(-cosh(d*x^2 + c) - sinh(d*x^2 + c)) - 2*(2*a*b*d*x^2 - (2*a*b*d*
x^2 - b^2)*cosh(d*x^2 + c)^2 - 2*(2*a*b*d*x^2 - b^2)*cosh(d*x^2 + c)*sinh(d
*x^2 + c) - (2*a*b*d*x^2 - b^2)*sinh(d*x^2 + c)^2 - b^2)*log(cosh(d*x^2 + c
) + sinh(d*x^2 + c) + 1) - 2*(2*a*b*c - (2*a*b*c - b^2)*cosh(d*x^2 + c)^2 -
 2*(2*a*b*c - b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) - (2*a*b*c - b^2)*sinh(d
*x^2 + c)^2 - b^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - 1) + 4*(a*b*d*x^
2 + a*b*c - (a*b*d*x^2 + a*b*c)*cosh(d*x^2 + c)^2 - 2*(a*b*d*x^2 + a*b*c)*c
osh(d*x^2 + c)*sinh(d*x^2 + c) - (a*b*d*x^2 + a*b*c)*sinh(d*x^2 + c)^2)*log
(-cosh(d*x^2 + c) - sinh(d*x^2 + c) + 1))/(d^2*cosh(d*x^2 + c)^2 + 2*d^2*co
sh(d*x^2 + c)*sinh(d*x^2 + c) + d^2*sinh(d*x^2 + c)^2 - d^2)
```

Sympy [F]

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^3\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**3*(a + b*csch(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int \left(b\operatorname{csch}\left(dx^2+c\right)+a\right)^2x^3\,dx$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/4*a^2*x^4 - 1/2*(2*x^2*e^(2*d*x^2 + 2*c)/(d*e^(2*d*x^2 + 2*c) - d) - log(
(e^(d*x^2 + c) + 1)*e^(-c))/d^2 - log((e^(d*x^2 + c) - 1)*e^(-c))/d^2)*b^2
+ 4*a*b*(integrate(1/2*x^3/(e^(d*x^2 + c) + 1), x) + integrate(1/2*x^3/(e^(
d*x^2 + c) - 1), x))
```

Giac [F]

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int \left(b\operatorname{csch}\left(dx^2+c\right)+a\right)^2x^3\,dx$$

```
[In] integrate(x^3*(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^3\,\left(a+\frac{b}{\sinh\left(d\,x^2+c\right)}\right)^2\,dx$$

```
[In] int(x^3*(a + b/sinh(c + d*x^2))^2,x)
[Out] int(x^3*(a + b/sinh(c + d*x^2))^2, x)
```

3.11 $\int x^2(a+b\operatorname{csch}(c+dx^2))^2\,dx$

Optimal result . . . 90
Rubi [N/A] . . . 90
Mathematica [N/A] . . . 91
Maple [N/A] (verified) . . . 91
Fricas [N/A] . . . 91
Sympy [N/A] . . . 91
Maxima [N/A] . . . 92
Giac [N/A] . . . 92
Mupad [N/A] . . . 92

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx=\operatorname{Int}\left(x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Int[x^2*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*Csch[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^2*(a + b*Csch[c + d*x^2])^2,x]
[Out] Integrate[x^2*(a + b*Csch[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\ \text{csch}\left(d\,x^2+c\right)\right)^2dx$$

```
[In] int(x^2*(a+b*csch(d*x^2+c))^2,x)
[Out] int(x^2*(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*csch(d*x^2 + c)^2 + 2*a*b*x^2*csch(d*x^2 + c) + a^2*x^2, x
)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\text{csch}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**2*(a + b*csch(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.44

$$\int x^2 \left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = \int \left(b \operatorname{csch}\left(dx^2 + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - b^2*x/(d*e^(2*d*x^2 + 2*c) - d) + integrate(1/2*(4*a*b*d*x^2
- b^2)/(d*e^(d*x^2 + c) + d), x) + integrate(1/2*(4*a*b*d*x^2 + b^2)/(d*e^(
d*x^2 + c) - d), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = \int \left(b \operatorname{csch}\left(dx^2 + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^2 + c) + a)^2*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2 \left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = \int x^2 \left(a + \frac{b}{\sinh\left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^2*(a + b/sinh(c + d*x^2))^2,x)
```

```
[Out] int(x^2*(a + b/sinh(c + d*x^2))^2, x)
```

3.12 $\int x(a+b\text{csch}(c+dx^2))^2\,dx$

Optimal result 93
Rubi [A] (verified) 93
Mathematica [A] (verified) 94
Maple [A] (verified) 95
Fricas [B] (verification not implemented) 95
Sympy [F] 96
Maxima [A] (verification not implemented) 96
Giac [A] (verification not implemented) 96
Mupad [B] (verification not implemented) 97

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x(a+b\text{csch}(c+dx^2))^2\,dx=\frac{a^2x^2}{2}-\frac{ab\text{arctanh}(\cosh(c+dx^2))}{d}-\frac{b^2\coth(c+dx^2)}{2d}$$

```
[Out] 1/2*a^2*x^2-a*b*arctanh(cosh(d*x^2+c))/d-1/2*b^2*coth(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5545, 3858, 3855, 3852, 8}

$$\int x(a+b\text{csch}(c+dx^2))^2\,dx=\frac{a^2x^2}{2}-\frac{ab\text{arctanh}(\cosh(c+dx^2))}{d}-\frac{b^2\coth(c+dx^2)}{2d}$$

```
[In] Int[x*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^2)/2 - (a*b*ArcTanh[Cosh[c + d*x^2]])/d - (b^2*Coth[c + d*x^2])/(2*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a + b\text{csch}(c + dx))^2 \, dx, x, x^2\right) \\
&= \frac{a^2x^2}{2} + (ab)\text{Subst}\left(\int \text{csch}(c+dx)\, dx, x, x^2\right) + \frac{1}{2}b^2\text{Subst}\left(\int \text{csch}^2(c+dx)\, dx, x, x^2\right) \\
&= \frac{a^2x^2}{2} - \frac{ab\text{arctanh}(\cosh(c + dx^2))}{d} - \frac{(ib^2)\,\text{Subst}\left(\int 1\, dx, x, -i\coth(c + dx^2)\right)}{2d} \\
&= \frac{a^2x^2}{2} - \frac{ab\text{arctanh}(\cosh(c + dx^2))}{d} - \frac{b^2\coth(c + dx^2)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int x\left(a + b\text{csch}\left(c + dx^2\right)\right)^2 \, dx = -\frac{b^2\coth\left(\frac{1}{2}(c + dx^2)\right) - 2a\left(ac + adx^2 - 2b\log\left(\cosh\left(\frac{1}{2}(c + dx^2)\right)\right) + 2b\log\left(\sinh\left(\frac{1}{2}(c + dx^2)\right)\right)\right) + b^2\tan}{4d}$$

```
[In] Integrate[x*(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] -1/4*(b^2*Coth[(c + d*x^2)/2] - 2*a*(a*c + a*d*x^2 - 2*b*Log[Cosh[(c + d*x^
2)/2]] + 2*b*Log[Sinh[(c + d*x^2)/2]]) + b^2*Tanh[(c + d*x^2)/2])/d
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a^{2}\left(d\,x^{2}+c\right)-4ab\,\operatorname{arctanh}\left({\mathrm e}^{d\,x^{2}+c}\right)-b^{2}\coth\left(d\,x^{2}+c\right)}{2d}$	44
default	$\frac{a^{2}\left(d\,x^{2}+c\right)-4ab\,\operatorname{arctanh}\left({\mathrm e}^{d\,x^{2}+c}\right)-b^{2}\coth\left(d\,x^{2}+c\right)}{2d}$	44
parts	$\frac{a^{2}x^{2}}{2}-\frac{b^{2}\coth\left(d\,x^{2}+c\right)}{2d}+\frac{ab\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{d}$	44
parallelrisch	$\frac{2a^{2}d\,x^{2}-\coth\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)b^{2}-\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)b^{2}+4\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)ab}{4d}$	64
risch	$\frac{a^{2}x^{2}}{2}-\frac{b^{2}}{d\left({\mathrm e}^{2d\,x^{2}+2c}-1\right)}+\frac{ab\ln\left({\mathrm e}^{d\,x^{2}+c}-1\right)}{d}-\frac{ab\ln\left({\mathrm e}^{d\,x^{2}+c}+1\right)}{d}$	68

```
[In] int(x*(a+b*csch(d*x^2+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2/d*(a^2*(d*x^2+c)-4*a*b*arctanh(exp(d*x^2+c))-b^2*coth(d*x^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 6.02

$$\int x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{a^2dx^2\cosh\left(dx^2+c\right)^2+2\,a^2dx^2\cosh\left(dx^2+c\right)\sinh\left(dx^2+c\right)+a^2dx^2\sinh\left(dx^2+c\right)^2-a^2dx^2-2\,b^2-2\,(}{}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*d*x^2*cosh(d*x^2 + c)^2 + 2*a^2*d*x^2*cosh(d*x^2 + c)*sinh(d*x^2 +
 c) + a^2*d*x^2*sinh(d*x^2 + c)^2 - a^2*d*x^2 - 2*b^2 - 2*(a*b*cosh(d*x^2 +
 c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^2 + c)^2 - a*b
)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + 1) + 2*(a*b*cosh(d*x^2 + c)^2 + 2
*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^2 + c)^2 - a*b)*log(cos
h(d*x^2 + c) + sinh(d*x^2 + c) - 1))/(d*cosh(d*x^2 + c)^2 + 2*d*cosh(d*x^2
+ c)*sinh(d*x^2 + c) + d*sinh(d*x^2 + c)^2 - d)
```

Sympy [F]

$$\int x\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)^2\,dx=\int x\left(a+b\,\mathrm{csch}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x*(a + b*csch(c + d*x**2))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)^2\,dx=\frac{1}{2}\,a^2x^2+\frac{ab\log\left(\tanh\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)\right)}{d}+\frac{b^2}{d\left(e^{\left(-2\,dx^2-2\,c\right)}-1\right)}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 + a*b*log(tanh(1/2*d*x^2 + 1/2*c))/d + b^2/(d*(e^(-2*d*x^2 - 2*
c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.67

$$\int x\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)^2\,dx=\frac{(dx^2+c)a^2}{2\,d}-\frac{ab\log\left(e^{\left(dx^2+c\right)}+1\right)}{d}+\frac{ab\log\left(\left|e^{\left(dx^2+c\right)}-1\right|\right)}{d}-\frac{b^2}{d\left(e^{\left(2\,dx^2+2\,c\right)}-1\right)}$$

```
[In] integrate(x*(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] 1/2*(d*x^2 + c)*a^2/d - a*b*log(e^(d*x^2 + c) + 1)/d + a*b*log(abs(e^(d*x^2
 + c) - 1))/d - b^2/(d*(e^(2*d*x^2 + 2*c) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.80

$$\int x\big(a+b\operatorname{csch}\big(c+dx^2\big)\big)^2\,dx=\frac{a^2\,x^2}{2}-\frac{b^2}{d\,\left(\mathrm{e}^{2\,d\,x^2+2\,c}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x^2}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}$$

```
[In] int(x*(a + b/sinh(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 - b^2/(d*(exp(2*c + 2*d*x^2) - 1)) - (2*atan((a*b*exp(d*x^2)*ex
p(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2)
```

3.13 $\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx$

Optimal result 98
Rubi [N/A] 98
Mathematica [N/A] 99
Maple [N/A] (verified) 99
Fricas [N/A] 99
Sympy [N/A] 99
Maxima [N/A] 100
Giac [N/A] 100
Mupad [N/A] 100

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*csch(d*x^2+c))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx$$

```
[In] Int[(a + b*Csch[c + d*x^2])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*x^2])^2/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 58.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*x^2])^2/x,x]
[Out] Integrate[(a + b*Csch[c + d*x^2])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\,\operatorname{csch}\left(d\,x^{2}+c\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*csch(d*x^2+c))^2/x,x)
[Out] int((a+b*csch(d*x^2+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx=\int \frac{\left(b\operatorname{csch}\left(dx^2+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x,x, algorithm="fricas")
[Out] integral((b^2*csch(d*x^2 + c)^2 + 2*a*b*csch(d*x^2 + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x**2+c))**2/x,x)
[Out] Integral((a + b*csch(c + d*x**2))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 6.22

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx = \int \frac{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - b^2/(d*x^2*e^(2*d*x^2 + 2*c) - d*x^2) + integrate((2*a*b*d*x^2
 + b^2)/(d*x^3*e^(d*x^2 + c) + d*x^3), x) + integrate((2*a*b*d*x^2 - b^2)/(
d*x^3*e^(d*x^2 + c) - d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx = \int \frac{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^2 + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x}\,dx = \int \frac{\left(a+\frac{b}{\sinh(d\,x^2+c)}\right)^2}{x}\,dx$$

```
[In] int((a + b/sinh(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b/sinh(c + d*x^2))^2/x, x)
```

3.14 $\int \frac{(a+b\,\mathrm{csch}(c+dx^2))^2}{x^2}\,dx$

Optimal result . . . 101
Rubi [N/A] . . . 101
Mathematica [N/A] . . . 102
Maple [N/A] (verified) . . . 102
Fricas [N/A] . . . 102
Sympy [N/A] . . . 102
Maxima [N/A] . . . 103
Giac [N/A] . . . 103
Mupad [N/A] . . . 103

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\mathrm{csch}(c+dx^2))^2}{x^2}\,dx=\mathrm{Int}\left(\frac{(a+b\mathrm{csch}(c+dx^2))^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*csch(d*x^2+c))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\mathrm{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\mathrm{csch}(c+dx^2))^2}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*x^2])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*x^2])^2/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\mathrm{csch}(c+dx^2))^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 33.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*x^2])^2/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*x^2])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\ \text{csch}\left(d\,x^2+c\right))^2}{x^2}dx$$

```
[In] int((a+b*csch(d*x^2+c))^2/x^2,x)
[Out] int((a+b*csch(d*x^2+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*csch(d*x^2 + c)^2 + 2*a*b*csch(d*x^2 + c) + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x**2+c))**2/x**2,x)
[Out] Integral((a + b*csch(c + d*x**2))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 6.56

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\,\text{csch}\,(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2/(d*x^3*e^(2*d*x^2 + 2*c) - d*x^3) - a^2/x + integrate(1/2*(4*a*b*d*x^2
 + 3*b^2)/(d*x^4*e^(d*x^2 + c) + d*x^4), x) + integrate(1/2*(4*a*b*d*x^2 -
3*b^2)/(d*x^4*e^(d*x^2 + c) - d*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\,\text{csch}\,(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\text{csch}(c+dx^2))^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sinh(d\,x^2+c)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/sinh(c + d*x^2))^2/x^2,x)
```

```
[Out] int((a + b/sinh(c + d*x^2))^2/x^2, x)
```

3.15 $\int x\text{csch}^7(a+bx^2)\,dx$

Optimal result 104
Rubi [A] (verified) 104
Mathematica [A] (verified) 106
Maple [A] (verified) 106
Fricas [B] (verification not implemented) 107
Sympy [F] 108
Maxima [B] (verification not implemented) 109
Giac [A] (verification not implemented) 109
Mupad [B] (verification not implemented) 110

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x\text{csch}^7\left(a+bx^2\right)\,dx=\frac{5\text{arctanh}(\cosh\left(a+bx^2\right))}{32b}-\frac{5\coth\left(a+bx^2\right)\text{csch}(a+bx^2)}{32b}+\frac{5\coth\left(a+bx^2\right)\text{csch}^3(a+bx^2)}{48b}-\frac{\coth\left(a+bx^2\right)\text{csch}^5(a+bx^2)}{12b}$$

```
[Out] 5/32*arctanh(cosh(b*x^2+a))/b-5/32*coth(b*x^2+a)*csch(b*x^2+a)/b+5/48*coth(
b*x^2+a)*csch(b*x^2+a)^3/b-1/12*coth(b*x^2+a)*csch(b*x^2+a)^5/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5545, 3853, 3855}

$$\int x\text{csch}^7\left(a+bx^2\right)\,dx=\frac{5\text{arctanh}(\cosh\left(a+bx^2\right))}{32b}-\frac{\coth\left(a+bx^2\right)\text{csch}^5(a+bx^2)}{12b}+\frac{5\coth\left(a+bx^2\right)\text{csch}^3(a+bx^2)}{48b}-\frac{5\coth\left(a+bx^2\right)\text{csch}(a+bx^2)}{32b}$$

```
[In] Int[x*Csch[a + b*x^2]^7,x]
```

```
[Out] (5*ArcTanh[Cosh[a + b*x^2]])/(32*b) - (5*Coth[a + b*x^2]*Csch[a + b*x^2])/(
32*b) + (5*Coth[a + b*x^2]*Csch[a + b*x^2]^3)/(48*b) - (Coth[a + b*x^2]*Csc
h[a + b*x^2]^5)/(12*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \operatorname{csch}^7(a+bx)\,dx, x, x^2\right) \\
&= -\frac{\coth\left(a+bx^2\right)\operatorname{csch}^5\left(a+bx^2\right)}{12b} - \frac{5}{12}\text{Subst}\left(\int \operatorname{csch}^5(a+bx)\,dx, x, x^2\right) \\
&= \frac{5\coth\left(a+bx^2\right)\operatorname{csch}^3\left(a+bx^2\right)}{48b} - \frac{\coth\left(a+bx^2\right)\operatorname{csch}^5\left(a+bx^2\right)}{12b} \\
&\quad + \frac{5}{16}\text{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx, x, x^2\right) \\
&= -\frac{5\coth\left(a+bx^2\right)\operatorname{csch}\left(a+bx^2\right)}{32b} + \frac{5\coth\left(a+bx^2\right)\operatorname{csch}^3\left(a+bx^2\right)}{48b} \\
&\quad - \frac{\coth\left(a+bx^2\right)\operatorname{csch}^5\left(a+bx^2\right)}{12b} - \frac{5}{32}\text{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, x^2\right) \\
&= \frac{5\operatorname{arctanh}\left(\cosh\left(a+bx^2\right)\right)}{32b} - \frac{5\coth\left(a+bx^2\right)\operatorname{csch}\left(a+bx^2\right)}{32b} \\
&\quad + \frac{5\coth\left(a+bx^2\right)\operatorname{csch}^3\left(a+bx^2\right)}{48b} - \frac{\coth\left(a+bx^2\right)\operatorname{csch}^5\left(a+bx^2\right)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86

$$\int x\operatorname{csch}^7\left(a+bx^2\right)\,dx=-\frac{5\operatorname{csch}^2\left(\frac{1}{2}(a+bx^2)\right)}{128b}+\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx^2)\right)}{128b}-\frac{\operatorname{csch}^6\left(\frac{1}{2}(a+bx^2)\right)}{768b}+\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b}-\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b}-\frac{5\operatorname{sech}^2\left(\frac{1}{2}(a+bx^2)\right)}{128b}-\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx^2)\right)}{128b}-\frac{\operatorname{sech}^6\left(\frac{1}{2}(a+bx^2)\right)}{768b}$$

```
[In] Integrate[x*Csch[a + b*x^2]^7,x]
```

```
[Out] (-5*Csch[(a + b*x^2)/2]^2)/(128*b) + Csch[(a + b*x^2)/2]^4/(128*b) - Csch[(
a + b*x^2)/2]^6/(768*b) + (5*Log[Cosh[(a + b*x^2)/2]])/(32*b) - (5*Log[Sinh
[(a + b*x^2)/2]])/(32*b) - (5*Sech[(a + b*x^2)/2]^2)/(128*b) - Sech[(a + b*
x^2)/2]^4/(128*b) - Sech[(a + b*x^2)/2]^6/(768*b)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\left(-\frac{\operatorname{csch}(b x^2+a)^5}{6}+\frac{5\operatorname{csch}(b x^2+a)^3}{24}-\frac{5\operatorname{csch}(b x^2+a)}{16}\right)\coth(b x^2+a)+\frac{5\operatorname{arctanh}\left(e^{b x^2+a}\right)}{8}}{2b}$
default	$\frac{\left(-\frac{\operatorname{csch}(b x^2+a)^5}{6}+\frac{5\operatorname{csch}(b x^2+a)^3}{24}-\frac{5\operatorname{csch}(b x^2+a)}{16}\right)\coth(b x^2+a)+\frac{5\operatorname{arctanh}\left(e^{b x^2+a}\right)}{8}}{2b}$
parallelrisch	$\frac{-\coth\left(\frac{b x^2}{2}+\frac{a}{2}\right)^6+\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^6+9\coth\left(\frac{b x^2}{2}+\frac{a}{2}\right)^4-9\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^4-45\coth\left(\frac{b x^2}{2}+\frac{a}{2}\right)^2+45\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^2-1}{768b}$
risch	$-\frac{e^{b x^2+a}\left(15 e^{10b x^2+10a}-85 e^{8b x^2+8a}+198 e^{6b x^2+6a}+198 e^{4b x^2+4a}-85 e^{2b x^2+2a}+15\right)}{48b\left(e^{2b x^2+2a}-1\right)^6}+\frac{5\ln\left(e^{b x^2+a}+1\right)}{32b}-$ 5 ln(

```
[In] int(x*csch(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((-1/6*csch(b*x^2+a)^5+5/24*csch(b*x^2+a)^3-5/16*csch(b*x^2+a))*coth(
b*x^2+a)+5/8*arctanh(exp(b*x^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2590 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 2590, normalized size of antiderivative = 28.78

$$\int x\text{csch}^7\left(a + bx^2\right)\, dx = \text{Too large to display}$$

```
[In] integrate(x*csch(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] -1/96*(30*cosh(b*x^2 + a)^11 + 330*cosh(b*x^2 + a)*sinh(b*x^2 + a)^10 + 30*
sinh(b*x^2 + a)^11 + 10*(165*cosh(b*x^2 + a)^2 - 17)*sinh(b*x^2 + a)^9 - 17
0*cosh(b*x^2 + a)^9 + 90*(55*cosh(b*x^2 + a)^3 - 17*cosh(b*x^2 + a))*sinh(b
*x^2 + a)^8 + 36*(275*cosh(b*x^2 + a)^4 - 170*cosh(b*x^2 + a)^2 + 11)*sinh(
b*x^2 + a)^7 + 396*cosh(b*x^2 + a)^7 + 84*(165*cosh(b*x^2 + a)^5 - 170*cosh
(b*x^2 + a)^3 + 33*cosh(b*x^2 + a))*sinh(b*x^2 + a)^6 + 36*(385*cosh(b*x^2
+ a)^6 - 595*cosh(b*x^2 + a)^4 + 231*cosh(b*x^2 + a)^2 + 11)*sinh(b*x^2 + a
)^5 + 396*cosh(b*x^2 + a)^5 + 180*(55*cosh(b*x^2 + a)^7 - 119*cosh(b*x^2 +
a)^5 + 77*cosh(b*x^2 + a)^3 + 11*cosh(b*x^2 + a))*sinh(b*x^2 + a)^4 + 10*(4
95*cosh(b*x^2 + a)^8 - 1428*cosh(b*x^2 + a)^6 + 1386*cosh(b*x^2 + a)^4 + 39
6*cosh(b*x^2 + a)^2 - 17)*sinh(b*x^2 + a)^3 - 170*cosh(b*x^2 + a)^3 + 6*(27
5*cosh(b*x^2 + a)^9 - 1020*cosh(b*x^2 + a)^7 + 1386*cosh(b*x^2 + a)^5 + 660
*cosh(b*x^2 + a)^3 - 85*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 - 15*(cosh(b*x^2
 + a)^12 + 12*cosh(b*x^2 + a)*sinh(b*x^2 + a)^11 + sinh(b*x^2 + a)^12 + 6*(
11*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^10 - 6*cosh(b*x^2 + a)^10 + 20*(1
1*cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^9 + 15*(33*cosh(b*
x^2 + a)^4 - 18*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^8 + 15*cosh(b*x^2 +
a)^8 + 24*(33*cosh(b*x^2 + a)^5 - 30*cosh(b*x^2 + a)^3 + 5*cosh(b*x^2 + a))
*sinh(b*x^2 + a)^7 + 4*(231*cosh(b*x^2 + a)^6 - 315*cosh(b*x^2 + a)^4 + 105
*cosh(b*x^2 + a)^2 - 5)*sinh(b*x^2 + a)^6 - 20*cosh(b*x^2 + a)^6 + 24*(33*c
osh(b*x^2 + a)^7 - 63*cosh(b*x^2 + a)^5 + 35*cosh(b*x^2 + a)^3 - 5*cosh(b*x
^2 + a))*sinh(b*x^2 + a)^5 + 15*(33*cosh(b*x^2 + a)^8 - 84*cosh(b*x^2 + a)^
6 + 70*cosh(b*x^2 + a)^4 - 20*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^4 + 15
*cosh(b*x^2 + a)^4 + 20*(11*cosh(b*x^2 + a)^9 - 36*cosh(b*x^2 + a)^7 + 42*c
osh(b*x^2 + a)^5 - 20*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a
)^3 + 6*(11*cosh(b*x^2 + a)^10 - 45*cosh(b*x^2 + a)^8 + 70*cosh(b*x^2 + a)^
6 - 50*cosh(b*x^2 + a)^4 + 15*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^2 - 6*
cosh(b*x^2 + a)^2 + 12*(cosh(b*x^2 + a)^11 - 5*cosh(b*x^2 + a)^9 + 10*cosh(
b*x^2 + a)^7 - 10*cosh(b*x^2 + a)^5 + 5*cosh(b*x^2 + a)^3 - cosh(b*x^2 + a)
)*sinh(b*x^2 + a) + 1)*log(cosh(b*x^2 + a) + sinh(b*x^2 + a) + 1) + 15*(cos
h(b*x^2 + a)^12 + 12*cosh(b*x^2 + a)*sinh(b*x^2 + a)^11 + sinh(b*x^2 + a)^1
2 + 6*(11*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^10 - 6*cosh(b*x^2 + a)^10
+ 20*(11*cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^9 + 15*(33*
cosh(b*x^2 + a)^4 - 18*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^8 + 15*cosh(b
*x^2 + a)^8 + 24*(33*cosh(b*x^2 + a)^5 - 30*cosh(b*x^2 + a)^3 + 5*cosh(b*x^
```

```
2 + a))*sinh(b*x^2 + a)^7 + 4*(231*cosh(b*x^2 + a)^6 - 315*cosh(b*x^2 + a)^
4 + 105*cosh(b*x^2 + a)^2 - 5)*sinh(b*x^2 + a)^6 - 20*cosh(b*x^2 + a)^6 + 2
4*(33*cosh(b*x^2 + a)^7 - 63*cosh(b*x^2 + a)^5 + 35*cosh(b*x^2 + a)^3 - 5*c
osh(b*x^2 + a))*sinh(b*x^2 + a)^5 + 15*(33*cosh(b*x^2 + a)^8 - 84*cosh(b*x^
2 + a)^6 + 70*cosh(b*x^2 + a)^4 - 20*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)
^4 + 15*cosh(b*x^2 + a)^4 + 20*(11*cosh(b*x^2 + a)^9 - 36*cosh(b*x^2 + a)^7
 + 42*cosh(b*x^2 + a)^5 - 20*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*
x^2 + a)^3 + 6*(11*cosh(b*x^2 + a)^10 - 45*cosh(b*x^2 + a)^8 + 70*cosh(b*x^
2 + a)^6 - 50*cosh(b*x^2 + a)^4 + 15*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)
^2 - 6*cosh(b*x^2 + a)^2 + 12*(cosh(b*x^2 + a)^11 - 5*cosh(b*x^2 + a)^9 + 1
0*cosh(b*x^2 + a)^7 - 10*cosh(b*x^2 + a)^5 + 5*cosh(b*x^2 + a)^3 - cosh(b*x
^2 + a))*sinh(b*x^2 + a) + 1)*log(cosh(b*x^2 + a) + sinh(b*x^2 + a) - 1) +
6*(55*cosh(b*x^2 + a)^10 - 255*cosh(b*x^2 + a)^8 + 462*cosh(b*x^2 + a)^6 +
330*cosh(b*x^2 + a)^4 - 85*cosh(b*x^2 + a)^2 + 5)*sinh(b*x^2 + a) + 30*cosh
(b*x^2 + a))/(b*cosh(b*x^2 + a)^12 + 12*b*cosh(b*x^2 + a)*sinh(b*x^2 + a)^1
1 + b*sinh(b*x^2 + a)^12 - 6*b*cosh(b*x^2 + a)^10 + 6*(11*b*cosh(b*x^2 + a)
^2 - b)*sinh(b*x^2 + a)^10 + 20*(11*b*cosh(b*x^2 + a)^3 - 3*b*cosh(b*x^2 +
a))*sinh(b*x^2 + a)^9 + 15*b*cosh(b*x^2 + a)^8 + 15*(33*b*cosh(b*x^2 + a)^4
 - 18*b*cosh(b*x^2 + a)^2 + b)*sinh(b*x^2 + a)^8 + 24*(33*b*cosh(b*x^2 + a)
^5 - 30*b*cosh(b*x^2 + a)^3 + 5*b*cosh(b*x^2 + a))*sinh(b*x^2 + a)^7 - 20*b
*cosh(b*x^2 + a)^6 + 4*(231*b*cosh(b*x^2 + a)^6 - 315*b*cosh(b*x^2 + a)^4 +
 105*b*cosh(b*x^2 + a)^2 - 5*b)*sinh(b*x^2 + a)^6 + 24*(33*b*cosh(b*x^2 + a
)^7 - 63*b*cosh(b*x^2 + a)^5 + 35*b*cosh(b*x^2 + a)^3 - 5*b*cosh(b*x^2 + a)
)*sinh(b*x^2 + a)^5 + 15*b*cosh(b*x^2 + a)^4 + 15*(33*b*cosh(b*x^2 + a)^8 -
 84*b*cosh(b*x^2 + a)^6 + 70*b*cosh(b*x^2 + a)^4 - 20*b*cosh(b*x^2 + a)^2 +
 b)*sinh(b*x^2 + a)^4 + 20*(11*b*cosh(b*x^2 + a)^9 - 36*b*cosh(b*x^2 + a)^7
 + 42*b*cosh(b*x^2 + a)^5 - 20*b*cosh(b*x^2 + a)^3 + 3*b*cosh(b*x^2 + a))*s
inh(b*x^2 + a)^3 - 6*b*cosh(b*x^2 + a)^2 + 6*(11*b*cosh(b*x^2 + a)^10 - 45*
b*cosh(b*x^2 + a)^8 + 70*b*cosh(b*x^2 + a)^6 - 50*b*cosh(b*x^2 + a)^4 + 15*
b*cosh(b*x^2 + a)^2 - b)*sinh(b*x^2 + a)^2 + 12*(b*cosh(b*x^2 + a)^11 - 5*b
*cosh(b*x^2 + a)^9 + 10*b*cosh(b*x^2 + a)^7 - 10*b*cosh(b*x^2 + a)^5 + 5*b*
cosh(b*x^2 + a)^3 - b*cosh(b*x^2 + a))*sinh(b*x^2 + a) + b)
```

Sympy [F]

$$\int x\text{csch}^7\left(a+bx^2\right)\,dx=\int x\,\text{csch}^7\left(a+bx^2\right)dx$$

```
[In] integrate(x*csch(b*x**2+a)**7,x)
[Out] Integral(x*csch(a + b*x**2)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(82) = 164.

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.28

$$\int x\text{csch}^7(a+bx^2)\,dx = \frac{5\,\log\left(e^{(-bx^2-a)}+1\right)}{32\,b} - \frac{5\,\log\left(e^{(-bx^2-a)}-1\right)}{32\,b} + \frac{15\,e^{(-bx^2-a)} - 85\,e^{(-3\,bx^2-3\,a)} + 198\,e^{(-5\,bx^2-5\,a)} + 198\,e^{(-7\,bx^2-7\,a)} - 85\,e^{(-9\,bx^2-9\,a)} + 15\,e^{(-11\,bx^2-11\,a)}}{48\,b(6\,e^{(-2\,bx^2-2\,a)} - 15\,e^{(-4\,bx^2-4\,a)} + 20\,e^{(-6\,bx^2-6\,a)} - 15\,e^{(-8\,bx^2-8\,a)} + 6\,e^{(-10\,bx^2-10\,a)} - e^{(-12\,bx^2-12\,a)} -}$$

```
[In] integrate(x*csch(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 5/32*log(e^(-b*x^2 - a) + 1)/b - 5/32*log(e^(-b*x^2 - a) - 1)/b + 1/48*(15*
e^(-b*x^2 - a) - 85*e^(-3*b*x^2 - 3*a) + 198*e^(-5*b*x^2 - 5*a) + 198*e^(-7
*b*x^2 - 7*a) - 85*e^(-9*b*x^2 - 9*a) + 15*e^(-11*b*x^2 - 11*a))/(b*(6*e^(-
2*b*x^2 - 2*a) - 15*e^(-4*b*x^2 - 4*a) + 20*e^(-6*b*x^2 - 6*a) - 15*e^(-8*b
*x^2 - 8*a) + 6*e^(-10*b*x^2 - 10*a) - e^(-12*b*x^2 - 12*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.76

$$\int x\text{csch}^7(a+bx^2)\,dx = \frac{5\,\log\left(e^{(bx^2+a)}+e^{(-bx^2-a)}+2\right)}{64\,b} - \frac{5\,\log\left(e^{(bx^2+a)}+e^{(-bx^2-a)}-2\right)}{64\,b} - \frac{15\left(e^{(bx^2+a)}+e^{(-bx^2-a)}\right)^5 - 160\left(e^{(bx^2+a)}+e^{(-bx^2-a)}\right)^3 + 528\,e^{(bx^2+a)} + 528\,e^{(-bx^2-a)}}{48\left(\left(e^{(bx^2+a)}+e^{(-bx^2-a)}\right)^2-4\right)^3 b}$$

```
[In] integrate(x*csch(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 5/64*log(e^(b*x^2 + a) + e^(-b*x^2 - a) + 2)/b - 5/64*log(e^(b*x^2 + a) + e
^(-b*x^2 - a) - 2)/b - 1/48*(15*(e^(b*x^2 + a) + e^(-b*x^2 - a))^5 - 160*(e
^(b*x^2 + a) + e^(-b*x^2 - a))^3 + 528*e^(b*x^2 + a) + 528*e^(-b*x^2 - a))/
(((e^(b*x^2 + a) + e^(-b*x^2 - a))^2 - 4)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.43

$$\int x\mathrm{csch}^7\left(a+bx^2\right)\,dx=\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^a\,\mathrm{e}^{b\,x^2}\,\sqrt{-b^2}}{b}\right)}{16\,\sqrt{-b^2}}-\frac{8\,\mathrm{e}^{3\,b\,x^2+3\,a}}{3\,b\,\left(5\,\mathrm{e}^{2\,b\,x^2+2\,a}-10\,\mathrm{e}^{4\,b\,x^2+4\,a}+10\,\mathrm{e}^{6\,b\,x^2+6\,a}-5\,\mathrm{e}^{8\,b\,x^2+8\,a}+\mathrm{e}^{10\,b\,x^2+10\,a}-1\right)}-\frac{\mathrm{e}^{b\,x^2+a}}{b\,\left(6\,\mathrm{e}^{4\,b\,x^2+4\,a}-4\,\mathrm{e}^{2\,b\,x^2+2\,a}-4\,\mathrm{e}^{6\,b\,x^2+6\,a}+\mathrm{e}^{8\,b\,x^2+8\,a}+1\right)}+\frac{5\,\mathrm{e}^{b\,x^2+a}}{24\,b\,\left(\mathrm{e}^{4\,b\,x^2+4\,a}-2\,\mathrm{e}^{2\,b\,x^2+2\,a}+1\right)}-\frac{16\,\mathrm{e}^{5\,b\,x^2+5\,a}}{3\,b\,\left(15\,\mathrm{e}^{4\,b\,x^2+4\,a}-6\,\mathrm{e}^{2\,b\,x^2+2\,a}-20\,\mathrm{e}^{6\,b\,x^2+6\,a}+15\,\mathrm{e}^{8\,b\,x^2+8\,a}-6\,\mathrm{e}^{10\,b\,x^2+10\,a}+\mathrm{e}^{12\,b\,x^2+12\,a}+1\right)}-\frac{\mathrm{e}^{b\,x^2+a}}{6\,b\,\left(3\,\mathrm{e}^{2\,b\,x^2+2\,a}-3\,\mathrm{e}^{4\,b\,x^2+4\,a}+\mathrm{e}^{6\,b\,x^2+6\,a}-1\right)}-\frac{5\,\mathrm{e}^{b\,x^2+a}}{16\,b\,\left(\mathrm{e}^{2\,b\,x^2+2\,a}-1\right)}$$

```
[In] int(x/sinh(a + b*x^2)^7,x)
```

```
[Out] (5*atan((exp(a)*exp(b*x^2)*(-b^2)^(1/2))/b))/(16*(-b^2)^(1/2)) - (8*exp(3*a
 + 3*b*x^2))/(3*b*(5*exp(2*a + 2*b*x^2) - 10*exp(4*a + 4*b*x^2) + 10*exp(6*
a + 6*b*x^2) - 5*exp(8*a + 8*b*x^2) + exp(10*a + 10*b*x^2) - 1)) - exp(a +
b*x^2)/(b*(6*exp(4*a + 4*b*x^2) - 4*exp(2*a + 2*b*x^2) - 4*exp(6*a + 6*b*x^
2) + exp(8*a + 8*b*x^2) + 1)) + (5*exp(a + b*x^2))/(24*b*(exp(4*a + 4*b*x^2
) - 2*exp(2*a + 2*b*x^2) + 1)) - (16*exp(5*a + 5*b*x^2))/(3*b*(15*exp(4*a +
 4*b*x^2) - 6*exp(2*a + 2*b*x^2) - 20*exp(6*a + 6*b*x^2) + 15*exp(8*a + 8*b
*x^2) - 6*exp(10*a + 10*b*x^2) + exp(12*a + 12*b*x^2) + 1)) - exp(a + b*x^2
)/(6*b*(3*exp(2*a + 2*b*x^2) - 3*exp(4*a + 4*b*x^2) + exp(6*a + 6*b*x^2) -
1)) - (5*exp(a + b*x^2))/(16*b*(exp(2*a + 2*b*x^2) - 1))
```

3.16 $\int \frac{x^5}{a+b\text{csch}(c+dx^2)}\,dx$

Optimal result 111
Rubi [A] (verified) 111
Mathematica [A] (verified) 115
Maple [F] 115
Fricas [B] (verification not implemented) 115
Sympy [F] 116
Maxima [F] 116
Giac [F] 116
Mupad [F(-1)] 117

Optimal result

Integrand size = 18, antiderivative size = 325

$$\begin{aligned}\int \frac{x^5}{a+b\,\text{csch}\left(c+dx^2\right)}\,dx &= \frac{x^6}{6a}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}\\&-\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}\\&+\frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}-\frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}\end{aligned}$$

```
[Out] 1/6*x^6/a-1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^
(1/2)+1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2
)-b*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2
)+b*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2
)+b*polylog(3,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-b*
polylog(3,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

= {5545, 4276, 3403, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{x^5}{a+b\operatorname{csch}(c+dx^2)}\,dx = \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} - \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}+1\right)}{2ad\sqrt{a^2+b^2}} + \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{\sqrt{a^2+b^2}+b}+1\right)}{2ad\sqrt{a^2+b^2}} + \frac{x^6}{6a}$$

```
[In] Int[x^5/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] x^6/(6*a) - (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])])/(2*a*S
qrt[a^2 + b^2]*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])]
)/(2*a*Sqrt[a^2 + b^2]*d) - (b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt
[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (b*x^2*PolyLog[2, -((a*E^(c + d*x
^2))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (b*PolyLog[3, -((a*
E^(c + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (b*PolyLo
g[3, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\operatorname{csch}(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sinh(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a}
\end{aligned}$$

$$=\frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{\sqrt{a^2+b^2}}+\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{\sqrt{a^2+b^2}}$$

$$=\frac{x^6}{6a}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{b\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a\sqrt{a^2+b^2}d}-\frac{b\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a\sqrt{a^2+b^2}d}$$

$$=\frac{x^6}{6a}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}-\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a\sqrt{a^2+b^2}d^2}-\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a\sqrt{a^2+b^2}d^2}$$

$$=\frac{x^6}{6a}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}-\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a\sqrt{a^2+b^2}d^3}-\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$=\frac{x^6}{6a}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}-\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{bx^2\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}+\frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}-\frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{a+b\operatorname{csch}(c+dx^2)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}d^3x^6 - 3bd^2x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right) + 3bd^2x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right) - 6bdx^2\operatorname{PolyLog}\left(2,\frac{ae^{c+dx^2}}{-b+\sqrt{a^2+b^2}}\right)}{6a\sqrt{a^2+b^2}d^3}$$

```
[In] Integrate[x^5/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (Sqrt[a^2 + b^2]*d^3*x^6 - 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[
a^2 + b^2])] + 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])]
 - 6*b*d*x^2*PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[a^2 + b^2])] + 6*b*d*x
^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))] + 6*b*PolyLog[3,
(a*E^(c + d*x^2))/(-b + Sqrt[a^2 + b^2])] - 6*b*PolyLog[3, -((a*E^(c + d*x^
2))/(b + Sqrt[a^2 + b^2]))])/(6*a*Sqrt[a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{x^5}{a+b\,\operatorname{csch}(d\,x^2+c)}dx$$

```
[In] int(x^5/(a+b*csch(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*csch(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(287) = 574$.

Time = 0.27 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.11

$$\int \frac{x^5}{a+b\operatorname{csch}(c+dx^2)}\,dx$$
$$= \frac{(a^2+b^2)d^3x^6 - 6\,abdx^2\sqrt{\frac{a^2+b^2}{a^2}}\operatorname{Li}_2\left(\frac{b\cosh(dx^2+c)+b\sinh(dx^2+c)+(a\cosh(dx^2+c)+a\sinh(dx^2+c))\sqrt{\frac{a^2+b^2}{a^2}}-a}{a}+1\right)+6\,ab}{}$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*((a^2 + b^2)*d^3*x^6 - 6*a*b*d*x^2*sqrt((a^2 + b^2)/a^2)*dilog((b*cosh(
d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sq
rt((a^2 + b^2)/a^2) - a)/a + 1) + 6*a*b*d*x^2*sqrt((a^2 + b^2)/a^2)*dilog((
b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 +
```

```
 c))*sqrt((a^2 + b^2)/a^2) - a)/a + 1) + 3*a*b*c^2*sqrt((a^2 + b^2)/a^2)*lo
g(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2
*b) - 3*a*b*c^2*sqrt((a^2 + b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*
x^2 + c) - 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 6*a*b*sqrt((a^2 + b^2)/a^2)*p
olylog(3, (b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*s
inh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))/a) - 6*a*b*sqrt((a^2 + b^2)/a^2)*pol
ylog(3, (b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sin
h(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))/a) - 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt((a
^2 + b^2)/a^2)*log(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2
+ c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a) + 3*(a*b*d^2*x^4 -
a*b*c^2)*sqrt((a^2 + b^2)/a^2)*log(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c)
- (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a))/((
a^3 + a*b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{a + b\text{csch}(c + dx^2)}\,dx = \int \frac{x^5}{a + b\,\text{csch}(c + dx^2)}\,dx$$

```
[In] integrate(x**5/(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**5/(a + b*csch(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{a + b\text{csch}(c + dx^2)}\,dx = \int \frac{x^5}{b\,\text{csch}(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/6*x^6/a - 2*b*integrate(x^5*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) - a^2), x)
```

Giac [F]

$$\int \frac{x^5}{a + b\text{csch}(c + dx^2)}\,dx = \int \frac{x^5}{b\,\text{csch}(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^5/(b*csch(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^5}{a+\frac{b}{\sinh(d\,x^2+c)}}\,dx$$

```
[In] int(x^5/(a + b/sinh(c + d*x^2)),x)
[Out] int(x^5/(a + b/sinh(c + d*x^2)), x)
```

3.17 $\int \frac{x^4}{a+b\text{csch}(c+dx^2)}\,dx$

Optimal result 118
Rubi [N/A] 118
Mathematica [N/A] 119
Maple [N/A] (verified) 119
Fricas [N/A] 119
Sympy [N/A] 119
Maxima [N/A] 120
Giac [N/A] 120
Mupad [N/A] 120

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\text{Int}\left(\frac{x^4}{a+b\text{csch}\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*csch(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx$$

```
[In] Int[x^4/(a + b*Csch[c + d*x^2]),x]
[Out] Defer[Int][x^4/(a + b*Csch[c + d*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx$$

```
[In] Integrate[x^4/(a + b*Csch[c + d*x^2]),x]
[Out] Integrate[x^4/(a + b*Csch[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^{4}}{a+b\,\operatorname{csch}\left(d\,x^{2}+c\right)}dx$$

```
[In] int(x^4/(a+b*csch(d*x^2+c)),x)
[Out] int(x^4/(a+b*csch(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^{4}}{b\operatorname{csch}\left(dx^{2}+c\right)+a}\,dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^4/(b*csch(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^{4}}{a+b\operatorname{csch}\left(c+dx^{2}\right)}\,dx$$

```
[In] integrate(x**4/(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**4/(a + b*csch(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \frac{x^4}{a + b\text{csch}(c + dx^2)} dx = \int \frac{x^4}{b\,\text{csch}(dx^2 + c) + a} dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/a - 2*b*integrate(x^4*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) - a^2), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b\text{csch}(c + dx^2)} dx = \int \frac{x^4}{b\,\text{csch}(dx^2 + c) + a} dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*csch(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b\text{csch}(c + dx^2)} dx = \int \frac{x^4}{a + \frac{b}{\sinh(d\,x^2+c)}} dx$$

```
[In] int(x^4/(a + b/sinh(c + d*x^2)),x)
```

```
[Out] int(x^4/(a + b/sinh(c + d*x^2)), x)
```

3.18 $\int \frac{x^3}{a+b\text{csch}(c+dx^2)} \, dx$

Optimal result 121
Rubi [A] (verified) 121
Mathematica [A] (verified) 124
Maple [F] 124
Fricas [B] (verification not implemented) 124
Sympy [F] 125
Maxima [F] 125
Giac [F] 125
Mupad [F(-1)] 126

Optimal result

Integrand size = 18, antiderivative size = 225

$$\int \frac{x^3}{a+b\text{csch}(c+dx^2)} \, dx = \frac{x^4}{4a} - \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d} + \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d} - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d^2}$$

```
[Out] 1/4*x^4/a-1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^
(1/2)+1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2
)-1/2*b*polylog(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2
)+1/2*b*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5545, 4276, 3403, 2296, 2221, 2317, 2438}

$$\int \frac{x^3}{a+b\text{csch}(c+dx^2)} \, dx = -\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{2ad^2\sqrt{a^2+b^2}} + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{2ad^2\sqrt{a^2+b^2}} - \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}+1\right)}{2ad\sqrt{a^2+b^2}} + \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{\sqrt{a^2+b^2}+b}+1\right)}{2ad\sqrt{a^2+b^2}} + \frac{x^4}{4a}$$

```
[In] Int[x^3/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] x^4/(4*a) - (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])])/(2*a*S
qrt[a^2 + b^2]*d) + (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])]
)/(2*a*Sqrt[a^2 + b^2]*d) - (b*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[a^2
 + b^2]))])/(2*a*Sqrt[a^2 + b^2]*d^2) + (b*PolyLog[2, -((a*E^(c + d*x^2))/(
b + Sqrt[a^2 + b^2]))])/(2*a*Sqrt[a^2 + b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\text{csch}(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\sinh(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{a^2+b^2}}+\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{a^2+b^2}}\\
&= \frac{x^4}{4a}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}\\
&\quad+\frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{a^2+b^2}d}\\
&\quad-\frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{a^2+b^2}d}\\
&= \frac{x^4}{4a}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{2a\sqrt{a^2+b^2}d^2}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{2a\sqrt{a^2+b^2}d^2}
\end{aligned}
$$

$$= \frac{x^4}{4a} - \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d} + \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d} - \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d^2} + \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}d^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a + b\text{csch}\,(c + dx^2)}\,dx = \frac{dx^2\left(\sqrt{a^2+b^2}dx^2 - 2b\log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right) + 2b\log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)\right) - 2b\,\text{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b+\sqrt{a^2+b^2}}\right) + 2b\,\text{Pol}}{4a\sqrt{a^2+b^2}d^2}$$

```
[In] Integrate[x^3/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] (d*x^2*(Sqrt[a^2 + b^2]*d*x^2 - 2*b*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2
 + b^2])] + 2*b*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])]) - 2*b*Pol
yLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[a^2 + b^2])] + 2*b*PolyLog[2, -((a*E^(
c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(4*a*Sqrt[a^2 + b^2]*d^2)
```

Maple [F]

$$\int \frac{x^3}{a + b\;\text{csch}\,(d\,x^2 + c)}dx$$

```
[In] int(x^3/(a+b*csch(d*x^2+c)),x)
```

```
[Out] int(x^3/(a+b*csch(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(193) = 386.

Time = 0.28 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.24

$$\int \frac{x^3}{a + b\text{csch}\,(c + dx^2)}\,dx = \frac{(a^2+b^2)d^2x^4 - 2\,abc\sqrt{\frac{a^2+b^2}{a^2}}\log\left(2\,a\cosh\left(dx^2+c\right) + 2\,a\sinh\left(dx^2+c\right) + 2\,a\sqrt{\frac{a^2+b^2}{a^2}} + 2\,b\right) + 2\,abc\sqrt{\frac{a^2+b}{a^2}}}{}$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 + b^2)*d^2*x^4 - 2*a*b*c*sqrt((a^2 + b^2)/a^2)*log(2*a*cosh(d*x^2
 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 2*a*b*c*sq
rt((a^2 + b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) - 2*a*sqr
t((a^2 + b^2)/a^2) + 2*b) - 2*a*b*sqrt((a^2 + b^2)/a^2)*dilog((b*cosh(d*x^2
 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a
^2 + b^2)/a^2) - a)/a + 1) + 2*a*b*sqrt((a^2 + b^2)/a^2)*dilog((b*cosh(d*x^
2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((
a^2 + b^2)/a^2) - a)/a + 1) - 2*(a*b*d*x^2 + a*b*c)*sqrt((a^2 + b^2)/a^2)*l
og(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*
x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a^2 +
 b^2)/a^2)*log(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c)
 + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a))/((a^3 + a*b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{a + b\text{csch}\,(c + dx^2)}\,dx = \int \frac{x^3}{a + b\,\text{csch}\,(c + dx^2)}\,dx$$

```
[In] integrate(x**3/(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**3/(a + b*csch(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a + b\text{csch}\,(c + dx^2)}\,dx = \int \frac{x^3}{b\,\text{csch}\,(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
[Out] 1/4*x^4/a - 2*b*integrate(x^3*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) - a^2), x)
```

Giac [F]

$$\int \frac{x^3}{a + b\text{csch}\,(c + dx^2)}\,dx = \int \frac{x^3}{b\,\text{csch}\,(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*csch(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\operatorname{csch}(c + dx^2)}\,dx = \int \frac{x^3}{a + \frac{b}{\sinh(d\,x^2+c)}}\,dx$$

```
[In] int(x^3/(a + b/sinh(c + d*x^2)),x)
[Out] int(x^3/(a + b/sinh(c + d*x^2)), x)
```

3.19 $\int \frac{x^2}{a+b\text{csch}(c+dx^2)}\,dx$

Optimal result 127
Rubi [N/A] 127
Mathematica [N/A] 128
Maple [N/A] (verified) 128
Fricas [N/A] 128
Sympy [N/A] 128
Maxima [N/A] 129
Giac [N/A] 129
Mupad [N/A] 129

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\text{Int}\left(\frac{x^2}{a+b\text{csch}\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*csch(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\text{csch}\left(c+dx^2\right)}\,dx$$

```
[In] Int[x^2/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^2/(a + b*Csch[c + d*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2}{a+b\text{csch}\left(c+dx^2\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx$$

```
[In] Integrate[x^2/(a + b*Csch[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Csch[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\ \operatorname{csch}\left(d\,x^2+c\right)}dx$$

```
[In] int(x^2/(a+b*csch(d*x^2+c)),x)
[Out] int(x^2/(a+b*csch(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{b\operatorname{csch}\left(dx^2+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*csch(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx$$

```
[In] integrate(x**2/(a+b*csch(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*csch(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{a+b\operatorname{csch}(c+dx^2)}\,dx=\int \frac{x^2}{b\operatorname{csch}(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/a - 2*b*integrate(x^2*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) - a^2), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\operatorname{csch}(c+dx^2)}\,dx=\int \frac{x^2}{b\operatorname{csch}(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*csch(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\operatorname{csch}(c+dx^2)}\,dx=\int \frac{x^2}{a+\frac{b}{\sinh(d\,x^2+c)}}\,dx$$

```
[In] int(x^2/(a + b/sinh(c + d*x^2)),x)
```

```
[Out] int(x^2/(a + b/sinh(c + d*x^2)), x)
```

3.20 $\int \frac{x}{a+b\,\mathbf{csch}(c+dx^2)}\,dx$

Optimal result 130
Rubi [A] (verified) 130
Mathematica [A] (verified) 132
Maple [A] (verified) 132
Fricas [B] (verification not implemented) 132
Sympy [F] 133
Maxima [A] (verification not implemented) 133
Giac [A] (verification not implemented) 133
Mupad [B] (verification not implemented) 134

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{x}{a+b\operatorname{csch}(c+dx^2)}\,dx = \frac{x^2}{2a}+\frac{b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

```
[Out] 1/2*x^2/a+b*arctanh((a-b*tanh(1/2*d*x^2+1/2*c))/(a^2+b^2)^(1/2))/a/d/(a^2+b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5545, 3868, 2739, 632, 210}

$$\int \frac{x}{a+b\operatorname{csch}(c+dx^2)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Csch[c + d*x^2]),x]
```

```
[Out] x^2/(2*a) + (b*ArcTanh[(a - b*Tanh[(c + d*x^2)/2])/Sqrt[a^2 + b^2]])/(a*Sqr
t[a^2 + b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b\text{csch}(c+dx)}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a} + \frac{i\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, i\tanh\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a} - \frac{(2i)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2i\tanh\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a} + \frac{b\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx^2)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{x}{a+b\operatorname{csch}(c+dx^2)}\,dx = \frac{\frac{c}{d}+x^2-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}}{2a}$$

```
[In] Integrate[x/(a + b*Csch[c + d*x^2]),x]
[Out] (c/d + x^2 - (2*b*ArcTan[(a - b*Tanh[(c + d*x^2)/2])/Sqrt[-a^2 - b^2]])/(Sq
rt[-a^2 - b^2]*d))/(2*a)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$\frac{\frac{2b\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)-1\right)}{a}}{2d}$	89
default	$\frac{\frac{2b\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)-1\right)}{a}}{2d}$	89
risch	$\frac{x^2}{2a}+\frac{b\ln\left(\mathrm{e}^{d\,x^2+c}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{2\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{d\,x^2+c}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{2\sqrt{a^2+b^2}\,da}$	132

```
[In] int(x/(a+b*csch(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x^2+1/2*c)+2*a)/(
a^2+b^2)^(1/2))+1/a*ln(1+tanh(1/2*d*x^2+1/2*c))-1/a*ln(tanh(1/2*d*x^2+1/2*c
)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.55

$$\int \frac{x}{a+b\operatorname{csch}(c+dx^2)}\,dx$$
$$= \frac{(a^2+b^2)dx^2+\sqrt{a^2+b^2}b\log\left(\frac{a^2\cosh(dx^2+c)^2+a^2\sinh(dx^2+c)^2+2\,ab\cosh(dx^2+c)+a^2+2\,b^2+2\,(a^2\cosh(dx^2+c)+ab)\sinh(dx^2+c)}{a\cosh(dx^2+c)^2+a\sinh(dx^2+c)^2+2\,b\cosh(dx^2+c)+2\,(a\cosh(dx^2+c)+b)}\right.}{2\,(a^3+ab^2)d}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*d*x^2 + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x^2 + c)^2 + a^2
*sinh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x^
2 + c) + a*b)*sinh(d*x^2 + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x^2 + c) + a*si
nh(d*x^2 + c) + b))/(a*cosh(d*x^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cosh(d
*x^2 + c) + 2*(a*cosh(d*x^2 + c) + b)*sinh(d*x^2 + c) - a)))/((a^3 + a*b^2)
*d)
```

Sympy [F]

$$\int \frac{x}{a+b\text{csch}\left(c+dx^2\right)}\,dx=\int \frac{x}{a+b\operatorname{csch}\left(c+dx^2\right)}\,dx$$

```
[In] integrate(x/(a+b*csch(d*x**2+c)),x)
```

```
[Out] Integral(x/(a + b*csch(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{x}{a+b\text{csch}\left(c+dx^2\right)}\,dx=-\frac{b\log\left(\frac{ae^{\left(-dx^2-c\right)}-b-\sqrt{a^2+b^2}}{ae^{\left(-dx^2-c\right)}-b+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}ad}+\frac{dx^2+c}{2\,ad}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] -1/2*b*log((a*e^(-d*x^2 - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x^2 - c) - b +
 sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + 1/2*(d*x^2 + c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{x}{a+b\text{csch}\left(c+dx^2\right)}\,dx=-\frac{b\log\left(\frac{\left|2\,ae^{\left(dx^2+c\right)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{\left(dx^2+c\right)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\sqrt{a^2+b^2}ad}+\frac{dx^2+c}{2\,ad}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c)),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(2*a*e^(d*x^2 + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x^
2 + c) + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + 1/2*(d*x^2 + c)/
(a*d)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.92

$$\int \frac{x}{a+b\operatorname{csch}(c+dx^2)}\,dx$$
$$=\frac{x^2}{2a}-\frac{\operatorname{atan}\left(\frac{a\,d\,\sqrt{b^2}}{\sqrt{-a^4\,d^2-a^2\,b^2\,d^2}}-\frac{b\,e^{d\,x^2}\,e^c\,\sqrt{-a^4\,d^2-a^2\,b^2\,d^2}}{a^2\,d\,\sqrt{b^2}}+\frac{a^2\,b\,d\,e^{d\,x^2}\,e^c\,\sqrt{b^2}\,\sqrt{-a^4\,d^2-a^2\,b^2\,d^2}}{a^6\,d^2+a^4\,b^2\,d^2}\right)\sqrt{b^2}}{\sqrt{-a^4\,d^2-a^2\,b^2\,d^2}}$$

```
[In] int(x/(a + b/sinh(c + d*x^2)),x)
```

```
[Out] x^2/(2*a) - (atan((a*d*(b^2)^(1/2))/(- a^4*d^2 - a^2*b^2*d^2)^(1/2) - (b*ex
p(d*x^2)*exp(c)*(- a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^2*d*(b^2)^(1/2)) + (a^2
*b*d*exp(d*x^2)*exp(c)*(b^2)^(1/2)*(- a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^6*d^
2 + a^4*b^2*d^2))*(b^2)^(1/2))/(- a^4*d^2 - a^2*b^2*d^2)^(1/2)
```

3.21 $\int \frac{1}{x\left(a+b\,\mathbf{csch}\left(c+dx^2\right)\right)}\,dx$

Optimal result 135
Rubi [N/A] 135
Mathematica [N/A] 136
Maple [N/A] (verified) 136
Fricas [N/A] 136
Sympy [N/A] 136
Maxima [N/A] 137
Giac [N/A] 137
Mupad [N/A] 137

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csch(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Csch[c + d*x^2])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Csch[c + d*x^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csch[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Csch[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \operatorname{csch}\left(d\,x^{2}+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*csch(d*x^2+c)),x)
[Out] int(1/x/(a+b*csch(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*csch(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{csch}\left(c+dx^2\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x**2+c)),x)
[Out] Integral(1/(x*(a + b*csch(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\operatorname{csch}\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] -2*b*integrate(e^(d*x^2 + c)/(a^2*x*e^(2*d*x^2 + 2*c) + 2*a*b*x*e^(d*x^2 +
c) - a^2*x), x) + log(x)/a
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\operatorname{csch}\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csch(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\,\left(a+\frac{b}{\mathrm{sinh}\left(d\,x^2+c\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/sinh(c + d*x^2))),x)
```

```
[Out] int(1/(x*(a + b/sinh(c + d*x^2))), x)
```

3.22 $\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx$

Optimal result 138
Rubi [N/A] 138
Mathematica [N/A] 139
Maple [N/A] (verified) 139
Fricas [N/A] 139
Sympy [N/A] 139
Maxima [N/A] 140
Giac [N/A] 140
Mupad [N/A] 140

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\text{csch}(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csch(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csch[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\text{csch}(c+dx^2)}{x^2}\right)dx\\&=-\frac{a}{x}+b\int \frac{\text{csch}(c+dx^2)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\,\operatorname{csch}\left(d\,x^2+c\right)}{x^2}dx$$

```
[In] int((a+b*csch(d*x^2+c))/x^2,x)
[Out] int((a+b*csch(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx=\int \frac{b\operatorname{csch}\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*csch(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{csch}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\operatorname{csch}\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x**2+c))/x**2,x)
[Out] Integral((a + b*csch(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \frac{a+b\operatorname{csch}(c+dx^2)}{x^2}\,dx = \int \frac{b\operatorname{csch}(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x^2*(e^(d*x^2 + c) - e^(-d*x^2 - c))), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{csch}(c+dx^2)}{x^2}\,dx = \int \frac{b\operatorname{csch}(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(d*x^2+c))/x^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\operatorname{csch}(c+dx^2)}{x^2}\,dx = \int \frac{a+\frac{b}{\sinh(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/sinh(c + d*x^2))/x^2,x)
[Out] int((a + b/sinh(c + d*x^2))/x^2, x)
```

3.23 $\int \frac{x^5}{\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result 142
Rubi [A] (verified) 143
Mathematica [A] (verified) 152
Maple [F] 153
Fricas [B] (verification not implemented) 154
Sympy [F] 156
Maxima [F] 156
Giac [F] 156
Mupad [F(-1)] 156

Optimal result

Integrand size = 18, antiderivative size = 922

$$\int \frac{x^5}{(a+b\text{csch}(c+dx^2))^2}\,dx = -\frac{b^2x^4}{2a^2(a^2+b^2)d}+\frac{x^6}{6a^2}+\frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}+\frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}-\frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}+\frac{b^3x^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}-\frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}+\frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}-\frac{b^3x^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}+\frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}-\frac{b^3\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}+\frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}+\frac{b^3\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}-\frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}-\frac{b^2x^4\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}$$

```
[Out] -1/2*b^2*x^4/a^2/(a^2+b^2)/d+1/6*x^6/a^2+b^2*x^2*ln(1+a*exp(d*x^2+c)/(b-(a^
2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+1/2*b^3*x^4*ln(1+a*exp(d*x^2+c)/(b-(a^2+b^
2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+b^2*x^2*ln(1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(
1/2)))/a^2/(a^2+b^2)/d^2-1/2*b^3*x^4*ln(1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)
))/a^2/(a^2+b^2)^(3/2)/d+b^2*polylog(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))
/a^2/(a^2+b^2)/d^3+b^3*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a
^2/(a^2+b^2)^(3/2)/d^2+b^2*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a
^2/(a^2+b^2)/d^3-b^3*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2
/(a^2+b^2)^(3/2)/d^2-b^3*polylog(3,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2
```

```
/(a^2+b^2)^(3/2)/d^3+b^3*polylog(3,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2
/(a^2+b^2)^(3/2)/d^3-1/2*b^2*x^4*cosh(d*x^2+c)/a/(a^2+b^2)/d/(b+a*sinh(d*x^
2+c))-b*x^4*ln(1+a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+
b*x^4*ln(1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)-2*b*x^
2*polylog(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+2*
b*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2
)+2*b*polylog(3,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2
)-2*b*polylog(3,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\int \frac{x^5}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = \frac{x^6}{6a^2}-\frac{b\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^4}{a^2\sqrt{a^2+b^2}d}+\frac{b^3\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^4}{2a^2(a^2+b^2)^{3/2}d}+\frac{b\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^4}{a^2\sqrt{a^2+b^2}d}-\frac{b^3\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^4}{2a^2(a^2+b^2)^{3/2}d}-\frac{b^2x^4}{2a^2(a^2+b^2)d}-\frac{b^2\cosh(dx^2+c)x^4}{2a(a^2+b^2)d(b+a\sinh(dx^2+c))}+\frac{b^2\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^2}{a^2(a^2+b^2)d^2}+\frac{b^2\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^2}{a^2(a^2+b^2)d^2}-\frac{2b\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)x^2}{a^2\sqrt{a^2+b^2}d^2}+\frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)x^2}{a^2(a^2+b^2)^{3/2}d^2}+\frac{2b\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)x^2}{a^2\sqrt{a^2+b^2}d^2}-\frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)x^2}{a^2(a^2+b^2)^{3/2}d^2}+\frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}+\frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}+\frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}-\frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}-\frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}+\frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$

```
[In] Int[x^5/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] -1/2*(b^2*x^4)/(a^2*(a^2 + b^2)*d) + x^6/(6*a^2) + (b^2*x^2*Log[1 + (a*E^(c
 + d*x^2))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (b^3*x^4*Log[1 +
 (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])])/(2*a^2*(a^2 + b^2)^(3/2)*d) - (b
*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]
*d) + (b^2*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2
+ b^2)*d^2) - (b^3*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])])/(2
*a^2*(a^2 + b^2)^(3/2)*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2
+ b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (b^2*PolyLog[2, -((a*E^(c + d*x^2))/(b
- Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (b^3*x^2*PolyLog[2, -((a*E^(c
```

```
 + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (2*b*x^2*
PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2
]*d^2) + (b^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(a^2*
(a^2 + b^2)*d^3) - (b^3*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 +
b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (2*b*x^2*PolyLog[2, -((a*E^(c + d*x^
2))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (b^3*PolyLog[3, -(
(a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (2
*b*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
b^2]*d^3) + (b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(a
^2*(a^2 + b^2)^(3/2)*d^3) - (2*b*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[a
^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (b^2*x^4*Cosh[c + d*x^2])/(2*a*(a
^2 + b^2)*d*(b + a*Sinh[c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\text{csch}(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sinh(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^6}{6a^2}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x^2}{(b+a\sinh(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^6}{6a^2}-\frac{b^2x^4\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad+\frac{b^3\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{2a^2(a^2+b^2)}+\frac{b^2\text{Subst}\left(\int \frac{x\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{a(a^2+b^2)d}\\
&= -\frac{b^2x^4}{2a^2(a^2+b^2)d}+\frac{x^6}{6a^2}-\frac{b^2x^4\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}\\
&\quad+\frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2+b^2)}\\
&\quad-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{b^2\text{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, x^2\right)}{a(a^2+b^2)d}+\frac{b^2\text{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, x^2\right)}{a(a^2+b^2)d}
\end{aligned}$$

$$= -\frac{b^2x^4}{2a^2\left(a^2+b^2\right)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$+ \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$- \frac{b^2x^4\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)} + \frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\left(a^2+b^2\right)^{3/2}}$$

$$- \frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\left(a^2+b^2\right)^{3/2}}$$

$$- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\left(a^2+b^2\right)d^2}$$

$$- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\left(a^2+b^2\right)d^2}$$

$$+ \frac{(2b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{a^2+b^2}d}$$

$$- \frac{(2b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{a^2+b^2}d}$$

$$
\begin{aligned}
&= -\frac{b^2x^4}{2a^2\left(a^2+b^2\right)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d} \\
&- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} \\
&- \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&- \frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&- \frac{b^2x^4\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)} - \frac{b^2\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b-\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&- \frac{b^2\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&+ \frac{(2b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&- \frac{(2b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&- \frac{b^3\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a^2\left(a^2+b^2\right)^{3/2}d} \\
&+ \frac{b^3\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^2\right)}{a^2\left(a^2+b^2\right)^{3/2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2x^4}{2a^2\left(a^2+b^2\right)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d} \\
&- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d} \\
&+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&+ \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2} - \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&+ \frac{b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2} \\
&+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{b^2x^4\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)} \\
&+ \frac{(2b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^2}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&- \frac{(2b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^2}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&- \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2} \\
&+ \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2}
\end{aligned}
$$

$$= -\frac{b^2x^4}{2a^2(a^2+b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{b^3x^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{b^3x^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} - \frac{b^2x^4\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}$$
$$- \frac{b^3\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$+ \frac{b^3\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$

$$= -\frac{b^2x^4}{2a^2(a^2+b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$+ \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} + \frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$- \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} - \frac{b^2x^4\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}$$

Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 1502, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx$$

$$= \frac{\operatorname{csch}^2(c+dx^2)(b+a\sinh(c+dx^2))\left(\frac{6b^2x^4\operatorname{csch}(c)(b\cosh(c)+a\sinh(dx^2))}{(a^2+b^2)d} + 2x^6(b+a\sinh(c+dx^2)) - \frac{6be^{2c}\left(2bd^2\right.}{}\right.}{}$$

```
[In] Integrate[x^5/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (Csch[c + d*x^2]^2*(b + a*Sinh[c + d*x^2])*((6*b^2*x^4*Csch[c]*(b*Cosh[c] +
 a*Sinh[d*x^2]))/((a^2 + b^2)*d) + 2*x^6*(b + a*Sinh[c + d*x^2]) - (6*b*E^(
2*c)*(2*b*d^2*E^(2*c)*Sqrt[(a^2 + b^2)*E^(2*c)]*x^4 + 2*b*d*Sqrt[(a^2 + b^2
)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])] - 2*b*d*E^(2*c)*Sqrt[(a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^
2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*a^2*d^2*E^c*x^4*Log[1 + (a*E^(
```

```
2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - b^2*d^2*E^c*x^4*Log[1
+ (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*a^2*d^2*E^(3
*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] +
b^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)])] + 2*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))
/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*b*d*E^(2*c)*Sqrt[(a^2 + b^2)*E^(2
*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] +
 2*a^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)])] + b^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)])] - 2*a^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)])] - b^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*
x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*(-1 + E^(2*c))*(-(b*Sqrt[(a^
2 + b^2)*E^(2*c)]) + 2*a^2*d*E^c*x^2 + b^2*d*E^c*x^2)*PolyLog[2, -((a*E^(2*
c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*(-1 + E^(2*c))*(b*Sqr
t[(a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*x^2 + b^2*d*E^c*x^2)*PolyLog[2, -((a*E
^(2*c + d*x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*a^2*E^c*PolyLog[3
, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*b^2*E^c*P
olyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 4*a
^2*E^(3*c)*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)]))] - 2*b^2*E^(3*c)*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)]))] - 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*a^2*E^(3*c)*PolyLog[3, -((a*E^(2*c +
 d*x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*b^2*E^(3*c)*PolyLog[3, -
((a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])*(b + a*Sinh[c +
 d*x^2]))/(d^3*((a^2 + b^2)*E^(2*c))^(3/2)*(-1 + E^(2*c)))))/(12*a^2*(a + b
*Csch[c + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\ \mathrm{csch}\left(d\,x^2+c\right)\right)^2}dx$$

[In] int(x^5/(a+b*csch(d*x^2+c))^2,x)

[Out] int(x^5/(a+b*csch(d*x^2+c))^2,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3756 vs. 2(834) = 1668.

Time = 0.32 (sec) , antiderivative size = 3756, normalized size of antiderivative = 4.07

$$\int \frac{x^5}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*((a^5 + 2*a^3*b^2 + a*b^4)*d^3*x^6 + 6*(a^3*b^2 + a*b^4)*c^2 - ((a^5 +
 2*a^3*b^2 + a*b^4)*d^3*x^6 - 6*(a^3*b^2 + a*b^4)*d^2*x^4 + 6*(a^3*b^2 + a*
b^4)*c^2)*cosh(d*x^2 + c)^2 - ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*x^6 - 6*(a^3*b
^2 + a*b^4)*d^2*x^4 + 6*(a^3*b^2 + a*b^4)*c^2)*sinh(d*x^2 + c)^2 + 6*(2*a^4
*b + a^2*b^3 - (2*a^4*b + a^2*b^3)*cosh(d*x^2 + c)^2 - (2*a^4*b + a^2*b^3)*
sinh(d*x^2 + c)^2 - 2*(2*a^3*b^2 + a*b^4)*cosh(d*x^2 + c) - 2*(2*a^3*b^2 +
a*b^4 + (2*a^4*b + a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b
^2)/a^2)*polylog(3, (b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2
+ c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))/a) - 6*(2*a^4*b + a^2*b^3
- (2*a^4*b + a^2*b^3)*cosh(d*x^2 + c)^2 - (2*a^4*b + a^2*b^3)*sinh(d*x^2 +
c)^2 - 2*(2*a^3*b^2 + a*b^4)*cosh(d*x^2 + c) - 2*(2*a^3*b^2 + a*b^4 + (2*a^
4*b + a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2)*poly
log(3, (b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh
(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))/a) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^3*x
^6 - 3*(a^2*b^3 + b^5)*d^2*x^4 + 6*(a^2*b^3 + b^5)*c^2)*cosh(d*x^2 + c) + 6
*(a^3*b^2 + a*b^4 - (a^3*b^2 + a*b^4)*cosh(d*x^2 + c)^2 - (a^3*b^2 + a*b^4)
*sinh(d*x^2 + c)^2 - 2*(a^2*b^3 + b^5)*cosh(d*x^2 + c) - 2*(a^2*b^3 + b^5 +
 (a^3*b^2 + a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) + ((2*a^4*b + a^2*b^3)*
d*x^2*cosh(d*x^2 + c)^2 + (2*a^4*b + a^2*b^3)*d*x^2*sinh(d*x^2 + c)^2 + 2*(
2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c) - (2*a^4*b + a^2*b^3)*d*x^2 + 2*((
2*a^4*b + a^2*b^3)*d*x^2*cosh(d*x^2 + c) + (2*a^3*b^2 + a*b^4)*d*x^2)*sinh(
d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*dilog((b*cosh(d*x^2 + c) + b*sinh(d*x^2
+ c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a
 + 1) + 6*(a^3*b^2 + a*b^4 - (a^3*b^2 + a*b^4)*cosh(d*x^2 + c)^2 - (a^3*b^2
 + a*b^4)*sinh(d*x^2 + c)^2 - 2*(a^2*b^3 + b^5)*cosh(d*x^2 + c) - 2*(a^2*b^
3 + b^5 + (a^3*b^2 + a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) - ((2*a^4*b +
a^2*b^3)*d*x^2*cosh(d*x^2 + c)^2 + (2*a^4*b + a^2*b^3)*d*x^2*sinh(d*x^2 + c
)^2 + 2*(2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c) - (2*a^4*b + a^2*b^3)*d*x
^2 + 2*((2*a^4*b + a^2*b^3)*d*x^2*cosh(d*x^2 + c) + (2*a^3*b^2 + a*b^4)*d*x
^2)*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*dilog((b*cosh(d*x^2 + c) + b*si
nh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^
2) - a)/a + 1) + 3*(2*(a^3*b^2 + a*b^4)*c*cosh(d*x^2 + c)^2 + 2*(a^3*b^2 +
a*b^4)*c*sinh(d*x^2 + c)^2 + 4*(a^2*b^3 + b^5)*c*cosh(d*x^2 + c) - 2*(a^3*b
^2 + a*b^4)*c + 4*((a^3*b^2 + a*b^4)*c*cosh(d*x^2 + c) + (a^2*b^3 + b^5)*c)
*sinh(d*x^2 + c) - ((2*a^4*b + a^2*b^3)*c^2*cosh(d*x^2 + c)^2 + (2*a^4*b +
```

```
a^2*b^3)*c^2*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 + a*b^4)*c^2*cosh(d*x^2 + c)
- (2*a^4*b + a^2*b^3)*c^2 + 2*((2*a^4*b + a^2*b^3)*c^2*cosh(d*x^2 + c) + (2
*a^3*b^2 + a*b^4)*c^2)*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*log(2*a*cosh
(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 3*(2
*(a^3*b^2 + a*b^4)*c*cosh(d*x^2 + c)^2 + 2*(a^3*b^2 + a*b^4)*c*sinh(d*x^2 +
 c)^2 + 4*(a^2*b^3 + b^5)*c*cosh(d*x^2 + c) - 2*(a^3*b^2 + a*b^4)*c + 4*((a
^3*b^2 + a*b^4)*c*cosh(d*x^2 + c) + (a^2*b^3 + b^5)*c)*sinh(d*x^2 + c) + ((
2*a^4*b + a^2*b^3)*c^2*cosh(d*x^2 + c)^2 + (2*a^4*b + a^2*b^3)*c^2*sinh(d*x
^2 + c)^2 + 2*(2*a^3*b^2 + a*b^4)*c^2*cosh(d*x^2 + c) - (2*a^4*b + a^2*b^3)
*c^2 + 2*((2*a^4*b + a^2*b^3)*c^2*cosh(d*x^2 + c) + (2*a^3*b^2 + a*b^4)*c^2
)*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*log(2*a*cosh(d*x^2 + c) + 2*a*sin
h(d*x^2 + c) - 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 3*(2*(a^3*b^2 + a*b^4)*d*
x^2 - 2*((a^3*b^2 + a*b^4)*d*x^2 + (a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c)^2 -
 2*((a^3*b^2 + a*b^4)*d*x^2 + (a^3*b^2 + a*b^4)*c)*sinh(d*x^2 + c)^2 + 2*(a
^3*b^2 + a*b^4)*c - 4*((a^2*b^3 + b^5)*d*x^2 + (a^2*b^3 + b^5)*c)*cosh(d*x^
2 + c) - 4*((a^2*b^3 + b^5)*d*x^2 + (a^2*b^3 + b^5)*c + ((a^3*b^2 + a*b^4)*
d*x^2 + (a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c) - ((2*a^4*b +
 a^2*b^3)*d^2*x^4 - (2*a^4*b + a^2*b^3)*c^2 - ((2*a^4*b + a^2*b^3)*d^2*x^4
- (2*a^4*b + a^2*b^3)*c^2)*cosh(d*x^2 + c)^2 - ((2*a^4*b + a^2*b^3)*d^2*x^4
 - (2*a^4*b + a^2*b^3)*c^2)*sinh(d*x^2 + c)^2 - 2*((2*a^3*b^2 + a*b^4)*d^2*
x^4 - (2*a^3*b^2 + a*b^4)*c^2)*cosh(d*x^2 + c) - 2*((2*a^3*b^2 + a*b^4)*d^2
*x^4 - (2*a^3*b^2 + a*b^4)*c^2 + ((2*a^4*b + a^2*b^3)*d^2*x^4 - (2*a^4*b +
a^2*b^3)*c^2)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*log(
-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2
 + c))*sqrt((a^2 + b^2)/a^2) - a)/a) + 3*(2*(a^3*b^2 + a*b^4)*d*x^2 - 2*((a
^3*b^2 + a*b^4)*d*x^2 + (a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c)^2 - 2*((a^3*b^
2 + a*b^4)*d*x^2 + (a^3*b^2 + a*b^4)*c)*sinh(d*x^2 + c)^2 + 2*(a^3*b^2 + a*
b^4)*c - 4*((a^2*b^3 + b^5)*d*x^2 + (a^2*b^3 + b^5)*c)*cosh(d*x^2 + c) - 4*
((a^2*b^3 + b^5)*d*x^2 + (a^2*b^3 + b^5)*c + ((a^3*b^2 + a*b^4)*d*x^2 + (a^
3*b^2 + a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c) + ((2*a^4*b + a^2*b^3)*d
^2*x^4 - (2*a^4*b + a^2*b^3)*c^2 - ((2*a^4*b + a^2*b^3)*d^2*x^4 - (2*a^4*b
+ a^2*b^3)*c^2)*cosh(d*x^2 + c)^2 - ((2*a^4*b + a^2*b^3)*d^2*x^4 - (2*a^4*b
 + a^2*b^3)*c^2)*sinh(d*x^2 + c)^2 - 2*((2*a^3*b^2 + a*b^4)*d^2*x^4 - (2*a^
3*b^2 + a*b^4)*c^2)*cosh(d*x^2 + c) - 2*((2*a^3*b^2 + a*b^4)*d^2*x^4 - (2*a
^3*b^2 + a*b^4)*c^2 + ((2*a^4*b + a^2*b^3)*d^2*x^4 - (2*a^4*b + a^2*b^3)*c^
2)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*log(-(b*cosh(d*
x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt
((a^2 + b^2)/a^2) - a)/a) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^3*x^6 - 3*(a^2*b
^3 + b^5)*d^2*x^4 + 6*(a^2*b^3 + b^5)*c^2 + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*
x^6 - 6*(a^3*b^2 + a*b^4)*d^2*x^4 + 6*(a^3*b^2 + a*b^4)*c^2)*cosh(d*x^2 + c
))*sinh(d*x^2 + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d^3*cosh(d*x^2 + c)^2 + (a
^7 + 2*a^5*b^2 + a^3*b^4)*d^3*sinh(d*x^2 + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^
2*b^5)*d^3*cosh(d*x^2 + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^3 + 2*((a^7 + 2*
a^5*b^2 + a^3*b^4)*d^3*cosh(d*x^2 + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d^3)
*sinh(d*x^2 + c))
```

Sympy [F]

$$\int \frac{x^5}{(a + b\text{csch}\,(c + dx^2))^2}\,dx = \int \frac{x^5}{(a + b\,\text{csch}\,(c + dx^2))^2}\,dx$$

```
[In] integrate(x**5/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**5/(a + b*csch(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{(a + b\text{csch}\,(c + dx^2))^2}\,dx = \int \frac{x^5}{(b\,\text{csch}\,(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] -1/6*((a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^6*e^(2*d*x^2) - 6*a*b^2*x^4 - (a^
3*d + a*b^2*d)*x^6 + 2*(3*b^3*x^4*e^c + (a^2*b*d*e^c + b^3*d*e^c)*x^6)*e^(d
*x^2))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2)
 - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*x^2)) - integrate(2*(2*a*b^2*x^2 -
(2*b^3*x^2*e^c + (2*a^2*b*d*e^c + b^3*d*e^c)*x^4)*e^(d*x^2))*x/(a^5*d + a^3
*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2) - 2*(a^4*b*d*e^c +
 a^2*b^3*d*e^c)*e^(d*x^2)), x)
```

Giac [F]

$$\int \frac{x^5}{(a + b\text{csch}\,(c + dx^2))^2}\,dx = \int \frac{x^5}{(b\,\text{csch}\,(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*csch(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a + b\text{csch}\,(c + dx^2))^2}\,dx = \int \frac{x^5}{\left(a + \frac{b}{\sinh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^5/(a + b/sinh(c + d*x^2))^2,x)
[Out] int(x^5/(a + b/sinh(c + d*x^2))^2, x)
```

3.24 $\int \frac{x^4}{\left(a+b\,\mathbf{csch}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result 157
Rubi [N/A] 157
Mathematica [N/A] 158
Maple [N/A] (verified) 158
Fricas [N/A] 158
Sympy [N/A] 158
Maxima [N/A] 159
Giac [N/A] 159
Mupad [N/A] 159

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{x^4}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^4}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[x^4/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4/(a + b*Csch[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x^4}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\text{csch}(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^4/(a + b*Csch[c + d*x^2])^2,x]
[Out] Integrate[x^4/(a + b*Csch[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+b\ \text{csch}(d\,x^2+c))^2}dx$$

```
[In] int(x^4/(a+b*csch(d*x^2+c))^2,x)
[Out] int(x^4/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\,\text{csch}(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^4/(b^2*csch(d*x^2 + c)^2 + 2*a*b*csch(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\,\text{csch}(c+dx^2))^2}\,dx$$

```
[In] integrate(x**4/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**4/(a + b*csch(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 16.89

$$\int \frac{x^4}{(a + b\text{csch}(c + dx^2))^2} dx = \int \frac{x^4}{(b\,\text{csch}(dx^2 + c) + a)^2} dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -1/5*((a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^5*e^(2*d*x^2) - 5*a*b^2*x^3 - (a^
3*d + a*b^2*d)*x^5 + (5*b^3*x^3*e^c + 2*(a^2*b*d*e^c + b^3*d*e^c)*x^5)*e^(d
*x^2))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2)
 - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*x^2)) - integrate((3*a*b^2*x^2 - (3
*b^3*x^2*e^c + 2*(2*a^2*b*d*e^c + b^3*d*e^c)*x^4)*e^(d*x^2))/(a^5*d + a^3*b
^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2) - 2*(a^4*b*d*e^c + a
^2*b^3*d*e^c)*e^(d*x^2)), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a + b\text{csch}(c + dx^2))^2} dx = \int \frac{x^4}{(b\,\text{csch}(dx^2 + c) + a)^2} dx$$

```
[In] integrate(x^4/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*csch(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a + b\text{csch}(c + dx^2))^2} dx = \int \frac{x^4}{\left(a + \frac{b}{\sinh(d\,x^2+c)}\right)^2} dx$$

```
[In] int(x^4/(a + b/sinh(c + d*x^2))^2,x)
```

```
[Out] int(x^4/(a + b/sinh(c + d*x^2))^2, x)
```

3.25 $\int \frac{x^3}{\left(a+b\,\text{csch}(c+dx^2)\right)^2}\,dx$

Optimal result 160
Rubi [A] (verified) 161
Mathematica [A] (verified) 165
Maple [F] 166
Fricas [B] (verification not implemented) 166
Sympy [F] 168
Maxima [F] 168
Giac [F] 168
Mupad [F(-1)] 169

Optimal result

Integrand size = 18, antiderivative size = 519

$$\begin{aligned}\int \frac{x^3}{(a+b\,\text{csch}(c+dx^2))^2}\,dx &= \frac{x^4}{4a^2}+\frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}\\
&-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}-\frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d}\\
&+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{b^2\log(b+a\sinh(c+dx^2))}{2a^2(a^2+b^2)d^2}\\
&+\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\\
&-\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d^2}+\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\\
&-\frac{b^2x^2\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}\end{aligned}$$

```
[Out] 1/4*x^4/a^2+1/2*b^2*ln(b+a*sinh(d*x^2+c))/a^2/(a^2+b^2)/d^2+1/2*b^3*x^2*ln(
1+a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-1/2*b^3*x^2*ln(
1+a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+1/2*b^3*polylog
(2,-a*exp(d*x^2+c)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-1/2*b^3*pol
ylog(2,-a*exp(d*x^2+c)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-1/2*b^2
*x^2*cosh(d*x^2+c)/a/(a^2+b^2)/d/(b+a*sinh(d*x^2+c))-b*x^2*ln(1+a*exp(d*x^2
+c)/(b-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+b*x^2*ln(1+a*exp(d*x^2+c)/(b
+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)-b*polylog(2,-a*exp(d*x^2+c)/(b-(a^
```

```
2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+b*polylog(2,-a*exp(d*x^2+c)/(b+(a^2+
b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5545, 4276, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{x^3}{(a+b\text{csch}(c+dx^2))^2}dx = -\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{b^2\log(a\sinh(c+dx^2)+b)}{2a^2d^2(a^2+b^2)}-\frac{bx^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}+1\right)}{a^2d\sqrt{a^2+b^2}}+\frac{bx^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{a^2+b^2}+b}+1\right)}{a^2d\sqrt{a^2+b^2}}-\frac{b^2x^2\cosh(c+dx^2)}{2ad(a^2+b^2)(a\sinh(c+dx^2)+b)}+\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{2a^2d^2(a^2+b^2)^{3/2}}-\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{2a^2d^2(a^2+b^2)^{3/2}}+\frac{b^3x^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}+1\right)}{2a^2d(a^2+b^2)^{3/2}}-\frac{b^3x^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{a^2+b^2}+b}+1\right)}{2a^2d(a^2+b^2)^{3/2}}+\frac{x^4}{4a^2}$$

```
[In] Int[x^3/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] x^4/(4*a^2) + (b^3*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])])/(2
*a^2*(a^2 + b^2)^(3/2)*d) - (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2
+ b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^3*x^2*Log[1 + (a*E^(c + d*x^2))/(b +
 Sqrt[a^2 + b^2])])/(2*a^2*(a^2 + b^2)^(3/2)*d) + (b*x^2*Log[1 + (a*E^(c +
d*x^2))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (b^2*Log[b + a*Si
nh[c + d*x^2]])/(2*a^2*(a^2 + b^2)*d^2) + (b^3*PolyLog[2, -((a*E^(c + d*x^2
))/(b - Sqrt[a^2 + b^2]))])/(2*a^2*(a^2 + b^2)^(3/2)*d^2) - (b*PolyLog[2, -
((a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (b^
3*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(2*a^2*(a^2 + b^2
)^(3/2)*d^2) + (b*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2]))])/(
a^2*Sqrt[a^2 + b^2]*d^2) - (b^2*x^2*Cosh[c + d*x^2])/(2*a*(a^2 + b^2)*d*(b
+ a*Sinh[c + d*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2296</u>

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

<u>Rule 2747</u>

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

<u>Rule 3403</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

<u>Rule 3405</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
```

```
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\text{csch}(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx}{a^2(b+a\sinh(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a^2}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x}{(b+a\sinh(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^4}{4a^2}-\frac{b^2x^2\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)}-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad+\frac{b^3\text{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2+b^2\right)}+\frac{b^2\text{Subst}\left(\int \frac{\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, x^2\right)}{2a\left(a^2+b^2\right)d}\\
&= \frac{x^4}{4a^2}-\frac{b^2x^2\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)}+\frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2\left(a^2+b^2\right)}\\
&\quad-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{b^2\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\sinh\left(c+dx^2\right)\right)}{2a^2\left(a^2+b^2\right)d^2}
\end{aligned}
$$

$$=\frac{x^4}{4a^2}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+\frac{b^2\log\left(b+a\sinh\left(c+dx^2\right)\right)}{2a^2\left(a^2+b^2\right)d^2}-\frac{b^2x^2\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)}$$
$$+\frac{b^3\text{Subst}\left(\int\frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{a\left(a^2+b^2\right)^{3/2}}$$
$$-\frac{b^3\text{Subst}\left(\int\frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{a\left(a^2+b^2\right)^{3/2}}$$
$$+\frac{b\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{a^2+b^2}d}$$
$$-\frac{b\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{a^2+b^2}d}$$

$$=\frac{x^4}{4a^2}+\frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$-\frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2\left(a^2+b^2\right)^{3/2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{b^2\log\left(b+a\sinh\left(c+dx^2\right)\right)}{2a^2\left(a^2+b^2\right)d^2}$$
$$-\frac{b^2x^2\cosh\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+dx^2\right)\right)}+\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^2}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$-\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^2}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$-\frac{b^3\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{2a^2\left(a^2+b^2\right)^{3/2}d}$$
$$+\frac{b^3\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^2\right)}{2a^2\left(a^2+b^2\right)^{3/2}d}$$

$$
\begin{aligned}
&= \frac{x^4}{4a^2} + \frac{b^3 x^2 \log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} - \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&\quad - \frac{b^3 x^2 \log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} + \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{b^2 \log(b+a\sinh(c+dx^2))}{2a^2(a^2+b^2)d^2} \\
&\quad - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&\quad - \frac{b^2x^2\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))} - \frac{b^3\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{2a^2(a^2+b^2)^{3/2}d^2} \\
&\quad + \frac{b^3\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{2a^2(a^2+b^2)^{3/2}d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^4}{4a^2} + \frac{b^3 x^2 \log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} - \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&\quad - \frac{b^3 x^2 \log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} + \frac{bx^2 \log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&\quad + \frac{b^2 \log(b+a\sinh(c+dx^2))}{2a^2(a^2+b^2)d^2} + \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d^2} \\
&\quad - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d^2} \\
&\quad + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{b^2x^2\cosh(c+dx^2)}{2a(a^2+b^2)d(b+a\sinh(c+dx^2))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.42

$$
\int \frac{x^3}{(a+b\text{csch}(c+dx^2))^2}\,dx
$$

$$
= \frac{\text{csch}^2(c+dx^2)(b+a\sinh(c+dx^2))\left(-\frac{2ab^2dx^2\cosh(c+dx^2)}{a^2+b^2}+(-c+dx^2)(c+dx^2)(b+a\sinh(c+dx^2))-\right.}{}
$$

```
[In] Integrate[x^3/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (Csch[c + d*x^2]^2*(b + a*Sinh[c + d*x^2])*((-2*a*b^2*d*x^2*Cosh[c + d*x^2]
)/(a^2 + b^2) + (-c + d*x^2)*(c + d*x^2)*(b + a*Sinh[c + d*x^2]) - (2*b*(a^
2 + b^2)*(-(b*Sqrt[-(a^2 + b^2)^2]*(c + d*x^2)) + 2*b^2*Sqrt[a^2 + b^2]*Arc
Tan[(b + a*E^(c + d*x^2))/Sqrt[-a^2 - b^2]] + 2*b^2*Sqrt[-a^2 - b^2]*ArcTan
h[(b + a*E^(c + d*x^2))/Sqrt[a^2 + b^2]] - 4*a^2*Sqrt[-a^2 - b^2]*c*ArcTanh
[(b + a*E^(c + d*x^2))/Sqrt[a^2 + b^2]] - 2*b^2*Sqrt[-a^2 - b^2]*c*ArcTanh[
(b + a*E^(c + d*x^2))/Sqrt[a^2 + b^2]] - 2*a^2*Sqrt[-a^2 - b^2]*(c + d*x^2)
*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])] - b^2*Sqrt[-a^2 - b^2]*(c
 + d*x^2)*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[a^2 + b^2])] + 2*a^2*Sqrt[-a^
2 - b^2]*(c + d*x^2)*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2])] + b^2
*Sqrt[-a^2 - b^2]*(c + d*x^2)*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[a^2 + b^2
])] + b*Sqrt[-(a^2 + b^2)^2]*Log[2*b*E^(c + d*x^2) + a*(-1 + E^(2*(c + d*x^
2)))] - Sqrt[-a^2 - b^2]*(2*a^2 + b^2)*PolyLog[2, (a*E^(c + d*x^2))/(-b + S
qrt[a^2 + b^2])] + Sqrt[-a^2 - b^2]*(2*a^2 + b^2)*PolyLog[2, -((a*E^(c + d*
x^2))/(b + Sqrt[a^2 + b^2]))])*(b + a*Sinh[c + d*x^2]))/(-(a^2 + b^2)^2)^(3
/2)))/(4*a^2*d^2*(a + b*Csch[c + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\,\mathrm{csch}\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(x^3/(a+b*csch(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(461) = 922$.

Time = 0.31 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.59

$$\int \frac{x^3}{\left(a+b\mathrm{csch}\left(c+dx^2\right)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*((a^5 + 2*a^3*b^2 + a*b^4)*d^2*x^4 - ((a^5 + 2*a^3*b^2 + a*b^4)*d^2*x^
4 - 4*(a^3*b^2 + a*b^4)*d*x^2 - 4*(a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c)^2 -
((a^5 + 2*a^3*b^2 + a*b^4)*d^2*x^4 - 4*(a^3*b^2 + a*b^4)*d*x^2 - 4*(a^3*b^2
 + a*b^4)*c)*sinh(d*x^2 + c)^2 - 2*(2*a^4*b + a^2*b^3 - (2*a^4*b + a^2*b^3)
*cosh(d*x^2 + c)^2 - (2*a^4*b + a^2*b^3)*sinh(d*x^2 + c)^2 - 2*(2*a^3*b^2 +
 a*b^4)*cosh(d*x^2 + c) - 2*(2*a^3*b^2 + a*b^4 + (2*a^4*b + a^2*b^3)*cosh(d
*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2)*dilog((b*cosh(d*x^2 + c)
+ b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b
^2)/a^2) - a)/a + 1) + 2*(2*a^4*b + a^2*b^3 - (2*a^4*b + a^2*b^3)*cosh(d*x^
```

```
2 + c)^2 - (2*a^4*b + a^2*b^3)*sinh(d*x^2 + c)^2 - 2*(2*a^3*b^2 + a*b^4)*co
sh(d*x^2 + c) - 2*(2*a^3*b^2 + a*b^4 + (2*a^4*b + a^2*b^3)*cosh(d*x^2 + c))
*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2)*dilog((b*cosh(d*x^2 + c) + b*sinh(d
*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) -
 a)/a + 1) - 2*((2*a^4*b + a^2*b^3)*d*x^2 - ((2*a^4*b + a^2*b^3)*d*x^2 + (2
*a^4*b + a^2*b^3)*c)*cosh(d*x^2 + c)^2 - ((2*a^4*b + a^2*b^3)*d*x^2 + (2*a^
4*b + a^2*b^3)*c)*sinh(d*x^2 + c)^2 + (2*a^4*b + a^2*b^3)*c - 2*((2*a^3*b^2
 + a*b^4)*d*x^2 + (2*a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c) - 2*((2*a^3*b^2 +
a*b^4)*d*x^2 + (2*a^3*b^2 + a*b^4)*c + ((2*a^4*b + a^2*b^3)*d*x^2 + (2*a^4*
b + a^2*b^3)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2)*log
(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^
2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a) + 2*((2*a^4*b + a^2*b^3)*d*x^2 - ((2*
a^4*b + a^2*b^3)*d*x^2 + (2*a^4*b + a^2*b^3)*c)*cosh(d*x^2 + c)^2 - ((2*a^4
*b + a^2*b^3)*d*x^2 + (2*a^4*b + a^2*b^3)*c)*sinh(d*x^2 + c)^2 + (2*a^4*b +
 a^2*b^3)*c - 2*((2*a^3*b^2 + a*b^4)*d*x^2 + (2*a^3*b^2 + a*b^4)*c)*cosh(d*
x^2 + c) - 2*((2*a^3*b^2 + a*b^4)*d*x^2 + (2*a^3*b^2 + a*b^4)*c + ((2*a^4*b
 + a^2*b^3)*d*x^2 + (2*a^4*b + a^2*b^3)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c)
)*sqrt((a^2 + b^2)/a^2)*log(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*co
sh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2) - a)/a) - 4*(a^3*b
^2 + a*b^4)*c - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*x^4 - 2*(a^2*b^3 + b^5)*d*
x^2 - 4*(a^2*b^3 + b^5)*c)*cosh(d*x^2 + c) + 2*(a^3*b^2 + a*b^4 - (a^3*b^2
+ a*b^4)*cosh(d*x^2 + c)^2 - (a^3*b^2 + a*b^4)*sinh(d*x^2 + c)^2 - 2*(a^2*b
^3 + b^5)*cosh(d*x^2 + c) - 2*(a^2*b^3 + b^5 + (a^3*b^2 + a*b^4)*cosh(d*x^2
 + c))*sinh(d*x^2 + c) + ((2*a^4*b + a^2*b^3)*c*cosh(d*x^2 + c)^2 + (2*a^4*
b + a^2*b^3)*c*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 + a*b^4)*c*cosh(d*x^2 + c)
- (2*a^4*b + a^2*b^3)*c + 2*((2*a^4*b + a^2*b^3)*c*cosh(d*x^2 + c) + (2*a^3
*b^2 + a*b^4)*c)*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2))*log(2*a*cosh(d*x^2
 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 2*(a^3*b^2
 + a*b^4 - (a^3*b^2 + a*b^4)*cosh(d*x^2 + c)^2 - (a^3*b^2 + a*b^4)*sinh(d*x
^2 + c)^2 - 2*(a^2*b^3 + b^5)*cosh(d*x^2 + c) - 2*(a^2*b^3 + b^5 + (a^3*b^2
 + a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) - ((2*a^4*b + a^2*b^3)*c*cosh(d*
x^2 + c)^2 + (2*a^4*b + a^2*b^3)*c*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 + a*b^4
)*c*cosh(d*x^2 + c) - (2*a^4*b + a^2*b^3)*c + 2*((2*a^4*b + a^2*b^3)*c*cosh
(d*x^2 + c) + (2*a^3*b^2 + a*b^4)*c)*sinh(d*x^2 + c))*sqrt((a^2 + b^2)/a^2)
)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) - 2*a*sqrt((a^2 + b^2)/a^2)
 + 2*b) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*x^4 - 2*(a^2*b^3 + b^5)*d*x^2 -
4*(a^2*b^3 + b^5)*c + ((a^5 + 2*a^3*b^2 + a*b^4)*d^2*x^4 - 4*(a^3*b^2 + a*b
^4)*d*x^2 - 4*(a^3*b^2 + a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c))/((a^7
+ 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x^2 + c)^2 + (a^7 + 2*a^5*b^2 + a^3*b^4)*
d^2*sinh(d*x^2 + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d^2*cosh(d*x^2 + c)
 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2 + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cos
h(d*x^2 + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d^2)*sinh(d*x^2 + c))
```

Sympy [F]

$$\int \frac{x^3}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = \int \frac{x^3}{(a+b\,\text{csch}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x**3/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**3/(a + b*csch(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = \int \frac{x^3}{(b\,\text{csch}\,(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] -4*a^2*b*d*integrate(x^3*e^(d*x^2 + c)/(a^5*d*e^(2*d*x^2 + 2*c) + a^3*b^2*d
*e^(2*d*x^2 + 2*c) + 2*a^4*b*d*e^(d*x^2 + c) + 2*a^2*b^3*d*e^(d*x^2 + c) -
a^5*d - a^3*b^2*d), x) - 2*b^3*d*integrate(x^3*e^(d*x^2 + c)/(a^5*d*e^(2*d*
x^2 + 2*c) + a^3*b^2*d*e^(2*d*x^2 + 2*c) + 2*a^4*b*d*e^(d*x^2 + c) + 2*a^2*
b^3*d*e^(d*x^2 + c) - a^5*d - a^3*b^2*d), x) + 1/2*a*b^2*(b*log((a*e^(d*x^2
 + c) + b - sqrt(a^2 + b^2))/(a*e^(d*x^2 + c) + b + sqrt(a^2 + b^2)))/((a^5
 + a^3*b^2)*sqrt(a^2 + b^2)*d^2) - 2*(d*x^2 + c)/((a^5 + a^3*b^2)*d^2) + lo
g(a*e^(2*d*x^2 + 2*c) + 2*b*e^(d*x^2 + c) - a)/((a^5 + a^3*b^2)*d^2)) - 1/2
*b^3*log((a*e^(d*x^2 + c) + b - sqrt(a^2 + b^2))/(a*e^(d*x^2 + c) + b + sqr
t(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d^2) - 1/4*((a^3*d*e^(2*c)
+ a*b^2*d*e^(2*c))*x^4*e^(2*d*x^2) - 4*a*b^2*x^2 - (a^3*d + a*b^2*d)*x^4 +
2*(2*b^3*x^2*e^c + (a^2*b*d*e^c + b^3*d*e^c)*x^4)*e^(d*x^2))/(a^5*d + a^3*b
^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2) - 2*(a^4*b*d*e^c + a
^2*b^3*d*e^c)*e^(d*x^2))
```

Giac [F]

$$\int \frac{x^3}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = \int \frac{x^3}{(b\,\text{csch}\,(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*csch(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = \int \frac{x^3}{\left(a+\frac{b}{\sinh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/sinh(c + d*x^2))^2,x)
[Out] int(x^3/(a + b/sinh(c + d*x^2))^2, x)
```

3.26 $\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result 170
Rubi [N/A] 170
Mathematica [N/A] 171
Maple [N/A] (verified) 171
Fricas [N/A] 171
Sympy [N/A] 171
Maxima [N/A] 172
Giac [N/A] 172
Mupad [N/A] 172

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = \operatorname{Int}\left(\frac{x^2}{(a+b\operatorname{csch}(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx$$

```
[In] Int[x^2/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2/(a + b*Csch[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^2/(a + b*Csch[c + d*x^2])^2,x]
[Out] Integrate[x^2/(a + b*Csch[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\ \text{csch}(d\,x^2+c))^2}dx$$

```
[In] int(x^2/(a+b*csch(d*x^2+c))^2,x)
[Out] int(x^2/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx=\int \frac{x^2}{(b\,\text{csch}(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*csch(d*x^2 + c)^2 + 2*a*b*csch(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\,\text{csch}(c+dx^2))^2}\,dx$$

```
[In] integrate(x**2/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(x**2/(a + b*csch(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 292, normalized size of antiderivative = 16.22

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\,\text{csch}(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] -1/3*((a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^3*e^(2*d*x^2) - 3*a*b^2*x - (a^3*
d + a*b^2*d)*x^3 + (3*b^3*x*e^c + 2*(a^2*b*d*e^c + b^3*d*e^c)*x^3)*e^(d*x^2
))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2) - 2
*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*x^2)) - integrate((a*b^2 - (b^3*e^c + 2
*(2*a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*x^2))/(a^5*d + a^3*b^2*d - (a^5*d*e^
(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*x^2) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^
(d*x^2)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\,\text{csch}(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*csch(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+b\text{csch}(c+dx^2))^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\sinh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^2/(a + b/sinh(c + d*x^2))^2,x)
[Out] int(x^2/(a + b/sinh(c + d*x^2))^2, x)
```

3.27 $\int \frac{x}{\left(a+b\,\mathrm{csch}\left(c+dx^2\right)\right)^2} \, dx$

Optimal result 173
Rubi [A] (verified) 173
Mathematica [A] (verified) 175
Maple [A] (verified) 176
Fricas [B] (verification not implemented) 176
Sympy [F] 177
Maxima [A] (verification not implemented) 177
Giac [A] (verification not implemented) 178
Mupad [B] (verification not implemented) 178

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x}{(a+b\,\mathrm{csch}(c+dx^2))^2} \, dx = \frac{x^2}{2a^2} + \frac{b(2a^2+b^2)\,\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d} - \frac{b^2\coth(c+dx^2)}{2a(a^2+b^2)\,d\,(a+b\,\mathrm{csch}(c+dx^2))}$$

```
[Out] 1/2*x^2/a^2+b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*d*x^2+1/2*c))/(a^2+b^2)^(1/
2))/a^2/(a^2+b^2)^(3/2)/d-1/2*b^2*coth(d*x^2+c)/a/(a^2+b^2)/d/(a+b*csch(d*x
^2+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5545, 3870, 4004, 3916, 2739, 632, 210}

$$\int \frac{x}{(a+b\,\mathrm{csch}(c+dx^2))^2} \, dx = \frac{b(2a^2+b^2)\,\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d\,(a^2+b^2)^{3/2}} - \frac{b^2\coth(c+dx^2)}{2ad\,(a^2+b^2)\,(a+b\,\mathrm{csch}(c+dx^2))} + \frac{x^2}{2a^2}$$

```
[In] Int[x/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] x^2/(2*a^2) + (b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x^2)/2])/Sqrt[a^2
 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x^2])/(2*a*(a^2 + b^2
)*d*(a + b*Csch[c + d*x^2]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
```

```
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx, x, x^2\right)\\
&= -\frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)} - \frac{\text{Subst}\left(\int \frac{-a^2-b^2+ab\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx, x, x^2\right)}{2a\left(a^2+b^2\right)}\\
&= \frac{x^2}{2a^2} - \frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)} - \frac{\left(b(2a^2+b^2)\right)\text{Subst}\left(\int \frac{\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2+b^2\right)}\\
&= \frac{x^2}{2a^2} - \frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)} - \frac{\left(2a^2+b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx, x, x^2\right)}{2a^2\left(a^2+b^2\right)}\\
&= \frac{x^2}{2a^2} - \frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)}\\
&\quad + \frac{\left(i(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{a^2\left(a^2+b^2\right)d}\\
&= \frac{x^2}{2a^2} - \frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)}\\
&\quad - \frac{\left(2i(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2i\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{a^2\left(a^2+b^2\right)d}\\
&= \frac{x^2}{2a^2} + \frac{b(2a^2+b^2)\,\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{b^2\coth\left(c+dx^2\right)}{2a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+dx^2\right)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.42

$$
\int \frac{x}{\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx = \frac{\text{csch}(c+dx^2)\left(-\frac{ab^2\coth(c+dx^2)}{a^2+b^2}+\left(c+dx^2\right)\left(a+b\text{csch}(c+dx^2)\right)+\frac{2b(2a^2+b^2)\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{-a^2-b^2}}\right)\left(a+b\text{cs}\right.}{\left(-a^2-b^2\right)^{3/2}}\right.}{2a^2d\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}
$$

```
[In] Integrate[x/(a + b*Csch[c + d*x^2])^2,x]
```

```
[Out] (Csch[c + d*x^2]*(-((a*b^2*Coth[c + d*x^2])/(a^2 + b^2)) + (c + d*x^2)*(a +
 b*Csch[c + d*x^2]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x^2)/2])
/Sqrt[-a^2 - b^2]]*(a + b*Csch[c + d*x^2]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh
[c + d*x^2]))/(2*a^2*d*(a + b*Csch[c + d*x^2])^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.67

method	result
derivativedivides	$-\frac{\frac{2b\left(\frac{\frac{a^2\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b}{2}+a\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\cdots}{2d}$
default	$-\frac{\frac{2b\left(\frac{\frac{a^2\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b}{2}+a\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\cdots}{2d}$
risch	$\frac{x^2}{2a^2}-\frac{b^2\left(-e^{dx^2+c}b+a\right)}{a^2(a^2+b^2)d\left(e^{2dx^2+2c}a+2e^{dx^2+c}b-a\right)}+\frac{b\ln\left(e^{dx^2+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}+\frac{b^3\ln\left(e^{dx^2+c}+\cdots\right)}{2(a^2+\cdots}$

```
[In] int(x/(a+b*csch(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2/a^2*b*((1/2*a^2/(a^2+b^2)*tanh(1/2*d*x^2+1/2*c)+1/2*b*a/(a^2+b^2)
)/(-1/2*tanh(1/2*d*x^2+1/2*c)^2*b+a*tanh(1/2*d*x^2+1/2*c)+1/2*b)-2*(2*a^2+b
^2)/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x^2+1/2*c)+2
*a)/(a^2+b^2)^(1/2)))-1/a^2*ln(tanh(1/2*d*x^2+1/2*c)-1)+1/a^2*ln(1+tanh(1/2
*d*x^2+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(106) = 212.

Time = 0.27 (sec) , antiderivative size = 711, normalized size of antiderivative = 6.29

$$\int \frac{x}{\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$
$$=\frac{(a^5+2a^3b^2+ab^4)dx^2\cosh\left(dx^2+c\right)^2+(a^5+2a^3b^2+ab^4)dx^2\sinh\left(dx^2+c\right)^2-2a^3b^2-2ab^4-(a^5+2a\cdots}{\cdots}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^5 + 2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c)^2 + (a^5 + 2*a^3*b^2 +
 a*b^4)*d*x^2*sinh(d*x^2 + c)^2 - 2*a^3*b^2 - 2*a*b^4 - (a^5 + 2*a^3*b^2 +
a*b^4)*d*x^2 - (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x^2 + c)^2 - (2*
a^3*b + a*b^3)*sinh(d*x^2 + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x^2 + c) - 2*
(2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt
(a^2 + b^2)*log((a^2*cosh(d*x^2 + c)^2 + a^2*sinh(d*x^2 + c)^2 + 2*a*b*cosh
(d*x^2 + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x^2 + c) + a*b)*sinh(d*x^2 + c) +
 2*sqrt(a^2 + b^2)*(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b))/(a*cosh(d*x
^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cosh(d*x^2 + c) + 2*(a*cosh(d*x^2 + c
) + b)*sinh(d*x^2 + c) - a)) + 2*(a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)
*d*x^2)*cosh(d*x^2 + c) + 2*(a^2*b^3 + b^5 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x^
2*cosh(d*x^2 + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*x^2)*sinh(d*x^2 + c))/((a^7
 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2 + c)^2 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d
*sinh(d*x^2 + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x^2 + c) - (a
^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2 +
 c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x^2 + c))
```

Sympy [F]

$$\int \frac{x}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = \int \frac{x}{(a+b\,\text{csch}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x/(a+b*csch(d*x**2+c))**2,x)
```

```
[Out] Integral(x/(a + b*csch(c + d*x**2))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.77

$$\int \frac{x}{(a+b\text{csch}\,(c+dx^2))^2}\,dx = -\frac{(2\,a^2b+b^3)\log\left(\frac{ae^{(-dx^2-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx^2-c)}-b+\sqrt{a^2+b^2}}\right)}{2\,(a^4+a^2b^2)\sqrt{a^2+b^2}d} - \frac{b^3e^{(-dx^2-c)}+ab^2}{(a^5+a^3b^2+2\,(a^4b+a^2b^3)e^{(-dx^2-c)}-(a^5+a^3b^2)e^{(-2\,dx^2-2\,c)})d} + \frac{dx^2+c}{2\,a^2d}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*b + b^3)*log((a*e^(-d*x^2 - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*
x^2 - c) - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - (b^3
*e^(-d*x^2 - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*e^(-d*x^2 -
c) - (a^5 + a^3*b^2)*e^(-2*d*x^2 - 2*c))*d) + 1/2*(d*x^2 + c)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

$$\int \frac{x}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = -\frac{(2\,a^2b+b^3)\log\left(\frac{\left|2\,ae^{\left(dx^2+c\right)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{\left(dx^2+c\right)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^4d+a^2b^2d)\sqrt{a^2+b^2}} + \frac{b^3e^{(dx^2+c)}-ab^2}{(a^4d+a^2b^2d)(ae^{(2\,dx^2+2\,c)}+2\,be^{(dx^2+c)}-a)} + \frac{dx^2+c}{2\,a^2d}$$

```
[In] integrate(x/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*b + b^3)*log(abs(2*a*e^(d*x^2 + c) + 2*b - 2*sqrt(a^2 + b^2))/a
bs(2*a*e^(d*x^2 + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4*d + a^2*b^2*d)*sqrt(
a^2 + b^2)) + (b^3*e^(d*x^2 + c) - a*b^2)/((a^4*d + a^2*b^2*d)*(a*e^(2*d*x^
2 + 2*c) + 2*b*e^(d*x^2 + c) - a)) + 1/2*(d*x^2 + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.57

$$\int \frac{x}{(a+b\operatorname{csch}(c+dx^2))^2}\,dx = \frac{x^2}{2\,a^2} - \frac{\frac{b^2}{d\,(a^3+a\,b^2)} - \frac{b^3\,\mathrm{e}^{d\,x^2+c}}{a\,d\,(a^3+a\,b^2)}}{2\,b\,\mathrm{e}^{d\,x^2+c}-a+a\,\mathrm{e}^{2\,d\,x^2+2\,c}} - \frac{b\,\ln\left(\frac{2\,b\,x\,\mathrm{e}^{d\,x^2+c}\,(2\,a^2+b^2)}{a^3\,(a^2+b^2)} - \frac{2\,b\,x\,(2\,a^2+b^2)\,\left(a-b\,\mathrm{e}^{d\,x^2+c}\right)}{a^3\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+b^2)}{2\,a^2\,d\,(a^2+b^2)^{3/2}} + \frac{b\,\ln\left(\frac{2\,b\,x\,\mathrm{e}^{d\,x^2+c}\,(2\,a^2+b^2)}{a^3\,(a^2+b^2)} + \frac{2\,b\,x\,(2\,a^2+b^2)\,\left(a-b\,\mathrm{e}^{d\,x^2+c}\right)}{a^3\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+b^2)}{2\,a^2\,d\,(a^2+b^2)^{3/2}}$$

```
[In] int(x/(a + b/sinh(c + d*x^2))^2,x)
```

```
[Out] x^2/(2*a^2) - (b^2/(d*(a*b^2 + a^3)) - (b^3*exp(c + d*x^2))/(a*d*(a*b^2 + a
^3)))/(2*b*exp(c + d*x^2) - a + a*exp(2*c + 2*d*x^2)) - (b*log((2*b*x*exp(c
 + d*x^2)*(2*a^2 + b^2))/(a^3*(a^2 + b^2)) - (2*b*x*(2*a^2 + b^2)*(a - b*ex
p(c + d*x^2)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(2*a^2*d*(a^2 + b^2)
^(3/2)) + (b*log((2*b*x*exp(c + d*x^2)*(2*a^2 + b^2))/(a^3*(a^2 + b^2)) + (
2*b*x*(2*a^2 + b^2)*(a - b*exp(c + d*x^2)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2
 + b^2))/(2*a^2*d*(a^2 + b^2)^(3/2))
```

3.28 $\int \frac{1}{x\left(a+b\operatorname{csch}(c+dx^2)\right)^2}\,dx$

Optimal result 179
Rubi [N/A] 179
Mathematica [N/A] 180
Maple [N/A] (verified) 180
Fricas [N/A] 180
Sympy [N/A] 180
Maxima [N/A] 181
Giac [N/A] 181
Mupad [N/A] 181

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Csch[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x*(a + b*Csch[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 32.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csch[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Csch[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{csch}\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*csch(d*x^2+c))^2,x)
[Out] int(1/x/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*csch(d*x^2 + c)^2 + 2*a*b*x*csch(d*x^2 + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(1/(x*(a + b*csch(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 246, normalized size of antiderivative = 13.67

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (b^3*e^(d*x^2 + c) - a*b^2)/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^2*e^(2*d
*x^2) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^2*e^(d*x^2) - (a^5*d + a^3*b^2*d)
*x^2) + log(x)/a^2 - integrate(2*(a*b^2 - (b^3*e^c - (2*a^2*b*d*e^c + b^3*d
*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^3*e^(2*d*x^2)
+ 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^3*e^(d*x^2) - (a^5*d + a^3*b^2*d)*x^3),
 x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*csch(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\mathrm{sinh}\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/sinh(c + d*x^2))^2),x)
```

```
[Out] int(1/(x*(a + b/sinh(c + d*x^2))^2), x)
```

3.29 $\int \frac{1}{x^2\left(a+b\operatorname{csch}(c+dx^2)\right)^2}\,dx$

Optimal result 182
Rubi [N/A] 182
Mathematica [N/A] 183
Maple [N/A] (verified) 183
Fricas [N/A] 183
Sympy [N/A] 184
Maxima [N/A] 184
Giac [N/A] 184
Mupad [N/A] 185

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Csch[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Csch[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} dx$$

```
[In] Integrate[1/(x^2*(a + b*Csch[c + d*x^2])^2),x]
[Out] Integrate[1/(x^2*(a + b*Csch[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b\ \text{csch}(d\,x^2 + c))^2} dx$$

```
[In] int(1/x^2/(a+b*csch(d*x^2+c))^2,x)
[Out] int(1/x^2/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} dx = \int \frac{1}{(b\,\text{csch}(dx^2 + c) + a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*csch(d*x^2 + c)^2 + 2*a*b*x^2*csch(d*x^2 + c) + a^2*x^2
), x)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} \, dx = \int \frac{1}{x^2 (a + b \operatorname{csch}(c + dx^2))^2} \, dx$$

```
[In] integrate(1/x**2/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(1/(x**2*(a + b*csch(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 313, normalized size of antiderivative = 17.39

$$\int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} \, dx = \int \frac{1}{(b \operatorname{csch}(dx^2 + c) + a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] -((a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^2*e^(2*d*x^2) + a*b^2 - (a^3*d + a*b^
2*d)*x^2 - (b^3*e^c - 2*(a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e
^(2*c) + a^3*b^2*d*e^(2*c))*x^3*e^(2*d*x^2) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^
c)*x^3*e^(d*x^2) - (a^5*d + a^3*b^2*d)*x^3) - integrate((3*a*b^2 - (3*b^3*e
^c - 2*(2*a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) + a^3*b^
2*d*e^(2*c))*x^4*e^(2*d*x^2) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^4*e^(d*x^2
) - (a^5*d + a^3*b^2*d)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\text{csch}(c + dx^2))^2} \, dx = \int \frac{1}{(b \operatorname{csch}(dx^2 + c) + a)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*csch(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 \left(a+b\text{csch}\left(c+dx^2\right)\right)^2} dx = \int \frac{1}{x^2 \left(a+\frac{b}{\sinh(d x^2+c)}\right)^2} dx$$

```
[In] int(1/(x^2*(a + b/sinh(c + d*x^2))^2),x)
[Out] int(1/(x^2*(a + b/sinh(c + d*x^2))^2), x)
```

3.30 $\int \frac{1}{x^3\left(a+b\,\mathbf{csch}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result 186
Rubi [N/A] 186
Mathematica [N/A] 187
Maple [N/A] (verified) 187
Fricas [N/A] 187
Sympy [N/A] 188
Maxima [N/A] 188
Giac [N/A] 188
Mupad [N/A] 189

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*csch(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Csch[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Csch[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\text{csch}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\text{csch}(c + dx^2))^2} \, dx = \int \frac{1}{x^3 (a + b\text{csch}(c + dx^2))^2} \, dx$$

```
[In] Integrate[1/(x^3*(a + b*Csch[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Csch[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b\ \text{csch}(d\,x^2 + c))^2} dx$$

```
[In] int(1/x^3/(a+b*csch(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*csch(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (a + b\text{csch}(c + dx^2))^2} \, dx = \int \frac{1}{(b\,\text{csch}(dx^2 + c) + a)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^3*csch(d*x^2 + c)^2 + 2*a*b*x^3*csch(d*x^2 + c) + a^2*x^3
), x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \left(a+b\text{csch}\left(c+dx^2\right)\right)^2} \, dx=\int \frac{1}{x^3 \left(a+b\,\text{csch}\left(c+dx^2\right)\right)^2} \, dx$$

```
[In] integrate(1/x**3/(a+b*csch(d*x**2+c))**2,x)
[Out] Integral(1/(x**3*(a + b*csch(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 315, normalized size of antiderivative = 17.50

$$\int \frac{1}{x^3 \left(a+b\text{csch}\left(c+dx^2\right)\right)^2} \, dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="maxima")
[Out] -1/2*((a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^2*e^(2*d*x^2) + 2*a*b^2 - (a^3*d
+ a*b^2*d)*x^2 - 2*(b^3*e^c - (a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*x^2))/((a
^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^4*e^(2*d*x^2) + 2*(a^4*b*d*e^c + a^2*b^
3*d*e^c)*x^4*e^(d*x^2) - (a^5*d + a^3*b^2*d)*x^4) - integrate(2*(2*a*b^2 -
(2*b^3*e^c - (2*a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) +
a^3*b^2*d*e^(2*c))*x^5*e^(2*d*x^2) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^5*e^
(d*x^2) - (a^5*d + a^3*b^2*d)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^3 \left(a+b\text{csch}\left(c+dx^2\right)\right)^2} \, dx=\int \frac{1}{\left(b\,\text{csch}\left(dx^2+c\right)+a\right)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*csch(d*x^2+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a + b\text{csch}(c + dx^2))^2}\,dx = \int \frac{1}{x^3 \left(a + \frac{b}{\sinh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(1/(x^3*(a + b/sinh(c + d*x^2))^2),x)
[Out] int(1/(x^3*(a + b/sinh(c + d*x^2))^2), x)
```

3.31 $\int x^3 \left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result . . . 191
Rubi [A] (verified) . . . 192
Mathematica [A] (verified) . . . 197
Maple [F] . . . 198
Fricas [F] . . . 198
Sympy [F] . . . 198
Maxima [A] (verification not implemented) . . . 198
Giac [F] . . . 199
Mupad [F(-1)] . . . 199

Optimal result

Integrand size = 18, antiderivative size = 356

$$\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{14bx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{14bx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{84bx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{84bx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{420bx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{420bx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{5040bx\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}+\frac{5040bx\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}+\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7}-\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7}-\frac{10080b\,\text{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)}{d^8}+\frac{10080b\,\text{PolyLog}\left(8,e^{c+d\sqrt{x}}\right)}{d^8}$$

```
[Out] 1/4*a*x^4-4*b*x^(7/2)*arctanh(exp(c+d*x^(1/2)))/d-14*b*x^3*polylog(2,-exp(c
+d*x^(1/2)))/d^2+14*b*x^3*polylog(2,exp(c+d*x^(1/2)))/d^2+84*b*x^(5/2)*poly
log(3,-exp(c+d*x^(1/2)))/d^3-84*b*x^(5/2)*polylog(3,exp(c+d*x^(1/2)))/d^3-4
20*b*x^2*polylog(4,-exp(c+d*x^(1/2)))/d^4+420*b*x^2*polylog(4,exp(c+d*x^(1/
2)))/d^4+1680*b*x^(3/2)*polylog(5,-exp(c+d*x^(1/2)))/d^5-1680*b*x^(3/2)*pol
ylog(5,exp(c+d*x^(1/2)))/d^5-5040*b*x*polylog(6,-exp(c+d*x^(1/2)))/d^6+5040
*b*x*polylog(6,exp(c+d*x^(1/2)))/d^6-10080*b*polylog(8,-exp(c+d*x^(1/2)))/d
^8+10080*b*polylog(8,exp(c+d*x^(1/2)))/d^8+10080*b*polylog(7,-exp(c+d*x^(1/
2)))*x^(1/2)/d^7-10080*b*polylog(7,exp(c+d*x^(1/2)))*x^(1/2)/d^7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {14, 5545, 4267, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=&\frac{ax^4}{4}-\frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{10080b\,\text{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)}{d^8}\\&+\frac{10080b\,\text{PolyLog}\left(8,e^{c+d\sqrt{x}}\right)}{d^8}\\&+\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7}\\&-\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7}\\&-\frac{5040bx\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}\\&+\frac{5040bx\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}\\&+\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{420bx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{420bx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{84bx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{84bx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{14bx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{14bx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}\end{aligned}$$

```
[In] Int[x^3*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 - (4*b*x^(7/2)*ArcTanh[E^(c + d*Sqrt[x])])/d - (14*b*x^3*PolyLog[
2, -E^(c + d*Sqrt[x])])/d^2 + (14*b*x^3*PolyLog[2, E^(c + d*Sqrt[x])])/d^2
+ (84*b*x^(5/2)*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (84*b*x^(5/2)*PolyLog
[3, E^(c + d*Sqrt[x])])/d^3 - (420*b*x^2*PolyLog[4, -E^(c + d*Sqrt[x])])/d^
4 + (420*b*x^2*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (1680*b*x^(3/2)*PolyLog
[5, -E^(c + d*Sqrt[x])])/d^5 - (1680*b*x^(3/2)*PolyLog[5, E^(c + d*Sqrt[x])
```

```
])/d^5 - (5040*b*x*PolyLog[6, -E^(c + d*Sqrt[x])])/d^6 + (5040*b*x*PolyLog[
6, E^(c + d*Sqrt[x])])/d^6 + (10080*b*Sqrt[x]*PolyLog[7, -E^(c + d*Sqrt[x])
])/d^7 - (10080*b*Sqrt[x]*PolyLog[7, E^(c + d*Sqrt[x])])/d^7 - (10080*b*Pol
yLog[8, -E^(c + d*Sqrt[x])])/d^8 + (10080*b*PolyLog[8, E^(c + d*Sqrt[x])])/
d^8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3+bx^3\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx\\
&= \frac{ax^4}{4}+b\int x^3\operatorname{csch}\left(c+d\sqrt{x}\right)\,dx\\
&= \frac{ax^4}{4}+(2b)\text{Subst}\left(\int x^7\operatorname{csch}(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{ax^4}{4}-\frac{4bx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{(14b)\text{Subst}\left(\int x^6\log\left(1-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(14b)\text{Subst}\left(\int x^6\log\left(1+e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{ax^4}{4}-\frac{4bx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{14bx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}\\
&\quad+\frac{14bx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{(84b)\text{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad-\frac{(84b)\text{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&= \frac{ax^4}{4}-\frac{4bx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{14bx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}\\
&\quad+\frac{14bx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}\\
&\quad-\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{(420b)\text{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}\\
&\quad+\frac{(420b)\text{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{14bx^3 \,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \,\text{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{84bx^{5/2} \,\text{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \,\text{PolyLog}\left(4, -e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{420bx^2 \,\text{PolyLog}\left(4, e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(1680b)\text{Subst}\left(\int x^3 \,\text{PolyLog}\left(4, -e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^4} \\
&\quad - \frac{(1680b)\text{Subst}\left(\int x^3 \,\text{PolyLog}\left(4, e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^4} \\
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{14bx^3 \,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \,\text{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{84bx^{5/2} \,\text{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \,\text{PolyLog}\left(4, -e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{420bx^2 \,\text{PolyLog}\left(4, e^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680bx^{3/2} \,\text{PolyLog}\left(5, -e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{1680bx^{3/2} \,\text{PolyLog}\left(5, e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{(5040b)\text{Subst}\left(\int x^2 \,\text{PolyLog}\left(5, -e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^5} \\
&\quad + \frac{(5040b)\text{Subst}\left(\int x^2 \,\text{PolyLog}\left(5, e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^5} \\
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{14bx^3 \,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \,\text{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{84bx^{5/2} \,\text{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \,\text{PolyLog}\left(4, -e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{420bx^2 \,\text{PolyLog}\left(4, e^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680bx^{3/2} \,\text{PolyLog}\left(5, -e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{1680bx^{3/2} \,\text{PolyLog}\left(5, e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{5040bx \,\text{PolyLog}\left(6, -e^{c+d\sqrt{x}}\right)}{d^6} + \frac{5040bx \,\text{PolyLog}\left(6, e^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad + \frac{(10080b)\text{Subst}\left(\int x \,\text{PolyLog}\left(6, -e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^6} \\
&\quad - \frac{(10080b)\text{Subst}\left(\int x \,\text{PolyLog}\left(6, e^{c+dx}\right) \,dx, x, \sqrt{x}\right)}{d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{14bx^3 \operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{84bx^{5/2} \operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{420bx^2 \operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040bx \operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{5040bx \operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{(10080b)\text{Subst}\left(\int \operatorname{PolyLog}\left(7,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7} \\
&+ \frac{(10080b)\text{Subst}\left(\int \operatorname{PolyLog}\left(7,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{14bx^3 \operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{84bx^{5/2} \operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{420bx^2 \operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040bx \operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{5040bx \operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} - \frac{(10080b)\text{Subst}\left(\int \frac{\operatorname{PolyLog}(7,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^8} \\
&+ \frac{(10080b)\text{Subst}\left(\int \frac{\operatorname{PolyLog}(7,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14bx^3 \operatorname{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{14bx^3 \operatorname{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{420bx^2 \operatorname{PolyLog}\left(4, -e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{420bx^2 \operatorname{PolyLog}\left(4, e^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, e^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040bx \operatorname{PolyLog}\left(6, -e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{5040bx \operatorname{PolyLog}\left(6, e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{10080b \operatorname{PolyLog}\left(8, -e^{c+d\sqrt{x}}\right)}{d^8} + \frac{10080b \operatorname{PolyLog}\left(8, e^{c+d\sqrt{x}}\right)}{d^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.03

$$
\int x^3\left(a + b\operatorname{csch}\left(c + d\sqrt{x}\right)\right)\,dx = \frac{ax^4}{4}
+ \frac{2b\left(d^7x^{7/2}\log\left(1 - e^{c+d\sqrt{x}}\right) - d^7x^{7/2}\log\left(1 + e^{c+d\sqrt{x}}\right) - 7d^6x^3\operatorname{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right) + 7d^6x^3\operatorname{PolyLog}\left(2\right.\right.}{}
$$

```
[In] Integrate[x^3*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 + (2*b*(d^7*x^(7/2)*Log[1 - E^(c + d*Sqrt[x])] - d^7*x^(7/2)*Log[
1 + E^(c + d*Sqrt[x])] - 7*d^6*x^3*PolyLog[2, -E^(c + d*Sqrt[x])] + 7*d^6*x
^3*PolyLog[2, E^(c + d*Sqrt[x])] + 42*d^5*x^(5/2)*PolyLog[3, -E^(c + d*Sqrt
[x])] - 42*d^5*x^(5/2)*PolyLog[3, E^(c + d*Sqrt[x])] - 210*d^4*x^2*PolyLog[
4, -E^(c + d*Sqrt[x])] + 210*d^4*x^2*PolyLog[4, E^(c + d*Sqrt[x])] + 840*d^
3*x^(3/2)*PolyLog[5, -E^(c + d*Sqrt[x])] - 840*d^3*x^(3/2)*PolyLog[5, E^(c
+ d*Sqrt[x])] - 2520*d^2*x*PolyLog[6, -E^(c + d*Sqrt[x])] + 2520*d^2*x*Poly
Log[6, E^(c + d*Sqrt[x])] + 5040*d*Sqrt[x]*PolyLog[7, -E^(c + d*Sqrt[x])] -
 5040*d*Sqrt[x]*PolyLog[7, E^(c + d*Sqrt[x])] - 5040*PolyLog[8, -E^(c + d*S
qrt[x])] + 5040*PolyLog[8, E^(c + d*Sqrt[x])]))/d^8
```

Maple [F]

$$\int x^3 \left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^3*(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x^3*(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^3 \left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^3*csch(d*sqrt(x) + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3 \left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**3*(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.98

$$\int x^3 \left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{1}{4}\,ax^4 - \frac{2\left(\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^7+7\,\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^6-42\,\log\left(e^{\left(d\sqrt{x}\right)}\right)^5\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})\right.}{} + \frac{2\left(\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^7+7\,\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^6-42\,\log\left(e^{\left(d\sqrt{x}\right)}\right)^5\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+\right.}{}$$

```
[In] integrate(x^3*(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 2*(log(e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^7 + 7*dilog(-e
^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^6 - 42*log(e^(d*sqrt(x)))^5*polylog(3,
 -e^(d*sqrt(x) + c)) + 210*log(e^(d*sqrt(x)))^4*polylog(4, -e^(d*sqrt(x) +
c)) - 840*log(e^(d*sqrt(x)))^3*polylog(5, -e^(d*sqrt(x) + c)) + 2520*log(e^
(d*sqrt(x)))^2*polylog(6, -e^(d*sqrt(x) + c)) - 5040*log(e^(d*sqrt(x)))*pol
ylog(7, -e^(d*sqrt(x) + c)) + 5040*polylog(8, -e^(d*sqrt(x) + c)))*b/d^8 +
2*(log(-e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^7 + 7*dilog(e^(d*sqrt(x)
+ c))*log(e^(d*sqrt(x)))^6 - 42*log(e^(d*sqrt(x)))^5*polylog(3, e^(d*sqrt(x
) + c)) + 210*log(e^(d*sqrt(x)))^4*polylog(4, e^(d*sqrt(x) + c)) - 840*log(
e^(d*sqrt(x)))^3*polylog(5, e^(d*sqrt(x) + c)) + 2520*log(e^(d*sqrt(x)))^2*
polylog(6, e^(d*sqrt(x) + c)) - 5040*log(e^(d*sqrt(x)))*polylog(7, e^(d*sqr
t(x) + c)) + 5040*polylog(8, e^(d*sqrt(x) + c)))*b/d^8
```

Giac [F]

$$\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^3*(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^3*(a + b/sinh(c + d*x^(1/2))), x)
```

3.32 $\int x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right) \, dx$

Optimal result 200
Rubi [A] (verified) 201
Mathematica [A] (verified) 205
Maple [F] 205
Fricas [F] 205
Sympy [F] 206
Maxima [A] (verification not implemented) 206
Giac [F] 206
Mupad [F(-1)] 207

Optimal result

Integrand size = 18, antiderivative size = 260

$$\int x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right) \, dx = \frac{ax^3}{3}-\frac{4bx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{10bx^2 \text{ PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{10bx^2 \text{ PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{40bx^{3/2} \text{ PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{40bx^{3/2} \text{ PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{120bx \text{ PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{120bx \text{ PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{240b\sqrt{x} \text{ PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{240b\sqrt{x} \text{ PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{240b \text{ PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}+\frac{240b \text{ PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}$$

```
[Out] 1/3*a*x^3-4*b*x^(5/2)*arctanh(exp(c+d*x^(1/2)))/d-10*b*x^2*polylog(2,-exp(c
+d*x^(1/2)))/d^2+10*b*x^2*polylog(2,exp(c+d*x^(1/2)))/d^2+40*b*x^(3/2)*poly
log(3,-exp(c+d*x^(1/2)))/d^3-40*b*x^(3/2)*polylog(3,exp(c+d*x^(1/2)))/d^3-1
20*b*x*polylog(4,-exp(c+d*x^(1/2)))/d^4+120*b*x*polylog(4,exp(c+d*x^(1/2)))
/d^4-240*b*polylog(6,-exp(c+d*x^(1/2)))/d^6+240*b*polylog(6,exp(c+d*x^(1/2)
))/d^6+240*b*polylog(5,-exp(c+d*x^(1/2)))*x^(1/2)/d^5-240*b*polylog(5,exp(c
+d*x^(1/2)))*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {14, 5545, 4267, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=&\ \frac{ax^3}{3}-\frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{240b\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}+\frac{240b\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}\\&+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{120bx\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{120bx\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{10bx^2\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{10bx^2\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}\end{aligned}$$

```
[In] Int[x^2*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 - (4*b*x^(5/2)*ArcTanh[E^(c + d*Sqrt[x])])/d - (10*b*x^2*PolyLog[
2, -E^(c + d*Sqrt[x])])/d^2 + (10*b*x^2*PolyLog[2, E^(c + d*Sqrt[x])])/d^2
+ (40*b*x^(3/2)*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (40*b*x^(3/2)*PolyLog
[3, E^(c + d*Sqrt[x])])/d^3 - (120*b*x*PolyLog[4, -E^(c + d*Sqrt[x])])/d^4
+ (120*b*x*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (240*b*Sqrt[x]*PolyLog[5, -
E^(c + d*Sqrt[x])])/d^5 - (240*b*Sqrt[x]*PolyLog[5, E^(c + d*Sqrt[x])])/d^5
 - (240*b*PolyLog[6, -E^(c + d*Sqrt[x])])/d^6 + (240*b*PolyLog[6, E^(c + d*
Sqrt[x])])/d^6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax^2 + bx^2\text{csch}\left(c + d\sqrt{x}\right)\right)\,dx$$

$$= \frac{ax^3}{3} + b\int x^2 \operatorname{csch}\left(c + d\sqrt{x}\right)\, dx$$

$$= \frac{ax^3}{3} + (2b)\text{Subst}\left(\int x^5 \operatorname{csch}(c + dx)\, dx, x, \sqrt{x}\right)$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 - e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d} + \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 + e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2 \operatorname{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} + \frac{10bx^2 \operatorname{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(40b)\text{Subst}\left(\int x^3 \operatorname{PolyLog}\left(2, -e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^2} - \frac{(40b)\text{Subst}\left(\int x^3 \operatorname{PolyLog}\left(2, e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2 \operatorname{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} + \frac{10bx^2 \operatorname{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(120b)\text{Subst}\left(\int x^2 \operatorname{PolyLog}\left(3, -e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^3} + \frac{(120b)\text{Subst}\left(\int x^2 \operatorname{PolyLog}\left(3, e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2 \operatorname{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} + \frac{10bx^2 \operatorname{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{120bx \operatorname{PolyLog}\left(4, -e^{c+d\sqrt{x}}\right)}{d^4} + \frac{120bx \operatorname{PolyLog}\left(4, e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(240b)\text{Subst}\left(\int x \operatorname{PolyLog}\left(4, -e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^4} - \frac{(240b)\text{Subst}\left(\int x \operatorname{PolyLog}\left(4, e^{c+dx}\right)\, dx, x, \sqrt{x}\right)}{d^4}$$

$$
\begin{aligned}
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{10bx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{120bx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{120bx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} \\
&+ \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{10bx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{120bx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{120bx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10bx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{10bx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{120bx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{120bx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{240b\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{240b\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}+\frac{2b\left(d^5x^{5/2}\log\left(1-e^{c+d\sqrt{x}}\right)-d^5x^{5/2}\log\left(1+e^{c+d\sqrt{x}}\right)-5d^4x^2\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)+5d^4x^2\,\text{PolyLog}\left(2\right.}{}$$

```
[In] Integrate[x^2*(a + b*Csch[c + d*Sqrt[x]]),x]
[Out] (a*x^3)/3 + (2*b*(d^5*x^(5/2)*Log[1 - E^(c + d*Sqrt[x])] - d^5*x^(5/2)*Log[
1 + E^(c + d*Sqrt[x])] - 5*d^4*x^2*PolyLog[2, -E^(c + d*Sqrt[x])] + 5*d^4*x
^2*PolyLog[2, E^(c + d*Sqrt[x])] + 20*d^3*x^(3/2)*PolyLog[3, -E^(c + d*Sqrt
[x])] - 20*d^3*x^(3/2)*PolyLog[3, E^(c + d*Sqrt[x])] - 60*d^2*x*PolyLog[4,
-E^(c + d*Sqrt[x])] + 60*d^2*x*PolyLog[4, E^(c + d*Sqrt[x])] + 120*d*Sqrt[x
]*PolyLog[5, -E^(c + d*Sqrt[x])] - 120*d*Sqrt[x]*PolyLog[5, E^(c + d*Sqrt[x
])] - 120*PolyLog[6, -E^(c + d*Sqrt[x])] + 120*PolyLog[6, E^(c + d*Sqrt[x])
]))/d^6
```

Maple [F]

$$\int x^2\left(a+b\ \text{csch}\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^2*(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x^2*(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^2*csch(d*sqrt(x) + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\text{csch}(c+d\sqrt{x}))\,dx=\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**2*(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00

$$\int x^2(a+b\text{csch}(c+d\sqrt{x}))\,dx=\frac{1}{3}\,ax^3-\frac{2\left(\log\left(e^{(d\sqrt{x}+c)}+1\right)\log\left(e^{(d\sqrt{x})}\right)^5+5\,\text{Li}_2\left(-e^{(d\sqrt{x}+c)}\right)\log\left(e^{(d\sqrt{x})}\right)^4-20\,\log\left(e^{(d\sqrt{x})}\right)^3\text{Li}_3(-e^{(d\sqrt{x}+c)})\right.}{d^6}+\frac{2\left(\log\left(-e^{(d\sqrt{x}+c)}+1\right)\log\left(e^{(d\sqrt{x})}\right)^5+5\,\text{Li}_2\left(e^{(d\sqrt{x}+c)}\right)\log\left(e^{(d\sqrt{x})}\right)^4-20\,\log\left(e^{(d\sqrt{x})}\right)^3\text{Li}_3(e^{(d\sqrt{x}+c)})+\right.}{d^6}$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/3*a*x^3 - 2*(log(e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^5 + 5*dilog(-e
^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^4 - 20*log(e^(d*sqrt(x)))^3*polylog(3,
 -e^(d*sqrt(x) + c)) + 60*log(e^(d*sqrt(x)))^2*polylog(4, -e^(d*sqrt(x) + c
)) - 120*log(e^(d*sqrt(x)))*polylog(5, -e^(d*sqrt(x) + c)) + 120*polylog(6,
 -e^(d*sqrt(x) + c)))*b/d^6 + 2*(log(-e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(
x)))^5 + 5*dilog(e^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^4 - 20*log(e^(d*sqrt
(x)))^3*polylog(3, e^(d*sqrt(x) + c)) + 60*log(e^(d*sqrt(x)))^2*polylog(4,
e^(d*sqrt(x) + c)) - 120*log(e^(d*sqrt(x)))*polylog(5, e^(d*sqrt(x) + c)) +
 120*polylog(6, e^(d*sqrt(x) + c)))*b/d^6
```

Giac [F]

$$\int x^2(a+b\text{csch}(c+d\sqrt{x}))\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\operatorname{csch}\left(c + d\sqrt{x}\right)\right) \, dx = \int x^2 \left(a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}\right) \, dx$$

```
[In] int(x^2*(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^2*(a + b/sinh(c + d*x^(1/2))), x)
```

3.33 $\int x\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result 208
Rubi [A] (verified) 208
Mathematica [A] (verified) 211
Maple [F] 212
Fricas [F] 212
Sympy [F] 212
Maxima [A] (verification not implemented) 212
Giac [F] 213
Mupad [F(-1)] 213

Optimal result

Integrand size = 16, antiderivative size = 164

$$\int x\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{4bx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{6bx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{6bx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{12b\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{12b\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}$$

```
[Out] 1/2*a*x^2-4*b*x^(3/2)*arctanh(exp(c+d*x^(1/2)))/d-6*b*x*polylog(2,-exp(c+d*
x^(1/2)))/d^2+6*b*x*polylog(2,exp(c+d*x^(1/2)))/d^2-12*b*polylog(4,-exp(c+d
*x^(1/2)))/d^4+12*b*polylog(4,exp(c+d*x^(1/2)))/d^4+12*b*polylog(3,-exp(c+d
*x^(1/2)))*x^(1/2)/d^3-12*b*polylog(3,exp(c+d*x^(1/2)))*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.438$, Rules used

= {14, 5545, 4267, 2611, 6744, 2320, 6724}

$$\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{4bx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{12b\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{12b\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{6bx\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{6bx\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[x*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 - (4*b*x^(3/2)*ArcTanh[E^(c + d*Sqrt[x])])/d - (6*b*x*PolyLog[2,
-E^(c + d*Sqrt[x])])/d^2 + (6*b*x*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (12*
b*Sqrt[x]*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (12*b*Sqrt[x]*PolyLog[3, E^
(c + d*Sqrt[x])])/d^3 - (12*b*PolyLog[4, -E^(c + d*Sqrt[x])])/d^4 + (12*b*P
olyLog[4, E^(c + d*Sqrt[x])])/d^4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax + bx\,\mathrm{csch}\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^2}{2} + b\int x\,\mathrm{csch}\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^2}{2} + (2b)\mathrm{Subst}\left(\int x^3\mathrm{csch}(c + dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{ax^2}{2} - \frac{4bx^{3/2}\mathrm{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(6b)\mathrm{Subst}\left(\int x^2\log\left(1 - e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(6b)\mathrm{Subst}\left(\int x^2\log\left(1 + e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^2}{2} - \frac{4bx^{3/2}\mathrm{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6bx\,\mathrm{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{6bx\,\mathrm{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(12b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2, -e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} \\
&\quad - \frac{(12b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2, e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6bx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6bx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} + \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6bx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6bx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6bx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6bx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{12b\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{12b\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^2}{2} + \frac{2b\left(d^3x^{3/2}\log\left(1-e^{c+d\sqrt{x}}\right) - d^3x^{3/2}\log\left(1+e^{c+d\sqrt{x}}\right) - 3d^2x\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right) + 3d^2x\,\text{PolyLog}\left(2,e\right.\right.}{}$$

```
[In] Integrate[x*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 + (2*b*(d^3*x^(3/2)*Log[1 - E^(c + d*Sqrt[x])] - d^3*x^(3/2)*Log[
1 + E^(c + d*Sqrt[x])] - 3*d^2*x*PolyLog[2, -E^(c + d*Sqrt[x])] + 3*d^2*x*P
olyLog[2, E^(c + d*Sqrt[x])] + 6*d*Sqrt[x]*PolyLog[3, -E^(c + d*Sqrt[x])] -
 6*d*Sqrt[x]*PolyLog[3, E^(c + d*Sqrt[x])] - 6*PolyLog[4, -E^(c + d*Sqrt[x]
)] + 6*PolyLog[4, E^(c + d*Sqrt[x])]))/d^4
```

Maple [F]

$$\int x\left(a+b \operatorname{csch}\left(c+d\sqrt{x}\right)\right) dx$$

```
[In] int(x*(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x*(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x*csch(d*sqrt(x) + c) + a*x, x)
```

Sympy [F]

$$\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x*(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{1}{2}\,ax^2 - \frac{2\left(\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^3+3\,\text{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^2-6\,\log\left(e^{\left(d\sqrt{x}\right)}\right)\text{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+\cdots}{d^4} + \frac{2\left(\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^3+3\,\text{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^2-6\,\log\left(e^{\left(d\sqrt{x}\right)}\right)\text{Li}_3(e^{\left(d\sqrt{x}+c\right)})+6\cdots}{d^4}$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 2*(log(e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^3 + 3*dilog(-e
^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^2 - 6*log(e^(d*sqrt(x)))*polylog(3, -e
^(d*sqrt(x) + c)) + 6*polylog(4, -e^(d*sqrt(x) + c)))*b/d^4 + 2*(log(-e^(d*
sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^3 + 3*dilog(e^(d*sqrt(x) + c))*log(e^(
d*sqrt(x)))^2 - 6*log(e^(d*sqrt(x)))*polylog(3, e^(d*sqrt(x) + c)) + 6*poly
log(4, e^(d*sqrt(x) + c)))*b/d^4
```

Giac [F]

$$\int x(a + b\text{csch}(c + d\sqrt{x}))\ dx = \int (b\,\text{csch}\left(d\sqrt{x} + c\right) + a)x\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a + b\text{csch}(c + d\sqrt{x}))\ dx = \int x\left(a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x*(a + b/sinh(c + d*x^(1/2))),x)
```

```
[Out] int(x*(a + b/sinh(c + d*x^(1/2))), x)
```

3.34 $\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x}\,dx$

Optimal result 214
Rubi [N/A] 214
Mathematica [N/A] 215
Maple [N/A] (verified) 215
Fricas [N/A] 215
Sympy [N/A] 215
Maxima [N/A] 216
Giac [N/A] 216
Mupad [N/A] 216

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\operatorname{csch}(c+d\sqrt{x})}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(csch(c+d*x^(1/2))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x}\,dx = \int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Csch[c + d*Sqrt[x]]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x}+\frac{b\operatorname{csch}(c+d\sqrt{x})}{x}\right)dx\\ &= a\log(x)+b\int \frac{\operatorname{csch}(c+d\sqrt{x})}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 34.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x,x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))/x,x)
[Out] int((a+b*csch(c+d*x^(1/2)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x,x, algorithm="fricas")
[Out] integral((b*csch(d*sqrt(x) + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x,x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(1/(x*e^(d*sqrt(x) + c) + x), x) + b*integrate(1/(x*e^(d*sqrt(x)
 + c) - x), x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}{x}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x,x)
```

```
[Out] int((a + b/sinh(c + d*x^(1/2)))/x, x)
```

3.35 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{x^2}\,dx$

Optimal result 217
Rubi [N/A] 217
Mathematica [N/A] 218
Maple [N/A] (verified) 218
Fricas [N/A] 218
Sympy [N/A] 218
Maxima [N/A] 219
Giac [N/A] 219
Mupad [N/A] 219

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\text{csch}\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csch(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csch[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\right)dx\\&=-\frac{a}{x}+b\int \frac{\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 32.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\ \text{csch}\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*csch(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*csch(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\,\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
[Out] b*integrate(1/(x^2*e^(d*sqrt(x) + c) + x^2), x) + b*integrate(1/(x^2*e^(d*s
qrt(x) + c) - x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x^2,x)
[Out] int((a + b/sinh(c + d*x^(1/2)))/x^2, x)
```

3.36 $\int x^3\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result 221
Rubi [A] (verified) 222
Mathematica [B] (verified) 233
Maple [F] 234
Fricas [F] 235
Sympy [F] 235
Maxima [A] (verification not implemented) 235
Giac [F] 236
Mupad [F(-1)] 236

Optimal result

Integrand size = 20, antiderivative size = 597

$$
\begin{aligned}
\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx = & -\frac{2b^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8abx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} \\
& -\frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d}+\frac{14b^2x^3\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
& -\frac{28abx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
& +\frac{28abx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} \\
& +\frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
& +\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
& -\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} \\
& -\frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
& -\frac{840abx^2\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
& +\frac{840abx^2\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} \\
& +\frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
& +\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
& -\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} \\
& -\frac{315b^2x\operatorname{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
& -\frac{10080abx\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} \\
& +\frac{10080abx\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} \\
& +\frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7} \\
& +\frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} \\
& -\frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} \\
& -\frac{315b^2\operatorname{PolyLog}\left(7,e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8} \\
& \quad 20160ab\operatorname{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)
\end{aligned}
$$

```
[Out] 14*b^2*x^3*ln(1-exp(2*c+2*d*x^(1/2)))/d^2+42*b^2*x^(5/2)*polylog(2,exp(2*c+
2*d*x^(1/2)))/d^3-105*b^2*x^2*polylog(3,exp(2*c+2*d*x^(1/2)))/d^4-315/2*b^2
*polylog(7,exp(2*c+2*d*x^(1/2)))/d^8-2*b^2*x^(7/2)/d+1/4*a^2*x^4-20160*a*b*
polylog(7,exp(c+d*x^(1/2)))*x^(1/2)/d^7-10080*a*b*x*polylog(6,-exp(c+d*x^(1
/2)))/d^6+10080*a*b*x*polylog(6,exp(c+d*x^(1/2)))/d^6+20160*a*b*polylog(7,-
exp(c+d*x^(1/2)))*x^(1/2)/d^7+168*a*b*x^(5/2)*polylog(3,-exp(c+d*x^(1/2)))/
d^3-168*a*b*x^(5/2)*polylog(3,exp(c+d*x^(1/2)))/d^3-840*a*b*x^2*polylog(4,-
exp(c+d*x^(1/2)))/d^4+840*a*b*x^2*polylog(4,exp(c+d*x^(1/2)))/d^4+3360*a*b*
x^(3/2)*polylog(5,-exp(c+d*x^(1/2)))/d^5-3360*a*b*x^(3/2)*polylog(5,exp(c+d
*x^(1/2)))/d^5-8*a*b*x^(7/2)*arctanh(exp(c+d*x^(1/2)))/d-28*a*b*x^3*polylog
(2,-exp(c+d*x^(1/2)))/d^2+28*a*b*x^3*polylog(2,exp(c+d*x^(1/2)))/d^2+210*b^
2*x^(3/2)*polylog(4,exp(2*c+2*d*x^(1/2)))/d^5-315*b^2*x*polylog(5,exp(2*c+2
*d*x^(1/2)))/d^6-20160*a*b*polylog(8,-exp(c+d*x^(1/2)))/d^8+20160*a*b*polyl
og(8,exp(c+d*x^(1/2)))/d^8+315*b^2*polylog(6,exp(2*c+2*d*x^(1/2)))*x^(1/2)/
d^7-2*b^2*x^(7/2)*coth(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5545, 4275, 4267, 2611, 6744, 2320, 6724, 4269, 3797, 2221}

$$\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{8abx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}$$
$$-\frac{20160ab\operatorname{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)}{d^8}$$
$$+\frac{20160ab\operatorname{PolyLog}\left(8,e^{c+d\sqrt{x}}\right)}{d^8}$$
$$+\frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7}$$
$$-\frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7}$$
$$-\frac{10080abx\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}$$
$$+\frac{10080abx\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}$$
$$+\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}$$
$$-\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$
$$-\frac{840abx^2\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+\frac{840abx^2\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}$$
$$-\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$
$$-\frac{28abx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+\frac{28abx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}$$
$$-\frac{315b^2\operatorname{PolyLog}\left(7,e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8}$$
$$+\frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7}$$
$$-\frac{315b^2x\operatorname{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}$$
$$+\frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}$$
$$-\frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}$$
$$+\frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

```
[In] Int[x^3*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x^(7/2))/d + (a^2*x^4)/4 - (8*a*b*x^(7/2)*ArcTanh[E^(c + d*Sqrt[x])
])/d - (2*b^2*x^(7/2)*Coth[c + d*Sqrt[x]])/d + (14*b^2*x^3*Log[1 - E^(2*(c
+ d*Sqrt[x]))])/d^2 - (28*a*b*x^3*PolyLog[2, -E^(c + d*Sqrt[x])])/d^2 + (28
*a*b*x^3*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (42*b^2*x^(5/2)*PolyLog[2, E^
(2*(c + d*Sqrt[x]))])/d^3 + (168*a*b*x^(5/2)*PolyLog[3, -E^(c + d*Sqrt[x])]
)/d^3 - (168*a*b*x^(5/2)*PolyLog[3, E^(c + d*Sqrt[x])])/d^3 - (105*b^2*x^2*
PolyLog[3, E^(2*(c + d*Sqrt[x]))])/d^4 - (840*a*b*x^2*PolyLog[4, -E^(c + d*
Sqrt[x])])/d^4 + (840*a*b*x^2*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (210*b^2
*x^(3/2)*PolyLog[4, E^(2*(c + d*Sqrt[x]))])/d^5 + (3360*a*b*x^(3/2)*PolyLog
[5, -E^(c + d*Sqrt[x])])/d^5 - (3360*a*b*x^(3/2)*PolyLog[5, E^(c + d*Sqrt[x
])])/d^5 - (315*b^2*x*PolyLog[5, E^(2*(c + d*Sqrt[x]))])/d^6 - (10080*a*b*x
*PolyLog[6, -E^(c + d*Sqrt[x])])/d^6 + (10080*a*b*x*PolyLog[6, E^(c + d*Sqr
t[x])])/d^6 + (315*b^2*Sqrt[x]*PolyLog[6, E^(2*(c + d*Sqrt[x]))])/d^7 + (20
160*a*b*Sqrt[x]*PolyLog[7, -E^(c + d*Sqrt[x])])/d^7 - (20160*a*b*Sqrt[x]*Po
lyLog[7, E^(c + d*Sqrt[x])])/d^7 - (315*b^2*PolyLog[7, E^(2*(c + d*Sqrt[x])
)])/(2*d^8) - (20160*a*b*PolyLog[8, -E^(c + d*Sqrt[x])])/d^8 + (20160*a*b*P
olyLog[8, E^(c + d*Sqrt[x])])/d^8
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^7(a+b\text{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^7+2abx^7\text{csch}(c+dx)+b^2x^7\text{csch}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^4}{4}+(4ab)\text{Subst}\left(\int x^7\text{csch}(c+dx)\,dx, x, \sqrt{x}\right) + \left(2b^2\right)\text{Subst}\left(\int x^7\text{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^4}{4}-\frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} - \frac{(28ab)\text{Subst}\left(\int x^6\log\left(1-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(28ab)\text{Subst}\left(\int x^6\log\left(1+e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{\left(14b^2\right)\text{Subst}\left(\int x^6\coth(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(168ab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(168ab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{\left(28b^2\right)\text{Subst}\left(\int \frac{e^{2(c+dx)}x^6}{1-e^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} + \frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(84b^2)\,\text{Subst}\left(\int x^5\log\left(1-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{840abx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{840abx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(3360ab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(3360ab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(210b^2)\,\text{Subst}\left(\int x^4\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{105b^2x^2\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{840abx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{840abx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(10080ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(10080ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(420b^2)\,\text{Subst}\left(\int x^3\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{14b^2x^3\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+ \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$
$$- \frac{105b^2x^2\,\text{PolyLog}\left(3,e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{840abx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+ \frac{840abx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{210b^2x^{3/2}\,\text{PolyLog}\left(4,e^{2(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$
$$- \frac{10080abx\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080abx\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}$$
$$+ \frac{(20160ab)\text{Subst}\left(\int x\,\text{PolyLog}\left(6,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$
$$- \frac{(20160ab)\text{Subst}\left(\int x\,\text{PolyLog}\left(6,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$
$$- \frac{(630b^2)\,\text{Subst}\left(\int x^2\,\text{PolyLog}\left(4,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{105b^2x^2\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{840abx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{840abx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{210b^2x^{3/2}\,\text{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{315b^2x\,\text{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{10080abx\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080abx\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} + \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} - \frac{(20160ab)\text{Subst}\left(\int \text{PolyLog}\left(7,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7} + \frac{(20160ab)\text{Subst}\left(\int \text{PolyLog}\left(7,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7} + \frac{(630b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(5,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$

$$= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{14b^2x^3\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28abx^3\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+ \frac{28abx^3\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$
$$- \frac{105b^2x^2\,\text{PolyLog}\left(3,e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{840abx^2\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+ \frac{840abx^2\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{210b^2x^{3/2}\,\text{PolyLog}\left(4,e^{2(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$
$$- \frac{315b^2x\,\text{PolyLog}\left(5,e^{2(c+d\sqrt{x})}\right)}{d^6} - \frac{10080abx\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}$$
$$+ \frac{10080abx\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} + \frac{315b^2\sqrt{x}\,\text{PolyLog}\left(6,e^{2(c+d\sqrt{x})}\right)}{d^7}$$
$$+ \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7}$$
$$- \frac{(20160ab)\text{Subst}\left(\int \frac{\text{PolyLog}(7,-x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^8}$$
$$+ \frac{(20160ab)\text{Subst}\left(\int \frac{\text{PolyLog}(7,x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^8}$$
$$- \frac{(315b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(6,e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^7}$$

$$
\begin{aligned}
&= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{28abx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{28abx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&+ \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{840abx^2\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{840abx^2\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{10080abx\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{10080abx\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} + \frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7} \\
&+ \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{20160ab\operatorname{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)}{d^8} + \frac{20160ab\operatorname{PolyLog}\left(8,e^{c+d\sqrt{x}}\right)}{d^8} \\
&- \frac{(315b^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(6,x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} \\
&- \frac{2b^2x^{7/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28abx^3\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{28abx^3\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2(c+d\sqrt{x})}\right)}{d^4} \\
&- \frac{840abx^2\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{840abx^2\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{10080abx\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080abx\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{c+d\sqrt{x}}\right)}{d^7} - \frac{315b^2\operatorname{PolyLog}\left(7,e^{2(c+d\sqrt{x})}\right)}{2d^8} \\
&- \frac{20160ab\operatorname{PolyLog}\left(8,-e^{c+d\sqrt{x}}\right)}{d^8} + \frac{20160ab\operatorname{PolyLog}\left(8,e^{c+d\sqrt{x}}\right)}{d^8}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1289 vs. $2(597) = 1194$.

Time = 7.40 (sec) , antiderivative size = 1289, normalized size of antiderivative = 2.16

$$
\begin{aligned}
\int x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx &= \frac{a^2x^4\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)}{4\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2} \\
&- \frac{2b\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\left(2bd^7x^{7/2}-7bd^6\left(-1+e^{2c}\right)x^3\log\left(1-e^{-c-d\sqrt{x}}\right)-2ad^7\left(-1+e^{2c}\right)x^{7/2}\log\left(1\right.\right.}{} \\
&+ \frac{b^2x^{7/2}\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{csch}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2} \\
&- \frac{b^2x^{7/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}
\end{aligned}
$$

[In] Integrate[x^3*(a + b*Csch[c + d*Sqrt[x]])^2,x]

[Out] (a^2*x^4*(a + b*Csch[c + d*Sqrt[x]])^2*Sinh[c + d*Sqrt[x]]^2)/(4*(b + a*Sin
h[c + d*Sqrt[x]])^2) - (2*b*(a + b*Csch[c + d*Sqrt[x]])^2*(2*b*d^7*x^(7/2)
- 7*b*d^6*(-1 + E^(2*c))*x^3*Log[1 - E^(-c - d*Sqrt[x])] - 2*a*d^7*(-1 + E^
(2*c))*x^(7/2)*Log[1 - E^(-c - d*Sqrt[x])] - 7*b*d^6*(-1 + E^(2*c))*x^3*Log
[1 + E^(-c - d*Sqrt[x])] + 2*a*d^7*(-1 + E^(2*c))*x^(7/2)*Log[1 + E^(-c - d
*Sqrt[x])] + 42*b*d^5*(-1 + E^(2*c))*x^(5/2)*PolyLog[2, -E^(-c - d*Sqrt[x])
] - 14*a*d^6*(-1 + E^(2*c))*x^3*PolyLog[2, -E^(-c - d*Sqrt[x])] + 42*b*d^5*
(-1 + E^(2*c))*x^(5/2)*PolyLog[2, E^(-c - d*Sqrt[x])] + 14*a*d^6*(-1 + E^(2
*c))*x^3*PolyLog[2, E^(-c - d*Sqrt[x])] + 210*b*d^4*(-1 + E^(2*c))*x^2*Poly
Log[3, -E^(-c - d*Sqrt[x])] - 84*a*d^5*(-1 + E^(2*c))*x^(5/2)*PolyLog[3, -E
^(-c - d*Sqrt[x])] + 210*b*d^4*(-1 + E^(2*c))*x^2*PolyLog[3, E^(-c - d*Sqrt
[x])] + 84*a*d^5*(-1 + E^(2*c))*x^(5/2)*PolyLog[3, E^(-c - d*Sqrt[x])] + 84
0*b*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[4, -E^(-c - d*Sqrt[x])] - 420*a*d^4*
(-1 + E^(2*c))*x^2*PolyLog[4, -E^(-c - d*Sqrt[x])] + 840*b*d^3*(-1 + E^(2*c
))*x^(3/2)*PolyLog[4, E^(-c - d*Sqrt[x])] + 420*a*d^4*(-1 + E^(2*c))*x^2*Po
lyLog[4, E^(-c - d*Sqrt[x])] + 2520*b*d^2*(-1 + E^(2*c))*x*PolyLog[5, -E^(-
c - d*Sqrt[x])] - 1680*a*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[5, -E^(-c - d*S
qrt[x])] + 2520*b*d^2*(-1 + E^(2*c))*x*PolyLog[5, E^(-c - d*Sqrt[x])] + 168
0*a*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[5, E^(-c - d*Sqrt[x])] + 5040*b*d*(-
1 + E^(2*c))*Sqrt[x]*PolyLog[6, -E^(-c - d*Sqrt[x])] - 5040*a*d^2*(-1 + E^(
2*c))*x*PolyLog[6, -E^(-c - d*Sqrt[x])] + 5040*b*d*(-1 + E^(2*c))*Sqrt[x]*P
olyLog[6, E^(-c - d*Sqrt[x])] + 5040*a*d^2*(-1 + E^(2*c))*x*PolyLog[6, E^(-
c - d*Sqrt[x])] + 5040*b*(-1 + E^(2*c))*PolyLog[7, -E^(-c - d*Sqrt[x])] - 1
0080*a*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[7, -E^(-c - d*Sqrt[x])] + 5040*b*(-
1 + E^(2*c))*PolyLog[7, E^(-c - d*Sqrt[x])] + 10080*a*d*(-1 + E^(2*c))*Sqrt
[x]*PolyLog[7, E^(-c - d*Sqrt[x])] - 10080*a*(-1 + E^(2*c))*PolyLog[8, -E^(
-c - d*Sqrt[x])] + 10080*a*(-1 + E^(2*c))*PolyLog[8, E^(-c - d*Sqrt[x])])*S
inh[c + d*Sqrt[x]]^2)/(d^8*(-1 + E^(2*c))*(b + a*Sinh[c + d*Sqrt[x]])^2) +
(b^2*x^(7/2)*Csch[c/2]*Csch[c/2 + (d*Sqrt[x])/2]*(a + b*Csch[c + d*Sqrt[x]]
)^2*Sinh[c + d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*Sinh[c + d*Sqrt[x]
])^2) - (b^2*x^(7/2)*(a + b*Csch[c + d*Sqrt[x]])^2*Sech[c/2]*Sech[c/2 + (d*
Sqrt[x])/2]*Sinh[c + d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*Sinh[c + d
*Sqrt[x]])^2)

Maple [F]

$$\int x^3 \left(a + b \operatorname{csch}\left(c + d\sqrt{x}\right)\right)^2 dx$$

[In] int(x^3*(a+b*csch(c+d*x^(1/2)))^2,x)

[Out] int(x^3*(a+b*csch(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^3\,dx$$

[In] integrate(x^3*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2*x^3*csch(d*sqrt(x) + c)^2 + 2*a*b*x^3*csch(d*sqrt(x) + c) + a^2*x^3, x)

Sympy [F]

$$\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^3\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

[In] integrate(x**3*(a+b*csch(c+d*x**(1/2)))**2,x)

[Out] Integral(x**3*(a + b*csch(c + d*sqrt(x)))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.09

$$\int x^3\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{1}{4}\,a^2x^4-\frac{4\,b^2x^{\frac{7}{2}}}{de^{\left(2\,d\sqrt{x}+2\,c\right)}-d}$$
$$-\frac{4\left(d^7x^{\frac{7}{2}}\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+7\,d^6x^3\text{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-42\,d^5x^{\frac{5}{2}}\text{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+210\,d^4x^2\text{Li}_4(-e^{\left(d\sqrt{x}+c\right)}\right)}{}$$
$$+\frac{4\left(d^7x^{\frac{7}{2}}\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+7\,d^6x^3\text{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-42\,d^5x^{\frac{5}{2}}\text{Li}_3(e^{\left(d\sqrt{x}+c\right)})+210\,d^4x^2\text{Li}_4(e^{\left(d\sqrt{x}+c\right)})-\right.}{d^8}$$
$$+\frac{14\left(d^6x^3\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+6\,d^5x^{\frac{5}{2}}\text{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-30\,d^4x^2\text{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+120\,d^3x^{\frac{3}{2}}\text{Li}_4(-e^{\left(d\sqrt{x}+c\right.}\right.}{d^8}$$
$$+\frac{14\left(d^6x^3\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+6\,d^5x^{\frac{5}{2}}\text{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-30\,d^4x^2\text{Li}_3(e^{\left(d\sqrt{x}+c\right)})+120\,d^3x^{\frac{3}{2}}\text{Li}_4(e^{\left(d\sqrt{x}+c\right)})-\right.}{d^8}$$
$$-\frac{abd^8x^4+4\,b^2d^7x^{\frac{7}{2}}}{2\,d^8}+\frac{abd^8x^4-4\,b^2d^7x^{\frac{7}{2}}}{2\,d^8}$$

[In] integrate(x^3*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/4*a^2*x^4 - 4*b^2*x^(7/2)/(d*e^(2*d*sqrt(x) + 2*c) - d) - 4*(d^7*x^(7/2)*log(e^(d*sqrt(x) + c) + 1) + 7*d^6*x^3*dilog(-e^(d*sqrt(x) + c)) - 42*d^5*x

```
^(5/2)*polylog(3, -e^(d*sqrt(x) + c)) + 210*d^4*x^2*polylog(4, -e^(d*sqrt(x
) + c)) - 840*d^3*x^(3/2)*polylog(5, -e^(d*sqrt(x) + c)) + 2520*d^2*x*polyl
og(6, -e^(d*sqrt(x) + c)) - 5040*d*sqrt(x)*polylog(7, -e^(d*sqrt(x) + c)) +
 5040*polylog(8, -e^(d*sqrt(x) + c)))*a*b/d^8 + 4*(d^7*x^(7/2)*log(-e^(d*sq
rt(x) + c) + 1) + 7*d^6*x^3*dilog(e^(d*sqrt(x) + c)) - 42*d^5*x^(5/2)*polyl
og(3, e^(d*sqrt(x) + c)) + 210*d^4*x^2*polylog(4, e^(d*sqrt(x) + c)) - 840*
d^3*x^(3/2)*polylog(5, e^(d*sqrt(x) + c)) + 2520*d^2*x*polylog(6, e^(d*sqrt
(x) + c)) - 5040*d*sqrt(x)*polylog(7, e^(d*sqrt(x) + c)) + 5040*polylog(8,
e^(d*sqrt(x) + c)))*a*b/d^8 + 14*(d^6*x^3*log(e^(d*sqrt(x) + c) + 1) + 6*d^
5*x^(5/2)*dilog(-e^(d*sqrt(x) + c)) - 30*d^4*x^2*polylog(3, -e^(d*sqrt(x) +
 c)) + 120*d^3*x^(3/2)*polylog(4, -e^(d*sqrt(x) + c)) - 360*d^2*x*polylog(5
, -e^(d*sqrt(x) + c)) + 720*d*sqrt(x)*polylog(6, -e^(d*sqrt(x) + c)) - 720*
polylog(7, -e^(d*sqrt(x) + c)))*b^2/d^8 + 14*(d^6*x^3*log(-e^(d*sqrt(x) + c
) + 1) + 6*d^5*x^(5/2)*dilog(e^(d*sqrt(x) + c)) - 30*d^4*x^2*polylog(3, e^(
d*sqrt(x) + c)) + 120*d^3*x^(3/2)*polylog(4, e^(d*sqrt(x) + c)) - 360*d^2*x
*polylog(5, e^(d*sqrt(x) + c)) + 720*d*sqrt(x)*polylog(6, e^(d*sqrt(x) + c)
) - 720*polylog(7, e^(d*sqrt(x) + c)))*b^2/d^8 - 1/2*(a*b*d^8*x^4 + 4*b^2*d
^7*x^(7/2))/d^8 + 1/2*(a*b*d^8*x^4 - 4*b^2*d^7*x^(7/2))/d^8
```

Giac [F]

$$\int x^3 \left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2 \, dx=\int \left(b \operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2 \, dx=\int x^3 \left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)^2 \,dx$$

```
[In] int(x^3*(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^3*(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.37 $\int x^2 \left(a+b\,\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result 238
Rubi [A] (verified) 239
Mathematica [B] (verified) 247
Maple [F] 248
Fricas [F] 248
Sympy [F] 248
Maxima [A] (verification not implemented) 248
Giac [F] 250
Mupad [F(-1)] 250

Optimal result

Integrand size = 20, antiderivative size = 441

$$\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2b^2x^{5/2}}{d}+\frac{a^2x^3}{3}-\frac{8abx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d}+\frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{30b^2x\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{15b^2\operatorname{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{480ab\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}+\frac{480ab\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}$$

```
[Out] -2*b^2*x^(5/2)/d+1/3*a^2*x^3-8*a*b*x^(5/2)*arctanh(exp(c+d*x^(1/2)))/d-2*b^
2*x^(5/2)*coth(c+d*x^(1/2))/d+10*b^2*x^2*ln(1-exp(2*c+2*d*x^(1/2)))/d^2-20*
a*b*x^2*polylog(2,-exp(c+d*x^(1/2)))/d^2+20*a*b*x^2*polylog(2,exp(c+d*x^(1/
2)))/d^2+20*b^2*x^(3/2)*polylog(2,exp(2*c+2*d*x^(1/2)))/d^3+80*a*b*x^(3/2)*
```

```
polylog(3,-exp(c+d*x^(1/2)))/d^3-80*a*b*x^(3/2)*polylog(3,exp(c+d*x^(1/2)))
/d^3-30*b^2*x*polylog(3,exp(2*c+2*d*x^(1/2)))/d^4-240*a*b*x*polylog(4,-exp(
c+d*x^(1/2)))/d^4+240*a*b*x*polylog(4,exp(c+d*x^(1/2)))/d^4-15*b^2*polylog(
5,exp(2*c+2*d*x^(1/2)))/d^6-480*a*b*polylog(6,-exp(c+d*x^(1/2)))/d^6+480*a*
b*polylog(6,exp(c+d*x^(1/2)))/d^6+30*b^2*polylog(4,exp(2*c+2*d*x^(1/2)))*x^
(1/2)/d^5+480*a*b*polylog(5,-exp(c+d*x^(1/2)))*x^(1/2)/d^5-480*a*b*polylog(
5,exp(c+d*x^(1/2)))*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5545, 4275, 4267, 2611, 6744, 2320, 6724, 4269, 3797, 2221}

$$\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^3}{3}-\frac{8abx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{480ab\,\text{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6}+\frac{480ab\,\text{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}+\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{240abx\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{240abx\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{20abx^2\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{20abx^2\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}-\frac{15b^2\,\text{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{30b^2\sqrt{x}\,\text{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{30b^2x\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d}-\frac{2b^2x^{5/2}}{d}$$

```
[In] Int[x^2*(a + b*Csch[c + d*Sqrt[x]])^2,x]
[Out] (-2*b^2*x^(5/2))/d + (a^2*x^3)/3 - (8*a*b*x^(5/2)*ArcTanh[E^(c + d*Sqrt[x])
])/d - (2*b^2*x^(5/2)*Coth[c + d*Sqrt[x]])/d + (10*b^2*x^2*Log[1 - E^(2*(c
```

```
+ d*Sqrt[x]))])/d^2 - (20*a*b*x^2*PolyLog[2, -E^(c + d*Sqrt[x])])/d^2 + (20
*a*b*x^2*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (20*b^2*x^(3/2)*PolyLog[2, E^
(2*(c + d*Sqrt[x]))])/d^3 + (80*a*b*x^(3/2)*PolyLog[3, -E^(c + d*Sqrt[x])])
/d^3 - (80*a*b*x^(3/2)*PolyLog[3, E^(c + d*Sqrt[x])])/d^3 - (30*b^2*x*PolyL
og[3, E^(2*(c + d*Sqrt[x]))])/d^4 - (240*a*b*x*PolyLog[4, -E^(c + d*Sqrt[x]
)])/d^4 + (240*a*b*x*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (30*b^2*Sqrt[x]*P
olyLog[4, E^(2*(c + d*Sqrt[x]))])/d^5 + (480*a*b*Sqrt[x]*PolyLog[5, -E^(c +
 d*Sqrt[x])])/d^5 - (480*a*b*Sqrt[x]*PolyLog[5, E^(c + d*Sqrt[x])])/d^5 - (
15*b^2*PolyLog[5, E^(2*(c + d*Sqrt[x]))])/d^6 - (480*a*b*PolyLog[6, -E^(c +
 d*Sqrt[x])])/d^6 + (480*a*b*PolyLog[6, E^(c + d*Sqrt[x])])/d^6
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^5(a+b\text{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^5+2abx^5\text{csch}(c+dx)+b^2x^5\text{csch}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} + (4ab)\text{Subst}\left(\int x^5\text{csch}(c+dx)\,dx, x, \sqrt{x}\right) + (2b^2)\,\text{Subst}\left(\int x^5\text{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} - \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1+e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(10b^2)\,\text{Subst}\left(\int x^4\coth(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} - \frac{20abx^2\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(80ab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(80ab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(20b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x^4}{1-e^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{(40b^2)\,\text{Subst}\left(\int x^3\log\left(1-e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(480ab)\text{Subst}\left(\int x\operatorname{PolyLog}\left(4,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(480ab)\text{Subst}\left(\int x\operatorname{PolyLog}\left(4,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(60b^2)\,\text{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{30b^2x\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(480ab)\text{Subst}\left(\int\operatorname{PolyLog}\left(5,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(480ab)\text{Subst}\left(\int\operatorname{PolyLog}\left(5,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(60b^2)\,\text{Subst}\left(\int x\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+ \frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}$$
$$+ \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$
$$- \frac{30b^2x\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+ \frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}$$
$$+ \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$
$$- \frac{(480ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6}$$
$$+ \frac{(480ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6}$$
$$- \frac{\left(30b^2\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{30b^2x\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{480ab\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{480ab\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6} - \frac{(15b^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(4,x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}$$

$$= -\frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{5/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20abx^2\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20abx^2\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{30b^2x\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240abx\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{240abx\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{15b^2\operatorname{PolyLog}\left(5,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{480ab\operatorname{PolyLog}\left(6,-e^{c+d\sqrt{x}}\right)}{d^6} + \frac{480ab\operatorname{PolyLog}\left(6,e^{c+d\sqrt{x}}\right)}{d^6}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1017 vs. $2(441) = 882$.

Time = 7.15 (sec) , antiderivative size = 1017, normalized size of antiderivative = 2.31

$$\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^3\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)}{3\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$
$$-\frac{2b\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\left(2bd^5x^{5/2}-5bd^4(-1+e^{2c})\,x^2\log\left(1-e^{-c-d\sqrt{x}}\right)-2ad^5(-1+e^{2c})\,x^{5/2}\log\left(1\right.\right.}{}$$
$$+\frac{b^2x^{5/2}\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{csch}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$
$$-\frac{b^2x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[x^2*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (a^2*x^3*(a + b*Csch[c + d*Sqrt[x]])^2*Sinh[c + d*Sqrt[x]]^2)/(3*(b + a*Sin
h[c + d*Sqrt[x]])^2) - (2*b*(a + b*Csch[c + d*Sqrt[x]])^2*(2*b*d^5*x^(5/2)
- 5*b*d^4*(-1 + E^(2*c))*x^2*Log[1 - E^(-c - d*Sqrt[x])] - 2*a*d^5*(-1 + E^
(2*c))*x^(5/2)*Log[1 - E^(-c - d*Sqrt[x])] - 5*b*d^4*(-1 + E^(2*c))*x^2*Log
[1 + E^(-c - d*Sqrt[x])] + 2*a*d^5*(-1 + E^(2*c))*x^(5/2)*Log[1 + E^(-c - d
*Sqrt[x])] + 20*b*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[2, -E^(-c - d*Sqrt[x])
] - 10*a*d^4*(-1 + E^(2*c))*x^2*PolyLog[2, -E^(-c - d*Sqrt[x])] + 20*b*d^3*
(-1 + E^(2*c))*x^(3/2)*PolyLog[2, E^(-c - d*Sqrt[x])] + 10*a*d^4*(-1 + E^(2
*c))*x^2*PolyLog[2, E^(-c - d*Sqrt[x])] + 60*b*d^2*(-1 + E^(2*c))*x*PolyLog
[3, -E^(-c - d*Sqrt[x])] - 40*a*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[3, -E^(-
c - d*Sqrt[x])] + 60*b*d^2*(-1 + E^(2*c))*x*PolyLog[3, E^(-c - d*Sqrt[x])]
+ 40*a*d^3*(-1 + E^(2*c))*x^(3/2)*PolyLog[3, E^(-c - d*Sqrt[x])] + 120*b*d*
(-1 + E^(2*c))*Sqrt[x]*PolyLog[4, -E^(-c - d*Sqrt[x])] - 120*a*d^2*(-1 + E^
(2*c))*x*PolyLog[4, -E^(-c - d*Sqrt[x])] + 120*b*d*(-1 + E^(2*c))*Sqrt[x]*P
olyLog[4, E^(-c - d*Sqrt[x])] + 120*a*d^2*(-1 + E^(2*c))*x*PolyLog[4, E^(-c
 - d*Sqrt[x])] + 120*b*(-1 + E^(2*c))*PolyLog[5, -E^(-c - d*Sqrt[x])] - 240
*a*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[5, -E^(-c - d*Sqrt[x])] + 120*b*(-1 + E
^(2*c))*PolyLog[5, E^(-c - d*Sqrt[x])] + 240*a*d*(-1 + E^(2*c))*Sqrt[x]*Pol
yLog[5, E^(-c - d*Sqrt[x])] - 240*a*(-1 + E^(2*c))*PolyLog[6, -E^(-c - d*Sq
rt[x])] + 240*a*(-1 + E^(2*c))*PolyLog[6, E^(-c - d*Sqrt[x])])*Sinh[c + d*S
qrt[x]]^2)/(d^6*(-1 + E^(2*c))*(b + a*Sinh[c + d*Sqrt[x]])^2) + (b^2*x^(5/2
)*Csch[c/2]*Csch[c/2 + (d*Sqrt[x])/2]*(a + b*Csch[c + d*Sqrt[x]])^2*Sinh[c
+ d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*Sinh[c + d*Sqrt[x]])^2) - (b^
2*x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2*Sech[c/2]*Sech[c/2 + (d*Sqrt[x])/2]
*Sinh[c + d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*Sinh[c + d*Sqrt[x]])^
2)
```

Maple [F]

$$\int x^2\left(a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2dx$$

```
[In] int(x^2*(a+b*csch(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^2*(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*csch(d*sqrt(x) + c)^2 + 2*a*b*x^2*csch(d*sqrt(x) + c) + a^
2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*csch(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**2*(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{1}{3}\,a^2x^3-\frac{4\,b^2x^{\frac{5}{2}}}{de^{\left(2\,d\sqrt{x}+2\,c\right)}-d}$$

$$-\frac{4\left(d^5x^{\frac{5}{2}}\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+5\,d^4x^2\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-20\,d^3x^{\frac{3}{2}}\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+60\,d^2x\mathrm{Li}_4(-e^{\left(d\sqrt{x}+c\right)})-\right.}{d^6}$$

$$+\frac{4\left(d^5x^{\frac{5}{2}}\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+5\,d^4x^2\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-20\,d^3x^{\frac{3}{2}}\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+60\,d^2x\mathrm{Li}_4(e^{\left(d\sqrt{x}+c\right)})-12\right.}{d^6}$$

$$+\frac{10\left(d^4x^2\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+4\,d^3x^{\frac{3}{2}}\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-12\,d^2x\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+24\,d\sqrt{x}\mathrm{Li}_4(-e^{\left(d\sqrt{x}+c\right)})\right.}{d^6}$$

$$+\frac{10\left(d^4x^2\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+4\,d^3x^{\frac{3}{2}}\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-12\,d^2x\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+24\,d\sqrt{x}\mathrm{Li}_4(e^{\left(d\sqrt{x}+c\right)})-2\right.}{d^6}$$

$$-\frac{2\left(abd^6x^3+3\,b^2d^5x^{\frac{5}{2}}\right)}{3\,d^6}+\frac{2\left(abd^6x^3-3\,b^2d^5x^{\frac{5}{2}}\right)}{3\,d^6}$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - 4*b^2*x^(5/2)/(d*e^(2*d*sqrt(x) + 2*c) - d) - 4*(d^5*x^(5/2)*
log(e^(d*sqrt(x) + c) + 1) + 5*d^4*x^2*dilog(-e^(d*sqrt(x) + c)) - 20*d^3*x
^(3/2)*polylog(3, -e^(d*sqrt(x) + c)) + 60*d^2*x*polylog(4, -e^(d*sqrt(x) +
 c)) - 120*d*sqrt(x)*polylog(5, -e^(d*sqrt(x) + c)) + 120*polylog(6, -e^(d*
sqrt(x) + c)))*a*b/d^6 + 4*(d^5*x^(5/2)*log(-e^(d*sqrt(x) + c) + 1) + 5*d^4
*x^2*dilog(e^(d*sqrt(x) + c)) - 20*d^3*x^(3/2)*polylog(3, e^(d*sqrt(x) + c)
) + 60*d^2*x*polylog(4, e^(d*sqrt(x) + c)) - 120*d*sqrt(x)*polylog(5, e^(d*
sqrt(x) + c)) + 120*polylog(6, e^(d*sqrt(x) + c)))*a*b/d^6 + 10*(d^4*x^2*lo
g(e^(d*sqrt(x) + c) + 1) + 4*d^3*x^(3/2)*dilog(-e^(d*sqrt(x) + c)) - 12*d^2
*x*polylog(3, -e^(d*sqrt(x) + c)) + 24*d*sqrt(x)*polylog(4, -e^(d*sqrt(x) +
 c)) - 24*polylog(5, -e^(d*sqrt(x) + c)))*b^2/d^6 + 10*(d^4*x^2*log(-e^(d*s
qrt(x) + c) + 1) + 4*d^3*x^(3/2)*dilog(e^(d*sqrt(x) + c)) - 12*d^2*x*polylo
g(3, e^(d*sqrt(x) + c)) + 24*d*sqrt(x)*polylog(4, e^(d*sqrt(x) + c)) - 24*p
olylog(5, e^(d*sqrt(x) + c)))*b^2/d^6 - 2/3*(a*b*d^6*x^3 + 3*b^2*d^5*x^(5/2
))/d^6 + 2/3*(a*b*d^6*x^3 - 3*b^2*d^5*x^(5/2))/d^6
```

Giac [F]

$$\int x^2 \left(a + b\text{csch}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \,\text{csch}\left(d\sqrt{x} + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\text{csch}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^2*(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^2*(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.38 $\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result 251
Rubi [A] (verified) 252
Mathematica [A] (verified) 256
Maple [F] 257
Fricas [F] 257
Sympy [F] 257
Maxima [A] (verification not implemented) 257
Giac [F] 258
Mupad [F(-1)] 259

Optimal result

Integrand size = 18, antiderivative size = 287

$$\begin{aligned}\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=&-\frac{2b^2x^{3/2}}{d}+\frac{a^2x^2}{2}-\frac{8abx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d}+\frac{6b^2x\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{3b^2\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&-\frac{24ab\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{24ab\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}\end{aligned}$$

```
[Out] -2*b^2*x^(3/2)/d+1/2*a^2*x^2-8*a*b*x^(3/2)*arctanh(exp(c+d*x^(1/2)))/d-2*b^
2*x^(3/2)*coth(c+d*x^(1/2))/d+6*b^2*x*ln(1-exp(2*c+2*d*x^(1/2)))/d^2-12*a*b
*x*polylog(2,-exp(c+d*x^(1/2)))/d^2+12*a*b*x*polylog(2,exp(c+d*x^(1/2)))/d^
2-3*b^2*polylog(3,exp(2*c+2*d*x^(1/2)))/d^4-24*a*b*polylog(4,-exp(c+d*x^(1/
2)))/d^4+24*a*b*polylog(4,exp(c+d*x^(1/2)))/d^4+6*b^2*polylog(2,exp(2*c+2*d
```

```
*x^(1/2)))*x^(1/2)/d^3+24*a*b*polylog(3,-exp(c+d*x^(1/2)))*x^(1/2)/d^3-24*a
*b*polylog(3,exp(c+d*x^(1/2)))*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5545, 4275, 4267, 2611, 6744, 2320, 6724, 4269, 3797, 2221}

$$\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}-\frac{8abx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{24ab\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{24ab\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}-\frac{3b^2\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{6b^2x\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d}-\frac{2b^2x^{3/2}}{d}$$

```
[In] Int[x*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x^(3/2))/d + (a^2*x^2)/2 - (8*a*b*x^(3/2)*ArcTanh[E^(c + d*Sqrt[x])
])/d - (2*b^2*x^(3/2)*Coth[c + d*Sqrt[x]])/d + (6*b^2*x*Log[1 - E^(2*(c + d
*Sqrt[x]))])/d^2 - (12*a*b*x*PolyLog[2, -E^(c + d*Sqrt[x])])/d^2 + (12*a*b*
x*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (6*b^2*Sqrt[x]*PolyLog[2, E^(2*(c +
d*Sqrt[x]))])/d^3 + (24*a*b*Sqrt[x]*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (
24*a*b*Sqrt[x]*PolyLog[3, E^(c + d*Sqrt[x])])/d^3 - (3*b^2*PolyLog[3, E^(2*
(c + d*Sqrt[x]))])/d^4 - (24*a*b*PolyLog[4, -E^(c + d*Sqrt[x])])/d^4 + (24*
a*b*PolyLog[4, E^(c + d*Sqrt[x])])/d^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\text{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^3+2abx^3\text{csch}(c+dx)+b^2x^3\text{csch}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^2}{2}+(4ab)\text{Subst}\left(\int x^3\text{csch}(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad+\left(2b^2\right)\text{Subst}\left(\int x^3\text{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^2}{2}-\frac{8abx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{(12ab)\text{Subst}\left(\int x^2\log\left(1-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{(12ab)\text{Subst}\left(\int x^2\log\left(1+e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{\left(6b^2\right)\text{Subst}\left(\int x^2\coth(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

$$= -\frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d} - \frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(24ab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(24ab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(12b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x^2}{1-e^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} + \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(24ab)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(24ab)\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} - \frac{(6b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ab\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{24ab\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} - \frac{(3b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2(c+d\sqrt{x})}\right)}{d^4}$$

$$= -\frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^{3/2}\coth\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{12abx\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12abx\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{3b^2\,\text{PolyLog}\left(3,e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{24ab\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{24ab\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.33

$$\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{4b^2d^3x^{3/2}+a^2d^4x^2-4b^2d^3x^{3/2}\coth\left(c+d\sqrt{x}\right)+12b^2d^2x\log\left(1-e^{-c-d\sqrt{x}}\right)+8abd^3x^{3/2}\log\left(1-e^{-c-d\sqrt{x}}\right)}{}$$

```
[In] Integrate[x*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (4*b^2*d^3*x^(3/2) + a^2*d^4*x^2 - 4*b^2*d^3*x^(3/2)*Coth[c + d*Sqrt[x]] +
12*b^2*d^2*x*Log[1 - E^(-c - d*Sqrt[x])] + 8*a*b*d^3*x^(3/2)*Log[1 - E^(-c
- d*Sqrt[x])] + 12*b^2*d^2*x*Log[1 + E^(-c - d*Sqrt[x])] - 8*a*b*d^3*x^(3/2
)*Log[1 + E^(-c - d*Sqrt[x])] + 24*(-(b^2*d*Sqrt[x]) + a*b*d^2*x)*PolyLog[2
```

```
, -E^(-c - d*Sqrt[x])] - 24*b*d*(b + a*d*Sqrt[x])*Sqrt[x]*PolyLog[2, E^(-c
- d*Sqrt[x])] - 24*b^2*PolyLog[3, -E^(-c - d*Sqrt[x])] + 48*a*b*d*Sqrt[x]*P
olyLog[3, -E^(-c - d*Sqrt[x])] - 24*b^2*PolyLog[3, E^(-c - d*Sqrt[x])] - 48
*a*b*d*Sqrt[x]*PolyLog[3, E^(-c - d*Sqrt[x])] + 48*a*b*PolyLog[4, -E^(-c -
d*Sqrt[x])] - 48*a*b*PolyLog[4, E^(-c - d*Sqrt[x])])/(2*d^4)
```

Maple [F]

$$\int x\left(a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2dx$$

```
[In] int(x*(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(x*(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x*csch(d*sqrt(x) + c)^2 + 2*a*b*x*csch(d*sqrt(x) + c) + a^2*x,
 x)
```

Sympy [F]

$$\int x\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x*(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.20

$$\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{1}{2}a^2x^2 - \frac{4b^2x^{\frac{3}{2}}}{de^{\left(2d\sqrt{x}+2c\right)}-d} - \frac{4\left(d^3x^{\frac{3}{2}}\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+3d^2x\text{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-6d\sqrt{x}\text{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+6\text{Li}_4(-e^{\left(d\sqrt{x}+c\right)})\right)ab}{d^4} + \frac{4\left(d^3x^{\frac{3}{2}}\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+3d^2x\text{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-6d\sqrt{x}\text{Li}_3(e^{\left(d\sqrt{x}+c\right)})+6\text{Li}_4(e^{\left(d\sqrt{x}+c\right)})\right)ab}{d^4} + \frac{6\left(d^2x\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+2d\sqrt{x}\text{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-2\text{Li}_3(-e^{\left(d\sqrt{x}+c\right)})\right)b^2}{d^4} + \frac{6\left(d^2x\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+2d\sqrt{x}\text{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-2\text{Li}_3(e^{\left(d\sqrt{x}+c\right)})\right)b^2}{d^4} - \frac{abd^4x^2+2b^2d^3x^{\frac{3}{2}}}{d^4} + \frac{abd^4x^2-2b^2d^3x^{\frac{3}{2}}}{d^4}$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 - 4*b^2*x^(3/2)/(d*e^(2*d*sqrt(x) + 2*c) - d) - 4*(d^3*x^(3/2)*
log(e^(d*sqrt(x) + c) + 1) + 3*d^2*x*dilog(-e^(d*sqrt(x) + c)) - 6*d*sqrt(x
)*polylog(3, -e^(d*sqrt(x) + c)) + 6*polylog(4, -e^(d*sqrt(x) + c)))*a*b/d^
4 + 4*(d^3*x^(3/2)*log(-e^(d*sqrt(x) + c) + 1) + 3*d^2*x*dilog(e^(d*sqrt(x)
 + c)) - 6*d*sqrt(x)*polylog(3, e^(d*sqrt(x) + c)) + 6*polylog(4, e^(d*sqrt
(x) + c)))*a*b/d^4 + 6*(d^2*x*log(e^(d*sqrt(x) + c) + 1) + 2*d*sqrt(x)*dilo
g(-e^(d*sqrt(x) + c)) - 2*polylog(3, -e^(d*sqrt(x) + c)))*b^2/d^4 + 6*(d^2*
x*log(-e^(d*sqrt(x) + c) + 1) + 2*d*sqrt(x)*dilog(e^(d*sqrt(x) + c)) - 2*po
lylog(3, e^(d*sqrt(x) + c)))*b^2/d^4 - (a*b*d^4*x^2 + 2*b^2*d^3*x^(3/2))/d^
4 + (a*b*d^4*x^2 - 2*b^2*d^3*x^(3/2))/d^4
```

Giac [F]

$$\int x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx = \int \left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x*(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x*(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.39 $\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal result 260
Rubi [N/A] 260
Mathematica [F(-1)] 261
Maple [N/A] (verified) 261
Fricas [N/A] 261
Sympy [N/A] 261
Maxima [N/A] 262
Giac [N/A] 262
Mupad [N/A] 262

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\mathrm{Int}\left(\frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*csch(c+d*x^(1/2)))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*Sqrt[x]])^2/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\$Aborted$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x,x]
[Out] $Aborted
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\ \operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))^2/x,x)
[Out] int((a+b*csch(c+d*x^(1/2)))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2)/x, x
)
```

Sympy [N/A]

Not integrable

Time = 9.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2/x,x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.85

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - 4*b^2*sqrt(x)/(d*x*e^(2*d*sqrt(x) + 2*c) - d*x) + integrate((2
*a*b*d*x + b^2*sqrt(x))/(d*x^2*e^(d*sqrt(x) + c) + d*x^2), x) - integrate(-
(2*a*b*d*x - b^2*sqrt(x))/(d*x^2*e^(d*sqrt(x) + c) - d*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2}{x}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))^2/x,x)
```

```
[Out] int((a + b/sinh(c + d*x^(1/2)))^2/x, x)
```

3.40 $\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal result 263
Rubi [N/A] 263
Mathematica [N/A] 264
Maple [N/A] (verified) 264
Fricas [N/A] 264
Sympy [N/A] 265
Maxima [N/A] 265
Giac [N/A] 265
Mupad [N/A] 266

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\mathrm{Int}\left(\frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*csch(c+d*x^(1/2)))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 127.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}}{x^{2}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))^2/x^2,x)
[Out] int((a+b*csch(c+d*x^(1/2)))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2)/x^2,
 x)
```

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2/x**2,x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")
[Out] -(a^2*d*x*e^(2*d*sqrt(x) + 2*c) - a^2*d*x + 4*b^2*sqrt(x))/(d*x^2*e^(2*d*sq
rt(x) + 2*c) - d*x^2) + integrate((2*a*b*d*x + 3*b^2*sqrt(x))/(d*x^3*e^(d*s
qrt(x) + c) + d*x^3), x) - integrate(-(2*a*b*d*x - 3*b^2*sqrt(x))/(d*x^3*e^
(d*sqrt(x) + c) - d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{{\left(a+\frac{b}{\mathrm{sinh}\left(c+d\,\sqrt{x}\right)}\right)}^2}{x^2}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))^2/x^2,x)
[Out] int((a + b/sinh(c + d*x^(1/2)))^2/x^2, x)
```

3.41 $\int \frac{x^3}{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx$

Optimal result 268
Rubi [A] (verified) 269
Mathematica [A] (verified) 278
Maple [F] 279
Fricas [F] 279
Sympy [F] 279
Maxima [F] 280
Giac [F] 280
Mupad [F(-1)] 280

Optimal result

Integrand size = 20, antiderivative size = 897

$$\int \frac{x^3}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+ \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+ \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7}$$
$$- \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7}$$
$$- \frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8}$$
$$+ \frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8}$$

```
[Out] 1/4*x^4/a-2*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d/(a^2
+b^2)^(1/2)+2*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d/(a
^2+b^2)^(1/2)-14*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a
/d^2/(a^2+b^2)^(1/2)+14*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1
/2)))/a/d^2/(a^2+b^2)^(1/2)+84*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(
a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-84*b*x^(5/2)*polylog(3,-a*exp(c+d*x^
(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-420*b*x^2*polylog(4,-a*ex
p(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)+420*b*x^2*polylog
(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)+1680*b*x^
(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^5/(a^2+b^2)^(1
/2)-1680*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^5
/(a^2+b^2)^(1/2)-5040*b*x*polylog(6,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2))
)/a/d^6/(a^2+b^2)^(1/2)+5040*b*x*polylog(6,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)
^(1/2)))/a/d^6/(a^2+b^2)^(1/2)-10080*b*polylog(8,-a*exp(c+d*x^(1/2))/(b-(a^
2+b^2)^(1/2)))/a/d^8/(a^2+b^2)^(1/2)+10080*b*polylog(8,-a*exp(c+d*x^(1/2))/
(b+(a^2+b^2)^(1/2)))/a/d^8/(a^2+b^2)^(1/2)+10080*b*polylog(7,-a*exp(c+d*x^(
1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a/d^7/(a^2+b^2)^(1/2)-10080*b*polylog(7,
-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a/d^7/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

$= \{5545, 4276, 3403, 2296, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{x^3}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a} - \frac{2b\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a\sqrt{a^2+b^2}d} + \frac{2b\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a\sqrt{a^2+b^2}d} - \frac{14b\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^3}{a\sqrt{a^2+b^2}d^2} + \frac{14b\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^3}{a\sqrt{a^2+b^2}d^2} + \frac{84b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^{5/2}}{a\sqrt{a^2+b^2}d^3} - \frac{84b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^{5/2}}{a\sqrt{a^2+b^2}d^3} - \frac{420b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^2}{a\sqrt{a^2+b^2}d^4} + \frac{420b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^2}{a\sqrt{a^2+b^2}d^4} + \frac{1680b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^{3/2}}{a\sqrt{a^2+b^2}d^5} - \frac{1680b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^{3/2}}{a\sqrt{a^2+b^2}d^5} - \frac{5040b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x}{a\sqrt{a^2+b^2}d^6} + \frac{5040b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x}{a\sqrt{a^2+b^2}d^6} + \frac{10080b\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)\sqrt{x}}{a\sqrt{a^2+b^2}d^7} - \frac{10080b\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)\sqrt{x}}{a\sqrt{a^2+b^2}d^7} - \frac{10080b\,\text{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8} + \frac{10080b\,\text{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8}$$

```
[In] Int[x^3/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] x^4/(4*a) - (2*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]
)])/(a*Sqrt[a^2 + b^2]*d) + (2*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (14*b*x^3*PolyLog[2, -((a*E^(c
+ d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (14*b*x^3*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b
^2]*d^2) + (84*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
 b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (84*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (420*b*x^2*Pol
yLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]
*d^4) + (420*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])
)])/(a*Sqrt[a^2 + b^2]*d^4) + (1680*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt
[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^5) - (1680*b*x^(3/2)*Po
lyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2
]*d^5) - (5040*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])
)])/(a*Sqrt[a^2 + b^2]*d^6) + (5040*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/
(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^6) + (10080*b*Sqrt[x]*PolyLog
[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^7
) - (10080*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2
]))])/(a*Sqrt[a^2 + b^2]*d^7) - (10080*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))
/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^8) + (10080*b*PolyLog[8, -((
a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^8)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^7}{a + b\text{csch}(c + dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^7}{a} - \frac{bx^7}{a(b + a\sinh(c + dx))}\right) dx, x, \sqrt{x}\right)$$

$$= \frac{x^4}{4a} - \frac{(2b)\text{Subst}\left(\int \frac{x^7}{b+a\sinh(c+dx)} dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{-a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{(14b)\text{Subst}\left(\int x^6\log\left(1 + \frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d} - \frac{(14b)\text{Subst}\left(\int x^6\log\left(1 + \frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{14bx^3\,\text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2, -\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2} - \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2, -\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{14bx^3\,\text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{84bx^{5/2}\,\text{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{(420b)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3, -\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3} + \frac{(420b)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3, -\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{(1680b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$- \frac{(1680b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{(5040b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$+ \frac{(5040b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^5}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} + \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+ \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$- \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^6}$$

$$
\begin{aligned}
&= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&- \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} \\
&+ \frac{84bx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} \\
&- \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} \\
&+ \frac{1680bx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{1680bx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} \\
&- \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} + \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} \\
&+ \frac{10080b\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7} - \frac{10080b\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7} \\
&- \frac{(10080b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(7,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^7} \\
&+ \frac{(10080b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(7,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^7}
\end{aligned}
$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$

$$+ \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$

$$- \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} + \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$

$$+ \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7} - \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7}$$

$$- \frac{(10080b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(7,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^8}$$

$$+ \frac{(10080b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(7,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^8}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} + \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+ \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7} - \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^7}$$
$$- \frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8} + \frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^8}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 685, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}d^8x^4 - 8bd^7x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) + 8bd^7x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) - 56bd^6x^3\operatorname{PolyLog}\left(2,\frac{ae^{c+d\sqrt{x}}}{-b+\sqrt{a^2+}}\right.}{}$$

```
[In] Integrate[x^3/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 + b^2]*d^8*x^4 - 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[a^2 + b^2])] + 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2])] - 56*b*d^6*x^3*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt
[a^2 + b^2])] + 56*b*d^6*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))] + 336*b*d^5*x^(5/2)*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqr
t[a^2 + b^2])] - 336*b*d^5*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b +
Sqrt[a^2 + b^2]))] - 1680*b*d^4*x^2*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b +
Sqrt[a^2 + b^2])] + 1680*b*d^4*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b +
Sqrt[a^2 + b^2]))] + 6720*b*d^3*x^(3/2)*PolyLog[5, (a*E^(c + d*Sqrt[x]))/(-
b + Sqrt[a^2 + b^2])] - 6720*b*d^3*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]
```

```
))/(b + Sqrt[a^2 + b^2]))] - 20160*b*d^2*x*PolyLog[6, (a*E^(c + d*Sqrt[x]))
/(-b + Sqrt[a^2 + b^2])] + 20160*b*d^2*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))
/(b + Sqrt[a^2 + b^2]))] + 40320*b*d*Sqrt[x]*PolyLog[7, (a*E^(c + d*Sqrt[x]
))/(-b + Sqrt[a^2 + b^2])] - 40320*b*d*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[a^2 + b^2]))] - 40320*b*PolyLog[8, (a*E^(c + d*Sqrt[x]))/(
-b + Sqrt[a^2 + b^2])] + 40320*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[a^2 + b^2]))])/(4*a*Sqrt[a^2 + b^2]*d^8)
```

Maple [F]

$$\int \frac{x^3}{a + b \operatorname{csch}\left(c + d\sqrt{x}\right)} dx$$

```
[In] int(x^3/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x^3/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^3}{a + b\operatorname{csch}\left(c + d\sqrt{x}\right)} dx = \int \frac{x^3}{b \operatorname{csch}\left(d\sqrt{x} + c\right) + a} dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^3/(b*csch(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{a + b\operatorname{csch}\left(c + d\sqrt{x}\right)} dx = \int \frac{x^3}{a + b \operatorname{csch}\left(c + d\sqrt{x}\right)} dx$$

```
[In] integrate(x**3/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**3/(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{x^3}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/4*x^4/a - 2*b*integrate(x^3*e^(d*sqrt(x) + c)/(a^2*e^(2*d*sqrt(x) + 2*c)
+ 2*a*b*e^(d*sqrt(x) + c) - a^2), x)
```

Giac [F]

$$\int \frac{x^3}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^3/(b*csch(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a + \frac{b}{\sinh\left(c+d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^3/(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^3/(a + b/sinh(c + d*x^(1/2))), x)
```

3.42 $\int \frac{x^2}{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx$

Optimal result 282
Rubi [A] (verified) 283
Mathematica [A] (verified) 289
Maple [F] 289
Fricas [F] 289
Sympy [F] 290
Maxima [F] 290
Giac [F] 290
Mupad [F(-1)] 290

Optimal result

Integrand size = 20, antiderivative size = 673

$$\int \frac{x^2}{a+b\,\text{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{x^3}{3a}-\frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}+\frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$-\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$-\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$-\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$

```
[Out] 1/3*x^3/a-2*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d/(a^2
+b^2)^(1/2)+2*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d/(a
^2+b^2)^(1/2)-10*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a
/d^2/(a^2+b^2)^(1/2)+10*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1
/2)))/a/d^2/(a^2+b^2)^(1/2)+40*b*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(
a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-40*b*x^(3/2)*polylog(3,-a*exp(c+d*x^
(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-120*b*x*polylog(4,-a*exp(
c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)+120*b*x*polylog(4,-
a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)-240*b*polylog
```

```
(6,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^6/(a^2+b^2)^(1/2)+240*b*pol
ylog(6,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^6/(a^2+b^2)^(1/2)+240*b
*polylog(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a/d^5/(a^2+b^2)
^(1/2)-240*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a/d
^5/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5545, 4276, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^2}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx = -\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^6\sqrt{a^2+b^2}} + \frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^6\sqrt{a^2+b^2}} + \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} - \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} - \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} - \frac{2bx^{5/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}} + \frac{2bx^{5/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{a^2+b^2}+b}+1\right)}{ad\sqrt{a^2+b^2}} + \frac{x^3}{3a}$$

```
[In] Int[x^2/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] x^3/(3*a) - (2*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]
)])/(a*Sqrt[a^2 + b^2]*d) + (2*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (10*b*x^2*PolyLog[2, -((a*E^(c
+ d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (10*b*x^2*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b
^2]*d^2) + (40*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
 b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (40*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (120*b*x*PolyL
og[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d
^4) + (120*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/
(a*Sqrt[a^2 + b^2]*d^4) + (240*b*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))
/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^5) - (240*b*Sqrt[x]*PolyLog[
5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^5)
 - (240*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sq
rt[a^2 + b^2]*d^6) + (240*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^
2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^6)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^5}{a}-\frac{bx^5}{a(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^3}{3a}-\frac{(2b)\text{Subst}\left(\int \frac{x^5}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}
\end{aligned}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{(10b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d} - \frac{(10b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2} - \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3} + \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{(240b)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(4, -\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$- \frac{(240b)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(4, -\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5, -\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$+ \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5, -\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^5}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(5,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^6}$$
$$+ \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(5,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^6}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$
$$+ \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$
$$- \frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6} + \frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^6}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}d^6x^3-6bd^5x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)+6bd^5x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)-30bd^4x^2\,\text{PolyLog}\left(2,\frac{ae^{c+d}}{-b+\sqrt{a^2}}\right.}{}$$

```
[In] Integrate[x^2/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 + b^2]*d^6*x^3 - 6*b*d^5*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[a^2 + b^2])] + 6*b*d^5*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2])] - 30*b*d^4*x^2*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt
[a^2 + b^2])] + 30*b*d^4*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))] + 120*b*d^3*x^(3/2)*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqr
t[a^2 + b^2])] - 120*b*d^3*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b +
Sqrt[a^2 + b^2]))] - 360*b*d^2*x*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b + Sqr
t[a^2 + b^2])] + 360*b*d^2*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))] + 720*b*d*Sqrt[x]*PolyLog[5, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[
a^2 + b^2])] - 720*b*d*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt
[a^2 + b^2]))] - 720*b*PolyLog[6, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2 + b^
2])] + 720*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(3
*a*Sqrt[a^2 + b^2]*d^6)
```

Maple [F]

$$\int \frac{x^2}{a+b\ \text{csch}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^2/(a+b*csch(c+d*x^(1/2))),x)
```

```
[Out] int(x^2/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^2}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*csch(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a + b\,\text{csch}\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**2/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**2/(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{x^2}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/3*x^3/a - 2*b*integrate(x^2*e^(d*sqrt(x) + c)/(a^2*e^(2*d*sqrt(x) + 2*c)
+ 2*a*b*e^(d*sqrt(x) + c) - a^2), x)
```

Giac [F]

$$\int \frac{x^2}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^2/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^2/(b*csch(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a + \frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x^2/(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^2/(a + b/sinh(c + d*x^(1/2))), x)
```

3.43 $\int \frac{x}{a+b\text{csch}(c+d\sqrt{x})}\,dx$

Optimal result 291
Rubi [A] (verified) 292
Mathematica [A] (verified) 296
Maple [F] 296
Fricas [F] 296
Sympy [F] 297
Maxima [F] 297
Giac [F] 297
Mupad [F(-1)] 297

Optimal result

Integrand size = 18, antiderivative size = 449

$$\begin{aligned}\int \frac{x}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx &= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\ &\quad - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} \\ &\quad + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} \\ &\quad - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} \\ &\quad - \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}\end{aligned}$$

```
[Out] 1/2*x^2/a-2*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d/(a^2
+b^2)^(1/2)+2*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d/(a
^2+b^2)^(1/2)-6*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^
2/(a^2+b^2)^(1/2)+6*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/
a/d^2/(a^2+b^2)^(1/2)-12*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)
))/a/d^4/(a^2+b^2)^(1/2)+12*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1
/2)))/a/d^4/(a^2+b^2)^(1/2)+12*b*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)
^(1/2)))*x^(1/2)/a/d^3/(a^2+b^2)^(1/2)-12*b*polylog(3,-a*exp(c+d*x^(1/2))/(
b+(a^2+b^2)^(1/2)))*x^(1/2)/a/d^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5545, 4276, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx = -\frac{12b\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}}+\frac{12b\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}}+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{2bx^{3/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{2bx^{3/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{a^2+b^2}+b}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] x^2/(2*a) - (2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]
)])/(a*Sqrt[a^2 + b^2]*d) + (2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (6*b*x*PolyLog[2, -((a*E^(c + d
*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (6*b*x*PolyLo
g[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^
2) + (12*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])
)])/(a*Sqrt[a^2 + b^2]*d^3) - (12*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x
]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (12*b*PolyLog[4, -((
a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^4) + (12
*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
+ b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^3}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^3}{a} - \frac{bx^3}{a(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^2}{2a} - \frac{(2b)\text{Subst}\left(\int \frac{x^3}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^2}{2a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^2}{2a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{(6b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d} - \frac{(6b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2} - \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3} + \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^4}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}d^4x^2-4bd^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)+4bd^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)-12bd^2x\,\text{PolyLog}\left(2,\frac{ae^{c+d\sqrt{x}}}{-b+\sqrt{a^2+b^2}}\right.}{}$$

```
[In] Integrate[x/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 + b^2]*d^4*x^2 - 4*b*d^3*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[a^2 + b^2])] + 4*b*d^3*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2])] - 12*b*d^2*x*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a
^2 + b^2])] + 12*b*d^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 +
 b^2]))] + 24*b*d*Sqrt[x]*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2 +
 b^2])] - 24*b*d*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 +
 b^2]))] - 24*b*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2 + b^2])] +
24*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(2*a*Sqrt[
a^2 + b^2]*d^4)
```

Maple [F]

$$\int \frac{x}{a+b\ \text{csch}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x/(a+b*csch(c+d*x^(1/2))),x)
```

```
[Out] int(x/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(x/(b*csch(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{a + b\mathrm{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{a + b\operatorname{csch}\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x/(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{x}{a + b\mathrm{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\operatorname{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(x*e^(d*sqrt(x) + c)/(a^2*e^(2*d*sqrt(x) + 2*c) + 2*a*b*e^(d*
sqrt(x) + c) - a^2), x) + 1/2*x^2/a
```

Giac [F]

$$\int \frac{x}{a + b\mathrm{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\operatorname{csch}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x/(b*csch(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b\mathrm{csch}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}}\,dx$$

```
[In] int(x/(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x/(a + b/sinh(c + d*x^(1/2))), x)
```

3.44 $\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result 298
Rubi [N/A] 298
Mathematica [N/A] 299
Maple [N/A] (verified) 299
Fricas [N/A] 299
Sympy [N/A] 299
Maxima [N/A] 300
Giac [N/A] 300
Mupad [N/A] 300

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csch(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Csch[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csch[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\ \text{csch}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(1/x/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(1/(b*x*csch(d*sqrt(x) + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(1/(x*(a + b*csch(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x*e^(2*d*sqrt(x) + 2*c) + 2*a*b*x*e^(
d*sqrt(x) + c) - a^2*x), x) + log(x)/a
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(1/((b*csch(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/sinh(c + d*x^(1/2)))),x)
[Out] int(1/(x*(a + b/sinh(c + d*x^(1/2)))), x)
```

3.45 $\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^2}\,dx$

Optimal result 301
Rubi [N/A] 301
Mathematica [N/A] 302
Maple [N/A] (verified) 302
Fricas [N/A] 302
Sympy [N/A] 302
Maxima [N/A] 303
Giac [N/A] 303
Mupad [N/A] 303

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^2}\,dx = -\frac{a}{x}+b\text{Int}\left(\frac{\operatorname{csch}(c+d\sqrt{x})}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csch(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^2}\,dx = \int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^2}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Csch[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x^2}+\frac{b\operatorname{csch}(c+d\sqrt{x})}{x^2}\right)dx \\ &= -\frac{a}{x}+b\int \frac{\operatorname{csch}(c+d\sqrt{x})}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\ \text{csch}\left(c+d\sqrt{x}\right)}{x^{2}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*csch(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^{2}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*csch(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\,\text{csch}\left(c+d\sqrt{x}\right)}{x^{2}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^2} dx = \int \frac{b \operatorname{csch}\left(d\sqrt{x} + c\right) + a}{x^2} dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] b*integrate(1/(x^2*e^(d*sqrt(x) + c) + x^2), x) + b*integrate(1/(x^2*e^(d*s
qrt(x) + c) - x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^2} dx = \int \frac{b \operatorname{csch}\left(d\sqrt{x} + c\right) + a}{x^2} dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^2} dx = \int \frac{a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}}{x^2} dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x^2,x)
```

```
[Out] int((a + b/sinh(c + d*x^(1/2)))/x^2, x)
```

3.46 $\int \frac{x^3}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 305
Rubi [A] (verified) 307
Mathematica [A] (verified) 315
Maple [F] 316
Fricas [F] 317
Sympy [F] 317
Maxima [F] 317
Giac [F] 318
Mupad [F(-1)] 318

Optimal result

Integrand size = 20, antiderivative size = 2663

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2b^2x^{7/2}}{a^2\left(a^2+b^2\right)d}+\frac{x^4}{4a^2}+\frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}$$

$$+\frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}$$

$$-\frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}$$

$$-\frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}+\frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$+\frac{84b^2x^{5/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3}$$

$$+\frac{14b^3x^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$

$$-\frac{28bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$

$$+\frac{84b^2x^{5/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3}$$

$$-\frac{14b^3x^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$

$$+\frac{28bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$

$$-\frac{420b^2x^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^4}$$

$$-\frac{84b^3x^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$

$$+\frac{168bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$

$$-\frac{420b^2x^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^4}$$

$$+\frac{84b^3x^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$

$$-\frac{168bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$

$$1680b^2x^{3/2}\,\text{PolyLog}\left(4,\ ae^{c+d\sqrt{x}}\right.$$

```
[Out] 1/4*x^4/a^2+1680*b^2*x^(3/2)*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/
2)))/a^2/(a^2+b^2)/d^5+420*b^3*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^
2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^4+1680*b^2*x^(3/2)*polylog(4,-a*exp(c+d*x^
(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^5-420*b^3*x^2*polylog(4,-a*exp(
c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^4-5040*b^2*x*polylo
g(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^6-1680*b^3*x^(
3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)
/d^5-5040*b^2*x*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2
+b^2)/d^6+1680*b^3*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)
))/a^2/(a^2+b^2)^(3/2)/d^5+5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/2))/(b-(a^2
+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^6-5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/
2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^6-4*b*x^(7/2)*ln(1+a*exp(c+d
*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+4*b*x^(7/2)*ln(1+a*exp
(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)-28*b*x^3*polylog(2
,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+28*b*x^3*
polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-
2*b^2*x^(7/2)/a^2/(a^2+b^2)/d-20160*b*polylog(8,-a*exp(c+d*x^(1/2))/(b-(a^2
+b^2)^(1/2)))/a^2/d^8/(a^2+b^2)^(1/2)+20160*b*polylog(8,-a*exp(c+d*x^(1/2))
/(b+(a^2+b^2)^(1/2)))/a^2/d^8/(a^2+b^2)^(1/2)-10080*b^3*polylog(8,-a*exp(c+
d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^8-10080*b^2*polylog(7
,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^8+10080*b^3*polyl
og(8,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^8-10080
*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^8+1
68*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^3/(a^
2+b^2)^(1/2)-168*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)
))/a^2/d^3/(a^2+b^2)^(1/2)-840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+
b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2)+840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2)
)/(b+(a^2+b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2)+3360*b*x^(3/2)*polylog(5,-a*
exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^5/(a^2+b^2)^(1/2)-3360*b*x^(3/2
)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^5/(a^2+b^2)^(1/2
)-10080*b*x*polylog(6,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^6/(a^2
+b^2)^(1/2)+10080*b*x*polylog(6,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^
2/d^6/(a^2+b^2)^(1/2)+10080*b^2*polylog(6,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^
(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^7+10080*b^2*polylog(6,-a*exp(c+d*x^(1/2))/(
b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^7-10080*b^3*polylog(7,-a*exp(c+
d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^7+10080*b^3*p
olylog(7,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/
2)/d^7+20160*b*polylog(7,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a
^2/d^7/(a^2+b^2)^(1/2)-20160*b*polylog(7,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(
1/2)))*x^(1/2)/a^2/d^7/(a^2+b^2)^(1/2)+14*b^2*x^3*ln(1+a*exp(c+d*x^(1/2))/(
b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))
/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+14*b^2*x^3*ln(1+a*exp(c+d*x^(1/
2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2-2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(
1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+84*b^2*x^(5/2)*polylog(2,-
a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3+14*b^3*x^3*polylo
```

```
g(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2+84*b^2
*x^(5/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d
^3-14*b^3*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b
^2)^(3/2)/d^2-420*b^2*x^2*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2))
)/a^2/(a^2+b^2)/d^4-84*b^3*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^
2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-420*b^2*x^2*polylog(3,-a*exp(c+d*x^(1/2)
)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+84*b^3*x^(5/2)*polylog(3,-a*exp(c+
d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-2*b^2*x^(7/2)*cosh(
c+d*x^(1/2))/a/(a^2+b^2)/d/(b+a*sinh(c+d*x^(1/2)))
```

Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 2663, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 6744, 2320, 6724, 5680}

$$\int \frac{x^3}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{x^4}{4a^2} - \frac{4b\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a^2\sqrt{a^2+b^2}d} + \frac{2b^3\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a^2\left(a^2+b^2\right)^{3/2}d} + \frac{4b\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a^2\sqrt{a^2+b^2}d} - \frac{2b^3\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)x^{7/2}}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2b^2x^{7/2}}{a^2\left(a^2+b^2\right)d} - \frac{2b^2\cosh\left(c+d\sqrt{x}\right)x^{7/2}}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} + \frac{14b^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)x^3}{a^2\left(a^2+b^2\right)d^2} + \frac{14b^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)x^3}{a^2\left(a^2+b^2\right)d^2} - \frac{28b\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^3}{a^2\sqrt{a^2+b^2}d^2} + \frac{14b^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^3}{a^2\left(a^2+b^2\right)^{3/2}d^2} + \frac{28b\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^3}{a^2\sqrt{a^2+b^2}d^2} - \frac{14b^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^3}{a^2\left(a^2+b^2\right)^{3/2}d^2} + \frac{84b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\left(a^2+b^2\right)d^3} + \frac{84b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\left(a^2+b^2\right)d^3} + \frac{168b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\sqrt{a^2+b^2}d^3} - \frac{84b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\left(a^2+b^2\right)^{3/2}d^3} - \frac{168b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\sqrt{a^2+b^2}d^3} + \frac{84b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^{5/2}}{a^2\left(a^2+b^2\right)^{3/2}d^3} - \frac{420b^2\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)x^2}{a^2\left(a^2+b^2\right)d^4} - \frac{420b^2\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)x^2}{a^2\left(a^2+b^2\right)d^4}$$

```
[In] Int[x^3/(a + b*Csch[c + d*Sqrt[x]])^2,x]

[Out] (-2*b^2*x^(7/2))/(a^2*(a^2 + b^2)*d) + x^4/(4*a^2) + (14*b^2*x^3*Log[1 + (a
*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (2*b^3*
x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b
^2)^(3/2)*d) - (4*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b
^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (14*b^2*x^3*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (2*b^3*x^(7/2)*Log[1 + (a*E
^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) + (4*b*
x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2
 + b^2]*d) + (84*b^2*x^(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (14*b^3*x^3*PolyLog[2, -((a*E^(c + d*
Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (28*b*x^3*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
 b^2]*d^2) + (84*b^2*x^(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (14*b^3*x^3*PolyLog[2, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (28*b*x^3*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
 b^2]*d^2) - (420*b^2*x^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2
+ b^2]))])/(a^2*(a^2 + b^2)*d^4) - (84*b^3*x^(5/2)*PolyLog[3, -((a*E^(c + d
*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (168*b*x^
(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt
[a^2 + b^2]*d^3) - (420*b^2*x^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqr
t[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^4) + (84*b^3*x^(5/2)*PolyLog[3, -((a*E^
(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (16
8*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^
2*Sqrt[a^2 + b^2]*d^3) + (1680*b^2*x^(3/2)*PolyLog[4, -((a*E^(c + d*Sqrt[x]
))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^5) + (420*b^3*x^2*PolyLog[4,
 -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^
4) - (840*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])
/(a^2*Sqrt[a^2 + b^2]*d^4) + (1680*b^2*x^(3/2)*PolyLog[4, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^5) - (420*b^3*x^2*PolyLo
g[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2
)*d^4) + (840*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]
))])/(a^2*Sqrt[a^2 + b^2]*d^4) - (5040*b^2*x*PolyLog[5, -((a*E^(c + d*Sqrt[
x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^6) - (1680*b^3*x^(3/2)*Pol
yLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(
3/2)*d^5) + (3360*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^
2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^5) - (5040*b^2*x*PolyLog[5, -((a*E^(c +
 d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^6) + (1680*b^3*x^(
3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2
+ b^2)^(3/2)*d^5) - (3360*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^5) + (10080*b^2*Sqrt[x]*PolyLog
[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^7)
+ (5040*b^3*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(
```

```
a^2*(a^2 + b^2)^(3/2)*d^6) - (10080*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/
(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^6) + (10080*b^2*Sqrt[x]*Pol
yLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d
^7) - (5040*b^3*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))
])/(a^2*(a^2 + b^2)^(3/2)*d^6) + (10080*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x
]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^6) - (10080*b^2*PolyLog
[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^8)
- (10080*b^3*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2
]))])/(a^2*(a^2 + b^2)^(3/2)*d^7) + (20160*b*Sqrt[x]*PolyLog[7, -((a*E^(c +
 d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^7) - (10080*b^
2*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b
^2)*d^8) + (10080*b^3*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[
a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^7) - (20160*b*Sqrt[x]*PolyLog[7, -(
(a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^7) +
(10080*b^3*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2
*(a^2 + b^2)^(3/2)*d^8) - (20160*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b -
Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^8) - (10080*b^3*PolyLog[8, -((a*
E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^8) + (
20160*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sq
rt[a^2 + b^2]*d^8) - (2*b^2*x^(7/2)*Cosh[c + d*Sqrt[x]])/(a*(a^2 + b^2)*d*(
b + a*Sinh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{(a+b\operatorname{csch}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^7}{a^2}+\frac{b^2x^7}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^7}{a^2(b+a\sinh(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^4}{4a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^7}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^7}{(b+a\sinh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^4}{4a^2}-\frac{2b^2x^{7/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad+\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^7}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}+\frac{(14b^2)\,\text{Subst}\left(\int \frac{x^6\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}\\
&= -\frac{2b^2x^{7/2}}{a^2\left(a^2+b^2\right)d}+\frac{x^4}{4a^2}-\frac{2b^2x^{7/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}\\
&\quad+\frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^7}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}\\
&\quad-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}\\
&\quad+\frac{(14b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^6}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}\\
&\quad+\frac{(14b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^6}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}
\end{aligned}$$

$$= -\frac{2b^2x^{7/2}}{a^2\left(a^2+b^2\right)d} + \frac{x^4}{4a^2} + \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}$$
$$- \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}$$
$$+ \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{2b^2x^{7/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^7}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(a^2+b^2\right)^{3/2}}$$
$$- \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^7}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(a^2+b^2\right)^{3/2}}$$
$$- \frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\left(a^2+b^2\right)d^2}$$
$$- \frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\left(a^2+b^2\right)d^2}$$
$$+ \frac{(28b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{(28b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$= -\frac{2b^2x^{7/2}}{a^2(a^2+b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$

$$+ \frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$+ \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$

$$+ \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$

$$- \frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$

$$+ \frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2x^{7/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$

$$- \frac{(420b^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$

$$- \frac{(420b^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$

$$+ \frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$

$$- \frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$

$$- \frac{(14b^3)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$

$$+ \frac{(14b^3)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 2841, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^3/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Csch[c + d*Sqrt[x]]^2*(b + a*Sinh[c + d*Sqrt[x]])*(x^4*(b + a*Sinh[c + d*S
qrt[x]]) - (8*b*E^c*(2*b*E^c*x^(7/2) + ((-1 + E^(2*c))*(-7*b*d^6*Sqrt[(a^2
+ b^2)*E^(2*c)]*x^3*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])] + 2*a^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])] + b^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d
*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 7*b*d^6*Sqrt[(a^2 + b^2)*
E^(2*c)]*x^3*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)])] - 2*a^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])] - b^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x
]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 7*d^5*(-6*b*Sqrt[(a^2 + b^2)*E^(
2*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^
(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 7*d^5*(6*b*Sqrt[
(a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(5/2)*Pol
yLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2
10*b*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]*x^2*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 84*a^2*d^5*E^c*x^(5/2)*PolyLog[3,
-((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 42*b^2*d^
5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))] + 210*b*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]*x^2*PolyLog[3, -((a*E^(
2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 84*a^2*d^5*E^c*x^
(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)]))] + 42*b^2*d^5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))] - 840*b*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]*x^(3/2
)*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
] + 420*a^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] + 210*b^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sq
rt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 840*b*d^3*Sqrt[(a^2 + b^2)*
E^(2*c)]*x^(3/2)*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)]))] - 420*a^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 210*b^2*d^4*E^c*x^2*PolyLog[4, -((
a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2520*b*d^2*S
qrt[(a^2 + b^2)*E^(2*c)]*x*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)]))] - 1680*a^2*d^3*E^c*x^(3/2)*PolyLog[5, -((a*E^(2*
c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 840*b^2*d^3*E^c*x^(
3/2)*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))] + 2520*b*d^2*Sqrt[(a^2 + b^2)*E^(2*c)]*x*PolyLog[5, -((a*E^(2*c + d*Sq
```

```
rt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 1680*a^2*d^3*E^c*x^(3/2)*Po
lyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] +
840*b^2*d^3*E^c*x^(3/2)*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 5040*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLo
g[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5040
*a^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)]))] + 2520*b^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 5040*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sq
rt[x]*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)]))] - 5040*a^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] - 2520*b^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*
Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5040*b*Sqrt[(a^2 + b^2)*E
^(2*c)]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] - 10080*a^2*d*E^c*Sqrt[x]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 5040*b^2*d*E^c*Sqrt[x]*PolyLog[7, -((a*
E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5040*b*Sqrt[(a
^2 + b^2)*E^(2*c)]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2
+ b^2)*E^(2*c)]))] + 10080*a^2*d*E^c*Sqrt[x]*PolyLog[7, -((a*E^(2*c + d*Sqr
t[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5040*b^2*d*E^c*Sqrt[x]*PolyL
og[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 100
80*a^2*E^c*PolyLog[8, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] + 5040*b^2*E^c*PolyLog[8, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqr
t[(a^2 + b^2)*E^(2*c)]))] - 10080*a^2*E^c*PolyLog[8, -((a*E^(2*c + d*Sqrt[x
]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 5040*b^2*E^c*PolyLog[8, -((a*E^
(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^7*E^c*Sqrt[(a
^2 + b^2)*E^(2*c)]))*(b + a*Sinh[c + d*Sqrt[x]]))/((a^2 + b^2)*d*(-1 + E^(2
*c))) + (8*b^2*x^(7/2)*Csch[c]*(b*Cosh[c] + a*Sinh[d*Sqrt[x]]))/((a^2 + b^2
)*d)))/(4*a^2*(a + b*Csch[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

[In] `int(x^3/(a+b*csch(c+d*x^(1/2)))^2,x)`

[Out] `int(x^3/(a+b*csch(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^3/(b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2),
 x)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**3/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3/(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/4*(16*a*b^2*x^(7/2) - (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^4*e^(2*d*sqrt(x
)) + (a^3*d + a*b^2*d)*x^4 - 2*(8*b^3*x^(7/2)*e^c + (a^2*b*d*e^c + b^3*d*e^
c)*x^4)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2
*c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))) - int
egrate(2*(7*a*b^2*x^(5/2) - (7*b^3*x^(5/2)*e^c + (2*a^2*b*d*e^c + b^3*d*e^c
)*x^3)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*
c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))), x)
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^3/(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.47 $\int \frac{x^2}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 320
Rubi [A] (verified) 322
Mathematica [A] (verified) 330
Maple [F] 331
Fricas [F] 331
Sympy [F] 331
Maxima [F] 332
Giac [F] 332
Mupad [F(-1)] 332

Optimal result

Integrand size = 20, antiderivative size = 1983

$$
\begin{aligned}
\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = &-\frac{2b^2x^{5/2}}{a^2(a^2+b^2)\,d}+\frac{x^3}{3a^2}+\frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2} \\
&+\frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d} \\
&-\frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2} \\
&-\frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d}+\frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&+\frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3} \\
&+\frac{10b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2} \\
&-\frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&+\frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3} \\
&-\frac{10b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2} \\
&+\frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&-\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^4} \\
&-\frac{40b^3x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3} \\
&+\frac{80bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&-\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^4} \\
&+\frac{40b^3x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3} \\
&-\frac{80bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&\quad 240b^2\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{\ldots}\right)
\end{aligned}
$$

```
[Out] 10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+2
*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/
2)/d+10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/
d^2-2*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2
)^(3/2)/d+40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))
/a^2/(a^2+b^2)/d^3+10*b^3*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1
/2)))/a^2/(a^2+b^2)^(3/2)/d^2+40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/
(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-10*b^3*x^2*polylog(2,-a*exp(c+d*x^(1
/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-120*b^2*x*polylog(3,-a*ex
p(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4-40*b^3*x^(3/2)*polylo
g(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-120*b^
2*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+40
*b^3*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^
2)^(3/2)/d^3+120*b^3*x*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a
^2/(a^2+b^2)^(3/2)/d^4-120*b^3*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)
^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^4-4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^
2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b
+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)-20*b*x^2*polylog(2,-a*exp(c+d*x^(1
/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+20*b*x^2*polylog(2,-a*exp
(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+80*b*x^(3/2)*pol
ylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)-80*
b*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b
^2)^(1/2)-240*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^
4/(a^2+b^2)^(1/2)-2*b^2*x^(5/2)/a^2/(a^2+b^2)/d+240*b^3*polylog(6,-a*exp(c+
d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^6-240*b^2*polylog(5,-
a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^6-240*b^3*polylog(6
,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^6-480*b*pol
ylog(6,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^6/(a^2+b^2)^(1/2)+480
*b*polylog(6,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^6/(a^2+b^2)^(1/
2)-240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)
/d^6+240*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^4/(a^
2+b^2)^(1/2)+240*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(
1/2)/a^2/(a^2+b^2)/d^5+240*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(
1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^5-240*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b-(
a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^5+240*b^3*polylog(5,-a*exp(c
+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^5+480*b*poly
log(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(a^2+b^2)^(1
/2)-480*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^
5/(a^2+b^2)^(1/2)+1/3*x^3/a^2-2*b^2*x^(5/2)*cosh(c+d*x^(1/2))/a/(a^2+b^2)/d
/(b+a*sinh(c+d*x^(1/2)))
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 1983, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 6744, 2320, 6724, 5680}

$$\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^{5/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2x^{5/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d}$$

$$+ \frac{10x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$

$$- \frac{10x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$

$$- \frac{40x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$

$$+ \frac{40x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$

$$+ \frac{120x\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4}$$

$$- \frac{120x\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4}$$

$$- \frac{240\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^5}$$

$$+ \frac{240\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^5}$$

$$+ \frac{240\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^6}$$

$$- \frac{240\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^6} - \frac{2x^{5/2}b^2}{a^2\left(a^2+b^2\right)d}$$

$$+ \frac{10x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2} + \frac{10x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2}$$

$$+ \frac{40x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$

$$+ \frac{40x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$

$$- \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4}$$

$$- \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4}$$

$$+ \frac{240\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{}$$

```
[In] Int[x^2/(a + b*Csch[c + d*Sqrt[x]])^2,x]

[Out] (-2*b^2*x^(5/2))/(a^2*(a^2 + b^2)*d) + x^3/(3*a^2) + (10*b^2*x^2*Log[1 + (a
*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (2*b^3*
x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b
^2)^(3/2)*d) - (4*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b
^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (10*b^2*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (2*b^3*x^(5/2)*Log[1 + (a*E
^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) + (4*b*
x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2
 + b^2]*d) + (40*b^2*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (10*b^3*x^2*PolyLog[2, -((a*E^(c + d*
Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (20*b*x^2*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
 b^2]*d^2) + (40*b^2*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (10*b^3*x^2*PolyLog[2, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (20*b*x^2*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
 b^2]*d^2) - (120*b^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
b^2]))])/(a^2*(a^2 + b^2)*d^4) - (40*b^3*x^(3/2)*PolyLog[3, -((a*E^(c + d*S
qrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (80*b*x^(3/
2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^
2 + b^2]*d^3) - (120*b^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2
 + b^2]))])/(a^2*(a^2 + b^2)*d^4) + (40*b^3*x^(3/2)*PolyLog[3, -((a*E^(c +
d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (80*b*x^
(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt
[a^2 + b^2]*d^3) + (240*b^2*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b -
 Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^5) + (120*b^3*x*PolyLog[4, -((a*E^(
c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^4) - (240
*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[
a^2 + b^2]*d^4) + (240*b^2*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b +
Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^5) - (120*b^3*x*PolyLog[4, -((a*E^(c
 + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^4) + (240*
b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a
^2 + b^2]*d^4) - (240*b^2*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2
+ b^2]))])/(a^2*(a^2 + b^2)*d^6) - (240*b^3*Sqrt[x]*PolyLog[5, -((a*E^(c +
d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^5) + (480*b*S
qrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqr
t[a^2 + b^2]*d^5) - (240*b^2*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^6) + (240*b^3*Sqrt[x]*PolyLog[5, -((a*E^(c
 + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^5) - (480*
b*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*
Sqrt[a^2 + b^2]*d^5) + (240*b^3*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqr
t[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^6) - (480*b*PolyLog[6, -((a*E^(c
+ d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^6) - (240*b^3
```

```
*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^
2)^(3/2)*d^6) + (480*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b
^2]))])/(a^2*Sqrt[a^2 + b^2]*d^6) - (2*b^2*x^(5/2)*Cosh[c + d*Sqrt[x]])/(a*
(a^2 + b^2)*d*(b + a*Sinh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
```

```
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^5}{(a + b\text{csch}(c + dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^5}{a^2} + \frac{b^2x^5}{a^2(b+a\sinh(c+dx))^2} - \frac{2bx^5}{a^2(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^3}{3a^2} - \frac{(4b)\text{Subst}\left(\int \frac{x^5}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^5}{(b+a\sinh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= \frac{x^3}{3a^2} - \frac{2b^2x^{5/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$+ \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^5}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)} + \frac{(10b^2)\,\text{Subst}\left(\int \frac{x^4\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b^2x^{5/2}}{a^2\left(a^2+b^2\right)d} + \frac{x^3}{3a^2} - \frac{2b^2x^{5/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^5}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}$$
$$- \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^4}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}$$
$$+ \frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^4}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b^2x^{5/2}}{a^2(a^2+b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{2b^2x^{5/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\,\mathrm{Subst}\left(\int \frac{e^{c+dx}x^5}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\,\mathrm{Subst}\left(\int \frac{e^{c+dx}x^5}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(40b^2)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{(40b^2)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{(20b)\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{(20b)\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$= -\frac{2b^2x^{5/2}}{a^2(a^2+b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+ \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2x^{5/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(120b^2)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{(120b^2)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{(80b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(80b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(10b^3)\operatorname{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{(10b^3)\operatorname{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 2085, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Result too large to show}$$

```
[In] Integrate[x^2/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Csch[c + d*Sqrt[x]]^2*(b + a*Sinh[c + d*Sqrt[x]])*(x^3*(b + a*Sinh[c + d*S
qrt[x]]) - (6*b*E^c*(2*b*E^c*x^(5/2) + ((-1 + E^(2*c))*(-5*b*d^4*Sqrt[(a^2
+ b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])] + 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])] + b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d
*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 5*b*d^4*Sqrt[(a^2 + b^2)*
E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)])] - 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])] - b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x
]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 5*d^3*(-4*b*Sqrt[(a^2 + b^2)*E^(
2*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^
(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 5*d^3*(4*b*Sqrt[
(a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*Pol
yLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6
0*b*d^2*Sqrt[(a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((
a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 20*b^2*d^3*E
^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] + 60*b*d^2*Sqrt[(a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c +
d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 40*a^2*d^3*E^c*x^(3/2)*
PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
+ 20*b^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt
[(a^2 + b^2)*E^(2*c)]))] - 120*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLo
g[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 120*
a^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))] + 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 120*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sqrt[x
]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
] - 120*a^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))] - 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]
))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 120*b*Sqrt[(a^2 + b^2)*E^(2*c)]*
PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
- 240*a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*
Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 120*b*Sqrt[(a^2 + b^2)*E^
(2*c)]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*
```

```
c)]))] + 240*a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))] + 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2
*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 240*a^2*E^c*PolyLo
g[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 120*
b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] - 240*a^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))] - 120*b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^5*E^c*Sqrt[(a^2 + b^2)*E^(2*c)]))*
(b + a*Sinh[c + d*Sqrt[x]]))/((a^2 + b^2)*d*(-1 + E^(2*c))) + (6*b^2*x^(5/2
)*Csch[c]*(b*Cosh[c] + a*Sinh[d*Sqrt[x]]))/((a^2 + b^2)*d)))/(3*a^2*(a + b*
Csch[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\ \operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x^2/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(x^2/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2),
 x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**2/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2/(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^2}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/3*(12*a*b^2*x^(5/2) - (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^3*e^(2*d*sqrt(x)) + (a^3*d + a*b^2*d)*x^3 - 2*(6*b^3*x^(5/2)*e^c + (a^2*b*d*e^c + b^3*d*e^c)*x^3)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))) - integrate(2*(5*a*b^2*x^(3/2) - (5*b^3*x^(3/2)*e^c + (2*a^2*b*d*e^c + b^3*d*e^c)*x^2)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))), x)

Giac [F]

$$\int \frac{x^2}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^2}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x^2/(b*csch(d*sqrt(x) + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

[In] int(x^2/(a + b/sinh(c + d*x^(1/2)))^2,x)

[Out] int(x^2/(a + b/sinh(c + d*x^(1/2)))^2, x)

3.48 $\int \frac{x}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result . . . 334
Rubi [A] (verified) . . . 335
Mathematica [A] (verified) . . . 343
Maple [F] . . . 344
Fricas [F] . . . 344
Sympy [F] . . . 344
Maxima [F] . . . 344
Giac [F] . . . 345
Mupad [F(-1)] . . . 345

Optimal result

Integrand size = 18, antiderivative size = 1303

$$
\begin{aligned}
\int \frac{x}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = &-\frac{2b^2x^{3/2}}{a^2\left(a^2+b^2\right)d}+\frac{x^2}{2a^2}+\frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} \\
&+\frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} \\
&-\frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} \\
&-\frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}+\frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} \\
&+\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&+\frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2} \\
&-\frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&+\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&-\frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2} \\
&+\frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&-\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^4} \\
&-\frac{12b^3\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^3} \\
&+\frac{24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&-\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^4} \\
&+\frac{12b^3\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^3} \\
&-\frac{24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}
\end{aligned}
$$

```
[Out] 1/2*x^2/a^2-2*b^2*x^(3/2)/a^2/(a^2+b^2)/d-24*b*polylog(4,-a*exp(c+d*x^(1/2)
)/(b-(a^2+b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2)+24*b*polylog(4,-a*exp(c+d*x^
(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2)-12*b^2*polylog(3,-a*exp
(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4-12*b^2*polylog(3,-a*ex
p(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+12*b^3*polylog(4,-a*e
xp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^4-12*b^3*polylog
(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^4+12*b^2*
polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^
3-12*b^3*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^
2+b^2)^(3/2)/d^3+12*b^3*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*
x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^3+24*b*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+
b^2)^(1/2)))*x^(1/2)/a^2/d^3/(a^2+b^2)^(1/2)-24*b*polylog(3,-a*exp(c+d*x^(1
/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^3/(a^2+b^2)^(1/2)+6*b^2*x*ln(1+a*ex
p(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+2*b^3*x^(3/2)*ln(1+a*
exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+6*b^2*x*ln(1+a*
exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2-2*b^3*x^(3/2)*ln(1+
a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+6*b^3*x*polyl
og(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-6*b^3
*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d
^2-4*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)
^(1/2)+4*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d/(a^2+
b^2)^(1/2)-12*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^
2/(a^2+b^2)^(1/2)+12*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))
/a^2/d^2/(a^2+b^2)^(1/2)+12*b^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^
(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^3-2*b^2*x^(3/2)*cosh(c+d*x^(1/2))/a/(a^2+b^
2)/d/(b+a*sinh(c+d*x^(1/2)))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 1303, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 6744, 2320, 6724, 5680}

$$\int \frac{x}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d}$$
$$+ \frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$
$$- \frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$
$$- \frac{12\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$
$$+ \frac{12\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$
$$+ \frac{12\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4}$$
$$- \frac{12\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4} - \frac{2x^{3/2}b^2}{a^2\left(a^2+b^2\right)d}$$
$$+ \frac{6x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2} + \frac{6x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2}$$
$$+ \frac{12\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$
$$+ \frac{12\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$
$$- \frac{12\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4}$$
$$- \frac{12\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4}$$
$$- \frac{2x^{3/2}\cosh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{4x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d} + \frac{4x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{24\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^3}$$

```
[In] Int[x/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x^(3/2))/(a^2*(a^2 + b^2)*d) + x^2/(2*a^2) + (6*b^2*x*Log[1 + (a*E^
(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (2*b^3*x^(
3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)
^(3/2)*d) - (4*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]
)])/(a^2*Sqrt[a^2 + b^2]*d) + (6*b^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (2*b^3*x^(3/2)*Log[1 + (a*E^(c +
d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) + (4*b*x^(3/2
)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2
]*d) + (12*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b
^2]))])/(a^2*(a^2 + b^2)*d^3) + (6*b^3*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))
/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (12*b*x*PolyLog[2,
-((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2)
+ (12*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))
])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (12*b*x*PolyLog[2, -((a*
E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (12
*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2
+ b^2)*d^4) - (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[
a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (24*b*Sqrt[x]*PolyLog[3, -((a*
E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (12
*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2
+ b^2)*d^4) + (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[
a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (24*b*Sqrt[x]*PolyLog[3, -((a*
E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (12
*b^3*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2
+ b^2)^(3/2)*d^4) - (24*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2
+ b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) - (12*b^3*PolyLog[4, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^4) + (24*b*PolyLog
[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d
^4) - (2*b^2*x^(3/2)*Cosh[c + d*Sqrt[x]])/(a*(a^2 + b^2)*d*(b + a*Sinh[c +
d*Sqrt[x]]))
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2296</u>

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
```

```
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(a+b\text{csch}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a^2}+\frac{b^2x^3}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^3}{a^2(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^3}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^3}{(b+a\sinh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^2}{2a^2}-\frac{2b^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad+\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^3}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}+\frac{(6b^2)\,\text{Subst}\left(\int \frac{x^2\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}
\end{aligned}
$$

$$= -\frac{2b^2x^{3/2}}{a^2(a^2+b^2)d} + \frac{x^2}{2a^2} - \frac{2b^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^3}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)}$$
$$- \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(6b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)d}$$
$$+ \frac{(6b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)d}$$

$$= -\frac{2b^2x^{3/2}}{a^2(a^2+b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{2b^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(12b^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{(12b^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{(12b)\operatorname{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{(12b)\operatorname{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$= -\frac{2b^2x^{3/2}}{a^2(a^2+b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+ \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(12b^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{(12b^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{(24b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(24b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(6b^3)\,\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{(6b^3)\,\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$

$$= -\frac{2b^2x^{3/2}}{a^2(a^2+b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+ \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} - \frac{2b^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(12b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)d^4}$$
$$- \frac{(12b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)d^4}$$
$$- \frac{(24b)\text{Subst}\left(\int\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$+ \frac{(24b)\text{Subst}\left(\int\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{(12b^3)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{(12b^3)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$

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Mathematica [A] (verified)

Time = 7.78 (sec) , antiderivative size = 1333, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{\text{csch}^2\left(c+d\sqrt{x}\right)\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)\left(x^2\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)-\frac{4be^c\left(2be^cx^{3/2}+\frac{e^{-c}\left(-1+e^{2c}\right)\left(-3bd^2\sqrt{\left(a^2+b^2\right)}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[x/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Csch[c + d*Sqrt[x]]^2*(b + a*Sinh[c + d*Sqrt[x]])*(x^2*(b + a*Sinh[c + d*S
qrt[x]]) - (4*b*E^c*(2*b*E^c*x^(3/2) + ((-1 + E^(2*c))*(-3*b*d^2*Sqrt[(a^2
+ b^2)*E^(2*c)]*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)])] + 2*a^2*d^3*E^c*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)])] + b^2*d^3*E^c*x^(3/2)*Log[1 + (a*E^(2*c + d*S
qrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*b*d^2*Sqrt[(a^2 + b^2)*E^
(2*c)]*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
)] - 2*a^2*d^3*E^c*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])] - b^2*d^3*E^c*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/
(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + (-6*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sq
rt[x] + 6*a^2*d^2*E^c*x + 3*b^2*d^2*E^c*x)*PolyLog[2, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 3*d*(2*b*Sqrt[(a^2 + b^2)*E^(2
*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(
2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*b*Sqrt[(a^2 + b
^2)*E^(2*c)]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))] - 12*a^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*b^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E
^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*b*Sqrt[(a^2 +
 b^2)*E^(2*c)]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)]))] + 12*a^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/
(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*b^2*d*E^c*Sqrt[x]*PolyLog[3, -((a
*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*a^2*E^c*Po
lyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] +
6*b^2*E^c*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))] - 12*a^2*E^c*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)]))] - 6*b^2*E^c*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*
```

```
E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^3*E^c*Sqrt[(a^2 + b^2)*E^(2*c)]))*(
b + a*Sinh[c + d*Sqrt[x]]))/((a^2 + b^2)*d*(-1 + E^(2*c))) + (4*b^2*x^(3/2)
*Csch[c]*(b*Cosh[c] + a*Sinh[d*Sqrt[x]]))/((a^2 + b^2)*d)))/(2*a^2*(a + b*C
sch[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x}{\left(a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(x/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x}{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2), x
)
```

Sympy [F]

$$\int \frac{x}{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x/(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/2*(8*a*b^2*x^(3/2) - (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x)
) + (a^3*d + a*b^2*d)*x^2 - 2*(4*b^3*x^(3/2)*e^c + (a^2*b*d*e^c + b^3*d*e^c
```

```
)*x^2)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*
c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))) - inte
grate(2*(3*a*b^2*sqrt(x) - (3*b^3*sqrt(x)*e^c + (2*a^2*b*d*e^c + b^3*d*e^c)
*x)*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))
*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))), x)
```

Giac [F]

$$\int \frac{x}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x/(b*csch(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x/(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x/(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.49 $\int \frac{1}{x\left(a+b\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 346
Rubi [N/A] 346
Mathematica [N/A] 347
Maple [N/A] (verified) 347
Fricas [N/A] 347
Sympy [N/A] 348
Maxima [N/A] 348
Giac [N/A] 348
Mupad [N/A] 349

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 103.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(1/x/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*csch(d*sqrt(x) + c)^2 + 2*a*b*x*csch(d*sqrt(x) + c) + a^2
*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x*(a + b*csch(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 249, normalized size of antiderivative = 12.45

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 4*(b^3*sqrt(x)*e^(d*sqrt(x) + c) - a*b^2*sqrt(x))/((a^5*d*e^(2*c) + a^3*b^2
*d*e^(2*c))*x*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x*e^(d*sqrt
(x)) - (a^5*d + a^3*b^2*d)*x) + log(x)/a^2 + integrate(-2*(a*b^2*sqrt(x) -
(b^3*sqrt(x)*e^c - (2*a^2*b*d*e^c + b^3*d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^
(2*c) + a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c + a^2*b^3*d
*e^c)*x^2*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csch(d*sqrt(x) + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\,{\left(a+\frac{b}{\mathrm{sinh}\left(c+d\,\sqrt{x}\right)}\right)}^2} \,d x$$

```
[In] int(1/(x*(a + b/sinh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x*(a + b/sinh(c + d*x^(1/2)))^2), x)
```

3.50 $\int \frac{1}{x^2\left(a+b\,\mathbf{csch}(c+d\sqrt{x})\right)^2}\,dx$

Optimal result 350
Rubi [N/A] 350
Mathematica [N/A] 351
Maple [N/A] (verified) 351
Fricas [N/A] 351
Sympy [N/A] 352
Maxima [N/A] 352
Giac [N/A] 352
Mupad [N/A] 353

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Csch[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 69.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^2*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^2*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 \left(a+b \ \text{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*csch(d*sqrt(x) + c)^2 + 2*a*b*x^2*csch(d*sqrt(x) + c) +
 a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**2*(a + b*csch(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 15.90

$$\int \frac{1}{x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -(4*a*b^2*sqrt(x) + (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x)) - (
a^3*d + a*b^2*d)*x - 2*(2*b^3*sqrt(x)*e^c - (a^2*b*d*e^c + b^3*d*e^c)*x)*e^
(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x)) + 2*(
a^4*b*d*e^c + a^2*b^3*d*e^c)*x^2*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*x^2) +
 integrate(-2*(3*a*b^2*sqrt(x) - (3*b^3*sqrt(x)*e^c - (2*a^2*b*d*e^c + b^3*
d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^3*e^(2*d*sq
rt(x)) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^3*e^(d*sqrt(x)) - (a^5*d + a^3*b
^2*d)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\,{\left(a+\frac{b}{\mathrm{sinh}\left(c+d\,\sqrt{x}\right)}\right)}^2} \,d x$$

```
[In] int(1/(x^2*(a + b/sinh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^2*(a + b/sinh(c + d*x^(1/2)))^2), x)
```

3.51 $\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result 354
Rubi [A] (verified) 355
Mathematica [A] (verified) 358
Maple [F] 358
Fricas [F] 358
Sympy [F] 359
Maxima [A] (verification not implemented) 359
Giac [F] 359
Mupad [F(-1)] 360

Optimal result

Integrand size = 20, antiderivative size = 214

$$\begin{aligned}\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx&=\frac{2}{5}ax^{5/2}-\frac{4bx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{24bx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{48b\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{48b\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}\end{aligned}$$

```
[Out] 2/5*a*x^(5/2)-4*b*x^2*arctanh(exp(c+d*x^(1/2)))/d-8*b*x^(3/2)*polylog(2,-ex
p(c+d*x^(1/2)))/d^2+8*b*x^(3/2)*polylog(2,exp(c+d*x^(1/2)))/d^2+24*b*x*poly
log(3,-exp(c+d*x^(1/2)))/d^3-24*b*x*polylog(3,exp(c+d*x^(1/2)))/d^3+48*b*po
lylog(5,-exp(c+d*x^(1/2)))/d^5-48*b*polylog(5,exp(c+d*x^(1/2)))/d^5-48*b*po
lylog(4,-exp(c+d*x^(1/2)))*x^(1/2)/d^4+48*b*polylog(4,exp(c+d*x^(1/2)))*x^(
1/2)/d^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {14, 5545, 4267, 2611, 6744, 2320, 6724}

$$\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{48b\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{48b\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{24bx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[x^(3/2)*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(5/2))/5 - (4*b*x^2*ArcTanh[E^(c + d*Sqrt[x])])/d - (8*b*x^(3/2)*Pol
yLog[2, -E^(c + d*Sqrt[x])])/d^2 + (8*b*x^(3/2)*PolyLog[2, E^(c + d*Sqrt[x]
)])/d^2 + (24*b*x*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (24*b*x*PolyLog[3,
E^(c + d*Sqrt[x])])/d^3 - (48*b*Sqrt[x]*PolyLog[4, -E^(c + d*Sqrt[x])])/d^4
 + (48*b*Sqrt[x]*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (48*b*PolyLog[5, -E^(
c + d*Sqrt[x])])/d^5 - (48*b*PolyLog[5, E^(c + d*Sqrt[x])])/d^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^{3/2} + bx^{3/2}\operatorname{csch}\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{2}{5}ax^{5/2} + b\int x^{3/2}\operatorname{csch}\left(c + d\sqrt{x}\right) dx \\
&= \frac{2}{5}ax^{5/2} + (2b)\operatorname{Subst}\left(\int x^4\operatorname{csch}(c + dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{2}{5}ax^{5/2} - \frac{4bx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(8b)\operatorname{Subst}\left(\int x^3\log\left(1 - e^{c+dx}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(8b)\operatorname{Subst}\left(\int x^3\log\left(1 + e^{c+dx}\right) dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

$$=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{(24b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}-\frac{(24b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{24bx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}+\frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{24bx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{(48b)\text{Subst}\left(\int \text{PolyLog}\left(4,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4}-\frac{(48b)\text{Subst}\left(\int \text{PolyLog}\left(4,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{24bx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}(4,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}(4,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^5}$$

$$=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{24bx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{24bx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48b\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{48b\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2\left(ad^5x^{5/2}+5bd^4x^2\log\left(1-e^{c+d\sqrt{x}}\right)-5bd^4x^2\log\left(1+e^{c+d\sqrt{x}}\right)-20bd^3x^{3/2}\operatorname{PolyLog}\left(\right.\right.}{}$$

[In] Integrate[x^(3/2)*(a + b*Csch[c + d*Sqrt[x]]),x]

[Out] (2*(a*d^5*x^(5/2) + 5*b*d^4*x^2*Log[1 - E^(c + d*Sqrt[x])] - 5*b*d^4*x^2*Log[1 + E^(c + d*Sqrt[x])] - 20*b*d^3*x^(3/2)*PolyLog[2, -E^(c + d*Sqrt[x])] + 20*b*d^3*x^(3/2)*PolyLog[2, E^(c + d*Sqrt[x])] + 60*b*d^2*x*PolyLog[3, -E^(c + d*Sqrt[x])] - 60*b*d^2*x*PolyLog[3, E^(c + d*Sqrt[x])] - 120*b*d*Sqrt[x]*PolyLog[4, -E^(c + d*Sqrt[x])] + 120*b*d*Sqrt[x]*PolyLog[4, E^(c + d*Sqrt[x])] + 120*b*PolyLog[5, -E^(c + d*Sqrt[x])] - 120*b*PolyLog[5, E^(c + d*Sqrt[x])]))/(5*d^5)

Maple [F]

$$\int x^{\frac{3}{2}}\left(a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)\right)dx$$

[In] int(x^(3/2)*(a+b*csch(c+d*x^(1/2))),x)

[Out] int(x^(3/2)*(a+b*csch(c+d*x^(1/2))),x)

Fricas [F]

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")

[Out] integral(b*x^(3/2)*csch(d*sqrt(x) + c) + a*x^(3/2), x)

Sympy [F]

$$\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{\frac{3}{2}}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**(3/2)*(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)*(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{5}\,ax^{\frac{5}{2}}$$
$$-\frac{2\left(\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^{4}+4\,\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^{3}-12\,\log\left(e^{\left(d\sqrt{x}\right)}\right)^{2}\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})\right.}{d^5}$$
$$+\frac{2\left(\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^{4}+4\,\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^{3}-12\,\log\left(e^{\left(d\sqrt{x}\right)}\right)^{2}\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+\right.}{d^5}$$

```
[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 2/5*a*x^(5/2) - 2*(log(e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^4 + 4*dilo
g(-e^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^3 - 12*log(e^(d*sqrt(x)))^2*polylo
g(3, -e^(d*sqrt(x) + c)) + 24*log(e^(d*sqrt(x)))*polylog(4, -e^(d*sqrt(x) +
 c)) - 24*polylog(5, -e^(d*sqrt(x) + c)))*b/d^5 + 2*(log(-e^(d*sqrt(x) + c)
 + 1)*log(e^(d*sqrt(x)))^4 + 4*dilog(e^(d*sqrt(x) + c))*log(e^(d*sqrt(x)))^
3 - 12*log(e^(d*sqrt(x)))^2*polylog(3, e^(d*sqrt(x) + c)) + 24*log(e^(d*sqr
t(x)))*polylog(4, e^(d*sqrt(x) + c)) - 24*polylog(5, e^(d*sqrt(x) + c)))*b/
d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{3/2}\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(3/2)*(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^(3/2)*(a + b/sinh(c + d*x^(1/2))), x)
```

3.52 $\int \sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result 361
Rubi [A] (verified) 361
Mathematica [A] (verified) 363
Maple [F] 364
Fricas [F] 364
Sympy [F] 364
Maxima [A] (verification not implemented) 364
Giac [F] 365
Mupad [F(-1)] 365

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4bx\,\mathrm{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{4b\,\mathrm{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{4b\,\mathrm{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$

```
[Out] 2/3*a*x^(3/2)-4*b*x*arctanh(exp(c+d*x^(1/2)))/d+4*b*polylog(3,-exp(c+d*x^(1
/2)))/d^3-4*b*polylog(3,exp(c+d*x^(1/2)))/d^3-4*b*polylog(2,-exp(c+d*x^(1/2
)))*x^(1/2)/d^2+4*b*polylog(2,exp(c+d*x^(1/2)))*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {14, 5545, 4267, 2611, 2320, 6724}

$$\int \sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4bx\,\mathrm{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{4b\,\mathrm{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{4b\,\mathrm{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(3/2))/3 - (4*b*x*ArcTanh[E^(c + d*Sqrt[x])])/d - (4*b*Sqrt[x]*PolyL
og[2, -E^(c + d*Sqrt[x])])/d^2 + (4*b*Sqrt[x]*PolyLog[2, E^(c + d*Sqrt[x])]
)/d^2 + (4*b*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (4*b*PolyLog[3, E^(c + d
*Sqrt[x])])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\sqrt{x} + b\sqrt{x}\operatorname{csch}\left(c + d\sqrt{x}\right)\right)\,dx \\
&= \frac{2}{3}ax^{3/2} + b\int \sqrt{x}\operatorname{csch}\left(c + d\sqrt{x}\right)\,dx \\
&= \frac{2}{3}ax^{3/2} + (2b)\text{Subst}\left(\int x^2\operatorname{csch}(c + dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(4b)\text{Subst}\left(\int x\log\left(1 - e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(4b)\text{Subst}\left(\int x\log\left(1 + e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2, -e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} \\
&\quad - \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2, e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^3} \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2, e^{c+d\sqrt{x}}\right)}{d^2} + \frac{4b\,\text{PolyLog}\left(3, -e^{c+d\sqrt{x}}\right)}{d^3} - \frac{4b\,\text{PolyLog}\left(3, e^{c+d\sqrt{x}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \sqrt{x}\left(a + b\operatorname{csch}\left(c + d\sqrt{x}\right)\right)\,dx$$
$$= \frac{2\left(ad^3x^{3/2} + 3bd^2x\log\left(1 - e^{c+d\sqrt{x}}\right) - 3bd^2x\log\left(1 + e^{c+d\sqrt{x}}\right) - 6bd\sqrt{x}\,\text{PolyLog}\left(2, -e^{c+d\sqrt{x}}\right) + 6bd\sqrt{x}\,\text{Po}\right.}{3d^3}$$

```
[In] Integrate[Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^3*x^(3/2) + 3*b*d^2*x*Log[1 - E^(c + d*Sqrt[x])] - 3*b*d^2*x*Log[1
+ E^(c + d*Sqrt[x])] - 6*b*d*Sqrt[x]*PolyLog[2, -E^(c + d*Sqrt[x])] + 6*b*d
*Sqrt[x]*PolyLog[2, E^(c + d*Sqrt[x])] + 6*b*PolyLog[3, -E^(c + d*Sqrt[x])]
 - 6*b*PolyLog[3, E^(c + d*Sqrt[x])]))/(3*d^3)
```

Maple [F]

$$\int \left(a+b \ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)\sqrt{x}dx$$

[In] int((a+b*csch(c+d*x^(1/2)))*x^(1/2),x)

[Out] int((a+b*csch(c+d*x^(1/2)))*x^(1/2),x)

Fricas [F]

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx = \int \left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

[In] integrate((a+b*csch(c+d*x^(1/2)))*x^(1/2),x, algorithm="fricas")

[Out] integral(b*sqrt(x)*csch(d*sqrt(x) + c) + a*sqrt(x), x)

Sympy [F]

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx = \int \sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx$$

[In] integrate((a+b*csch(c+d*x**(1/2)))*x**(1/2),x)

[Out] Integral(sqrt(x)*(a + b*csch(c + d*sqrt(x))), x)

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{2}{3}\,ax^{\frac{3}{2}} - \frac{2\left(\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^2+2\,\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)-2\,\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})\right)b}{d^3} + \frac{2\left(\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)^2+2\,\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)\log\left(e^{\left(d\sqrt{x}\right)}\right)-2\,\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})\right)b}{d^3}$$

[In] integrate((a+b*csch(c+d*x^(1/2)))*x^(1/2),x, algorithm="maxima")

[Out] 2/3*a*x^(3/2) - 2*(log(e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^2 + 2*dilog(-e^(d*sqrt(x) + c))*log(e^(d*sqrt(x))) - 2*polylog(3, -e^(d*sqrt(x) + c)))*b/d^3 + 2*(log(-e^(d*sqrt(x) + c) + 1)*log(e^(d*sqrt(x)))^2 + 2*dilog(e^(d*sqrt(x) + c))*log(e^(d*sqrt(x))) - 2*polylog(3, e^(d*sqrt(x) + c)))*b/d^3

Giac [F]

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\mathrm{csch}\,\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))*x^(1/2),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\,\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(1/2)*(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^(1/2)*(a + b/sinh(c + d*x^(1/2))), x)
```

3.53 $\int \frac{a+b\text{csch}(c+d\sqrt{x})}{\sqrt{x}} dx$

Optimal result 366
Rubi [A] (verified) 366
Mathematica [A] (verified) 367
Maple [A] (verified) 367
Fricas [B] (verification not implemented) 368
Sympy [A] (verification not implemented) 368
Maxima [A] (verification not implemented) 368
Giac [B] (verification not implemented) 369
Mupad [B] (verification not implemented) 369

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a+b\text{csch}(c+d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} - \frac{2b\text{arctanh}(\cosh(c+d\sqrt{x}))}{d}$$

```
[Out] -2*b*arctanh(cosh(c+d*x^(1/2)))/d+2*a*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 5545, 3855}

$$\int \frac{a+b\text{csch}(c+d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} - \frac{2b\text{arctanh}(\cosh(c+d\sqrt{x}))}{d}$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] - (2*b*ArcTanh[Cosh[c + d*Sqrt[x]]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{\sqrt{x}} + \frac{b\text{csch}\left(c + d\sqrt{x}\right)}{\sqrt{x}}\right) dx \\
&= 2a\sqrt{x} + b\int \frac{\text{csch}\left(c + d\sqrt{x}\right)}{\sqrt{x}} dx \\
&= 2a\sqrt{x} + (2b)\text{Subst}\left(\int \text{csch}(c + dx)\, dx, x, \sqrt{x}\right) \\
&= 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cosh\left(c + d\sqrt{x}\right)\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{\sqrt{x}} dx = \frac{2\left(a\left(c + d\sqrt{x}\right) - b\log\left(\cosh\left(\frac{1}{2}\left(c + d\sqrt{x}\right)\right)\right) + b\log\left(\sinh\left(\frac{1}{2}\left(c + d\sqrt{x}\right)\right)\right)\right)}{d}$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/Sqrt[x],x]
[Out] (2*(a*(c + d*Sqrt[x]) - b*Log[Cosh[(c + d*Sqrt[x])/2]] + b*Log[Sinh[(c + d*
Sqrt[x])/2]]))/d
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$2a\sqrt{x} + \frac{2b\ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{d}$	26
default	$2a\sqrt{x} + \frac{2b\ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{d}$	26
parts	$2a\sqrt{x} + \frac{2b\ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{d}$	26

```
[In] int((a+b*csch(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*x^(1/2)+2*b/d*ln(tanh(1/2*c+1/2*d*x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{a + b\text{csch}(c + d\sqrt{x})}{\sqrt{x}}\,dx = \frac{2\left(ad\sqrt{x} - b\log\left(\cosh\left(d\sqrt{x} + c\right) + \sinh\left(d\sqrt{x} + c\right) + 1\right) + b\log\left(\cosh\left(d\sqrt{x} + c\right) + \sinh\left(d\sqrt{x} + c\right) - 1\right)\right)}{d}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
[Out] 2*(a*d*sqrt(x) - b*log(cosh(d*sqrt(x) + c) + sinh(d*sqrt(x) + c) + 1) + b*l
og(cosh(d*sqrt(x) + c) + sinh(d*sqrt(x) + c) - 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a + b\text{csch}(c + d\sqrt{x})}{\sqrt{x}}\,dx = 2a\sqrt{x} + 2b\left(\begin{cases}\sqrt{x}\,\text{csch}\,(c) & \text{for}\ d = 0\\ \frac{\log\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*csch(c), Eq(d, 0)), (log(tanh(c/2 + d*
sqrt(x)/2))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{a + b\text{csch}(c + d\sqrt{x})}{\sqrt{x}}\,dx = 2\,a\sqrt{x} + \frac{2\,b\log\left(\tanh\left(\frac{1}{2}\,d\sqrt{x} + \frac{1}{2}\,c\right)\right)}{d}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
[Out] 2*a*sqrt(x) + 2*b*log(tanh(1/2*d*sqrt(x) + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{a + b\operatorname{csch}(c + d\sqrt{x})}{\sqrt{x}}\, dx = \frac{2\,(d\sqrt{x} + c)a}{d} - \frac{2\,b\log\left(e^{(d\sqrt{x}+c)} + 1\right)}{d} + \frac{2\,b\log\left(\left|e^{(d\sqrt{x}+c)} - 1\right|\right)}{d}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*(d*sqrt(x) + c)*a/d - 2*b*log(e^(d*sqrt(x) + c) + 1)/d + 2*b*log(abs(e^(d
*sqrt(x) + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{a + b\operatorname{csch}(c + d\sqrt{x})}{\sqrt{x}}\, dx = 2\,a\,\sqrt{x} - \frac{4\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x^(1/2),x)
```

```
[Out] 2*a*x^(1/2) - (4*atan((b*exp(d*x^(1/2))*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)
))*(b^2)^(1/2))/(-d^2)^(1/2)
```

3.54 $\int \frac{a+b\,\mathrm{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx$

Optimal result 370
Rubi [N/A] 370
Mathematica [N/A] 371
Maple [N/A] (verified) 371
Fricas [N/A] 371
Sympy [N/A] 371
Maxima [N/A] 372
Giac [N/A] 372
Mupad [N/A] 372

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+b\operatorname{Int}\left(\frac{\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}},x\right)$$

```
[Out] -2*a/x^(1/2)+b*Unintegrable(csch(c+d*x^(1/2))/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx=\int \frac{a+b\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/x^(3/2),x]
[Out] (-2*a)/Sqrt[x] + b*Defer[Int][Csch[c + d*Sqrt[x]]/x^(3/2), x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^{3/2}}+\frac{b\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}}\right)dx\\&=-\frac{2a}{\sqrt{x}}+b\int \frac{\operatorname{csch}(c+d\sqrt{x})}{x^{3/2}}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 55.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^(3/2),x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))/x^(3/2),x)
[Out] int((a+b*csch(c+d*x^(1/2)))/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*csch(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x**(3/2),x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(1/(x^(3/2)*e^(d*sqrt(x) + c) + x^(3/2)), x) + b*integrate(1/(x^
(3/2)*e^(d*sqrt(x) + c) - x^(3/2)), x) - 2*a/sqrt(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\,\text{csch}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}{x^{3/2}}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x^(3/2),x)
```

```
[Out] int((a + b/sinh(c + d*x^(1/2)))/x^(3/2), x)
```

3.55 $\int \frac{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal result 373
Rubi [N/A] 373
Mathematica [N/A] 374
Maple [N/A] (verified) 374
Fricas [N/A] 374
Sympy [N/A] 374
Maxima [N/A] 375
Giac [N/A] 375
Mupad [N/A] 375

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{2a}{3x^{3/2}}+b\mathrm{Int}\left(\frac{\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}},x\right)$$

```
[Out] -2/3*a/x^(3/2)+b*Unintegrable(csch(c+d*x^(1/2))/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) + b*Defer[Int][Csch[c + d*Sqrt[x]]/x^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \left(\frac{a}{x^{5/2}}+\frac{b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\right)dx$$

$$=-\frac{2a}{3x^{3/2}}+b\int \frac{\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 55.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^(5/2),x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))/x^(5/2),x)
[Out] int((a+b*csch(c+d*x^(1/2)))/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*csch(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))/x**(5/2),x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))/x**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(1/(x^(5/2)*e^(d*sqrt(x) + c) + x^(5/2)), x) + b*integrate(1/(x^
(5/2)*e^(d*sqrt(x) + c) - x^(5/2)), x) - 2/3*a/x^(3/2)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{csch}\left(c + d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}}{x^{5/2}}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))/x^(5/2),x)
```

```
[Out] int((a + b/sinh(c + d*x^(1/2)))/x^(5/2), x)
```

3.56 $\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result 376
Rubi [A] (verified) 377
Mathematica [B] (verified) 382
Maple [F] 383
Fricas [F] 383
Sympy [F] 383
Maxima [A] (verification not implemented) 384
Giac [F] 385
Mupad [F(-1)] 385

Optimal result

Integrand size = 22, antiderivative size = 363

$$
\begin{aligned}
\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2b^2x^2}{d}+\frac{2}{5}a^2x^{5/2} \\
&- \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{12b^2x\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&+ \frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{6b^2\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
&+ \frac{96ab\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{96ab\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}
\end{aligned}
$$

```
[Out] -2*b^2*x^2/d+2/5*a^2*x^(5/2)-8*a*b*x^2*arctanh(exp(c+d*x^(1/2)))/d-2*b^2*x^
2*coth(c+d*x^(1/2))/d+8*b^2*x^(3/2)*ln(1-exp(2*c+2*d*x^(1/2)))/d^2-16*a*b*x
^(3/2)*polylog(2,-exp(c+d*x^(1/2)))/d^2+16*a*b*x^(3/2)*polylog(2,exp(c+d*x^
(1/2)))/d^2+12*b^2*x*polylog(2,exp(2*c+2*d*x^(1/2)))/d^3+48*a*b*x*polylog(3
,-exp(c+d*x^(1/2)))/d^3-48*a*b*x*polylog(3,exp(c+d*x^(1/2)))/d^3+6*b^2*poly
log(4,exp(2*c+2*d*x^(1/2)))/d^5+96*a*b*polylog(5,-exp(c+d*x^(1/2)))/d^5-96*
a*b*polylog(5,exp(c+d*x^(1/2)))/d^5-12*b^2*polylog(3,exp(2*c+2*d*x^(1/2)))*
```

```
x^(1/2)/d^4-96*a*b*polylog(4,-exp(c+d*x^(1/2)))*x^(1/2)/d^4+96*a*b*polylog(
4,exp(c+d*x^(1/2)))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5545, 4275, 4267, 2611, 6744, 2320, 6724, 4269, 3797, 2221}

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{5}a^2x^{5/2}-\frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{96ab\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5}-\frac{96ab\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}-\frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}+\frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4}+\frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}-\frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{6b^2\operatorname{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{12b^2x\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d}-\frac{2b^2x^2}{d}$$

```
[In] Int[x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x^2)/d + (2*a^2*x^(5/2))/5 - (8*a*b*x^2*ArcTanh[E^(c + d*Sqrt[x])])
/d - (2*b^2*x^2*Coth[c + d*Sqrt[x]])/d + (8*b^2*x^(3/2)*Log[1 - E^(2*(c + d
*Sqrt[x]))])/d^2 - (16*a*b*x^(3/2)*PolyLog[2, -E^(c + d*Sqrt[x])])/d^2 + (1
6*a*b*x^(3/2)*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (12*b^2*x*PolyLog[2, E^(
2*(c + d*Sqrt[x]))])/d^3 + (48*a*b*x*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 -
(48*a*b*x*PolyLog[3, E^(c + d*Sqrt[x])])/d^3 - (12*b^2*Sqrt[x]*PolyLog[3, E
^(2*(c + d*Sqrt[x]))])/d^4 - (96*a*b*Sqrt[x]*PolyLog[4, -E^(c + d*Sqrt[x])]
)/d^4 + (96*a*b*Sqrt[x]*PolyLog[4, E^(c + d*Sqrt[x])])/d^4 + (6*b^2*PolyLog
[4, E^(2*(c + d*Sqrt[x]))])/d^5 + (96*a*b*PolyLog[5, -E^(c + d*Sqrt[x])])/d
^5 - (96*a*b*PolyLog[5, E^(c + d*Sqrt[x])])/d^5
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^4(a+b\text{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^4+2abx^4\text{csch}(c+dx)+b^2x^4\text{csch}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}+(4ab)\text{Subst}\left(\int x^4\text{csch}(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad+\left(2b^2\right)\text{Subst}\left(\int x^4\text{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}-\frac{8abx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1+e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{\left(8b^2\right)\text{Subst}\left(\int x^3\coth(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

$$= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} - \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(48ab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(48ab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(16b^2)\operatorname{Subst}\left(\int \frac{e^{2(c+dx)}x^3}{1-e^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(96ab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} + \frac{(96ab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(24b^2)\operatorname{Subst}\left(\int x^2\log\left(1-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b^2x\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} + \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(96ab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,-e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(96ab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,e^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(24b^2)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$
\begin{aligned}
&= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} \\
&\quad + \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b^2x\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(96ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,-x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{(96ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,x)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad + \frac{(12b^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d} \\
&\quad + \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{16abx^{3/2}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b^2x\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{48abx\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{96ab\sqrt{x}\operatorname{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{96ab\operatorname{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{96ab\operatorname{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5} + \frac{(6b^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx, x, e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}
\end{aligned}
$$

$$= -\frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x^2\coth\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{8b^2x^{3/2}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16abx^{3/2}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}$$
$$+ \frac{16abx^{3/2}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{12b^2x\,\text{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}$$
$$+ \frac{48abx\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{48abx\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$
$$- \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{96ab\sqrt{x}\,\text{PolyLog}\left(4,-e^{c+d\sqrt{x}}\right)}{d^4}$$
$$+ \frac{96ab\sqrt{x}\,\text{PolyLog}\left(4,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{6b^2\,\text{PolyLog}\left(4,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}$$
$$+ \frac{96ab\,\text{PolyLog}\left(5,-e^{c+d\sqrt{x}}\right)}{d^5} - \frac{96ab\,\text{PolyLog}\left(5,e^{c+d\sqrt{x}}\right)}{d^5}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 875 vs. 2(363) = 726.

Time = 7.02 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.41

$$\int x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{2a^2x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)}{5\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$
$$- \frac{4b\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\left(bd^4x^2-2bd^3\left(-1+e^{2c}\right)x^{3/2}\log\left(1-e^{-c-d\sqrt{x}}\right)-ad^4\left(-1+e^{2c}\right)x^2\log\left(1-e^{-c-d}\right.\right.}{}$$
$$+ \frac{b^2x^2\text{csch}\left(\frac{c}{2}\right)\text{csch}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$
$$- \frac{b^2x^2\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\text{sech}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sinh^2\left(c+d\sqrt{x}\right)\sinh\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*a^2*x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2*Sinh[c + d*Sqrt[x]]^2)/(5*(b +
 a*Sinh[c + d*Sqrt[x]])^2) - (4*b*(a + b*Csch[c + d*Sqrt[x]])^2*(b*d^4*x^2
- 2*b*d^3*(-1 + E^(2*c))*x^(3/2)*Log[1 - E^(-c - d*Sqrt[x])] - a*d^4*(-1 +
E^(2*c))*x^2*Log[1 - E^(-c - d*Sqrt[x])] - 2*b*d^3*(-1 + E^(2*c))*x^(3/2)*L
og[1 + E^(-c - d*Sqrt[x])] + a*d^4*(-1 + E^(2*c))*x^2*Log[1 + E^(-c - d*Sqr
t[x])] + 6*b*d^2*(-1 + E^(2*c))*x*PolyLog[2, -E^(-c - d*Sqrt[x])] - 4*a*d^3
*(-1 + E^(2*c))*x^(3/2)*PolyLog[2, -E^(-c - d*Sqrt[x])] + 6*b*d^2*(-1 + E^(
2*c))*x*PolyLog[2, E^(-c - d*Sqrt[x])] + 4*a*d^3*(-1 + E^(2*c))*x^(3/2)*Pol
```

```
yLog[2, E^(-c - d*Sqrt[x])] + 12*b*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[3, -E^(
-c - d*Sqrt[x])] - 12*a*d^2*(-1 + E^(2*c))*x*PolyLog[3, -E^(-c - d*Sqrt[x])
] + 12*b*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[3, E^(-c - d*Sqrt[x])] + 12*a*d^2
*(-1 + E^(2*c))*x*PolyLog[3, E^(-c - d*Sqrt[x])] + 12*b*(-1 + E^(2*c))*Poly
Log[4, -E^(-c - d*Sqrt[x])] - 24*a*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[4, -E^(
-c - d*Sqrt[x])] + 12*b*(-1 + E^(2*c))*PolyLog[4, E^(-c - d*Sqrt[x])] + 24*
a*d*(-1 + E^(2*c))*Sqrt[x]*PolyLog[4, E^(-c - d*Sqrt[x])] - 24*a*(-1 + E^(2
*c))*PolyLog[5, -E^(-c - d*Sqrt[x])] + 24*a*(-1 + E^(2*c))*PolyLog[5, E^(-c
 - d*Sqrt[x])])*Sinh[c + d*Sqrt[x]]^2)/(d^5*(-1 + E^(2*c))*(b + a*Sinh[c +
d*Sqrt[x]])^2) + (b^2*x^2*Csch[c/2]*Csch[c/2 + (d*Sqrt[x])/2]*(a + b*Csch[c
 + d*Sqrt[x]])^2*Sinh[c + d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*Sinh[
c + d*Sqrt[x]])^2) - (b^2*x^2*(a + b*Csch[c + d*Sqrt[x]])^2*Sech[c/2]*Sech[
c/2 + (d*Sqrt[x])/2]*Sinh[c + d*Sqrt[x]]^2*Sinh[(d*Sqrt[x])/2])/(d*(b + a*S
inh[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}dx$$

```
[In] int(x^(3/2)*(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(x^(3/2)*(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^{2}x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^(3/2)*csch(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*csch(d*sqrt(x) +
 c) + a^2*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int x^{\frac{3}{2}}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx$$

```
[In] integrate(x**(3/2)*(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x**(3/2)*(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.16

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{5}\,a^2x^{\frac{5}{2}}-\frac{4\,b^2x^2}{de^{\left(2\,d\sqrt{x}+2\,c\right)}-d}$$
$$-\frac{4\left(d^4x^2\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+4\,d^3x^{\frac{3}{2}}\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-12\,d^2x\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+24\,d\sqrt{x}\mathrm{Li}_4(-e^{\left(d\sqrt{x}+c\right)})-24}{d^5}$$
$$+\frac{4\left(d^4x^2\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+4\,d^3x^{\frac{3}{2}}\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-12\,d^2x\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+24\,d\sqrt{x}\mathrm{Li}_4(e^{\left(d\sqrt{x}+c\right)})-24\,\mathrm{Li}_5(}{d^5}$$
$$+\frac{8\left(d^3x^{\frac{3}{2}}\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)+3\,d^2x\mathrm{Li}_2\left(-e^{\left(d\sqrt{x}+c\right)}\right)-6\,d\sqrt{x}\mathrm{Li}_3(-e^{\left(d\sqrt{x}+c\right)})+6\,\mathrm{Li}_4(-e^{\left(d\sqrt{x}+c\right)})\right)b^2}{d^5}$$
$$+\frac{8\left(d^3x^{\frac{3}{2}}\log\left(-e^{\left(d\sqrt{x}+c\right)}+1\right)+3\,d^2x\mathrm{Li}_2\left(e^{\left(d\sqrt{x}+c\right)}\right)-6\,d\sqrt{x}\mathrm{Li}_3(e^{\left(d\sqrt{x}+c\right)})+6\,\mathrm{Li}_4(e^{\left(d\sqrt{x}+c\right)})\right)b^2}{d^5}$$
$$-\frac{2\left(2\,abd^5x^{\frac{5}{2}}+5\,b^2d^4x^2\right)}{5\,d^5}+\frac{2\left(2\,abd^5x^{\frac{5}{2}}-5\,b^2d^4x^2\right)}{5\,d^5}$$

```
[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 2/5*a^2*x^(5/2) - 4*b^2*x^2/(d*e^(2*d*sqrt(x) + 2*c) - d) - 4*(d^4*x^2*log(
e^(d*sqrt(x) + c) + 1) + 4*d^3*x^(3/2)*dilog(-e^(d*sqrt(x) + c)) - 12*d^2*x
*polylog(3, -e^(d*sqrt(x) + c)) + 24*d*sqrt(x)*polylog(4, -e^(d*sqrt(x) + c
)) - 24*polylog(5, -e^(d*sqrt(x) + c)))*a*b/d^5 + 4*(d^4*x^2*log(-e^(d*sqrt
(x) + c) + 1) + 4*d^3*x^(3/2)*dilog(e^(d*sqrt(x) + c)) - 12*d^2*x*polylog(3
, e^(d*sqrt(x) + c)) + 24*d*sqrt(x)*polylog(4, e^(d*sqrt(x) + c)) - 24*poly
log(5, e^(d*sqrt(x) + c)))*a*b/d^5 + 8*(d^3*x^(3/2)*log(e^(d*sqrt(x) + c) +
 1) + 3*d^2*x*dilog(-e^(d*sqrt(x) + c)) - 6*d*sqrt(x)*polylog(3, -e^(d*sqrt
(x) + c)) + 6*polylog(4, -e^(d*sqrt(x) + c)))*b^2/d^5 + 8*(d^3*x^(3/2)*log(
-e^(d*sqrt(x) + c) + 1) + 3*d^2*x*dilog(e^(d*sqrt(x) + c)) - 6*d*sqrt(x)*po
lylog(3, e^(d*sqrt(x) + c)) + 6*polylog(4, e^(d*sqrt(x) + c)))*b^2/d^5 - 2/
5*(2*a*b*d^5*x^(5/2) + 5*b^2*d^4*x^2)/d^5 + 2/5*(2*a*b*d^5*x^(5/2) - 5*b^2*
d^4*x^2)/d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{3/2}\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.57 $\int \sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result 386
Rubi [A] (verified) 387
Mathematica [B] (verified) 390
Maple [F] 391
Fricas [F] 391
Sympy [F] 391
Maxima [A] (verification not implemented) 392
Giac [F] 392
Mupad [F(-1)] 393

Optimal result

Integrand size = 22, antiderivative size = 209

$$\int \sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2b^2x}{d}+\frac{2}{3}a^2x^{3/2}-\frac{8abx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d}+\frac{4b^2\sqrt{x}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{8ab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8ab\sqrt{x}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}+\frac{2b^2\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{8ab\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3}-\frac{8ab\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$

```
[Out] -2*b^2*x/d+2/3*a^2*x^(3/2)-8*a*b*x*arctanh(exp(c+d*x^(1/2)))/d-2*b^2*x*coth
(c+d*x^(1/2))/d+2*b^2*polylog(2,exp(2*c+2*d*x^(1/2)))/d^3+8*a*b*polylog(3,-
exp(c+d*x^(1/2)))/d^3-8*a*b*polylog(3,exp(c+d*x^(1/2)))/d^3+4*b^2*ln(1-exp(
2*c+2*d*x^(1/2)))*x^(1/2)/d^2-8*a*b*polylog(2,-exp(c+d*x^(1/2)))*x^(1/2)/d^
2+8*a*b*polylog(2,exp(c+d*x^(1/2)))*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5545, 4275, 4267, 2611, 2320, 6724, 4269, 3797, 2221, 2317, 2438}

$$\int \sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2 dx = \frac{2}{3}a^2x^{3/2} - \frac{8abx\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} + \frac{8ab\operatorname{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{8ab\operatorname{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{8ab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ab\sqrt{x}\operatorname{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2\operatorname{PolyLog}\left(2,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{4b^2\sqrt{x}\log\left(1-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d} - \frac{2b^2x}{d}$$

```
[In] Int[Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x)/d + (2*a^2*x^(3/2))/3 - (8*a*b*x*ArcTanh[E^(c + d*Sqrt[x])])/d -
 (2*b^2*x*Coth[c + d*Sqrt[x]])/d + (4*b^2*Sqrt[x]*Log[1 - E^(2*(c + d*Sqrt[
x]))])/d^2 - (8*a*b*Sqrt[x]*PolyLog[2, -E^(c + d*Sqrt[x])])/d^2 + (8*a*b*Sq
rt[x]*PolyLog[2, E^(c + d*Sqrt[x])])/d^2 + (2*b^2*PolyLog[2, E^(2*(c + d*Sq
rt[x]))])/d^3 + (8*a*b*PolyLog[3, -E^(c + d*Sqrt[x])])/d^3 - (8*a*b*PolyLog
[3, E^(c + d*Sqrt[x])])/d^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^2(a+b\text{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^2+2abx^2\text{csch}(c+dx)+b^2x^2\text{csch}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}+(4ab)\text{Subst}\left(\int x^2\text{csch}(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad+\left(2b^2\right)\text{Subst}\left(\int x^2\text{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}-\frac{8abx\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{(8ab)\text{Subst}\left(\int x\log\left(1-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{(8ab)\text{Subst}\left(\int x\log\left(1+e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{\left(4b^2\right)\text{Subst}\left(\int x\coth(c+dx)\,dx, x, \sqrt{x}\right)}{d}\\
&= -\frac{2b^2x}{d}+\frac{2}{3}a^2x^{3/2}-\frac{8abx\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2}+\frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2}\\
&\quad+\frac{(8ab)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}\\
&\quad-\frac{(8ab)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}\\
&\quad-\frac{\left(8b^2\right)\text{Subst}\left(\int \frac{e^{2(c+dx)}x}{1-e^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

$$= -\frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{(8ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(8ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(4b^2)\,\text{Subst}\left(\int \log\left(1-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ab\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{8ab\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2(c+d\sqrt{x})}\right)}{d^3}$$

$$= -\frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\text{arctanh}\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{2b^2x\coth\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,-e^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ab\sqrt{x}\,\text{PolyLog}\left(2,e^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2\,\text{PolyLog}\left(2,e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\,\text{PolyLog}\left(3,-e^{c+d\sqrt{x}}\right)}{d^3} - \frac{8ab\,\text{PolyLog}\left(3,e^{c+d\sqrt{x}}\right)}{d^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(209) = 418$.

Time = 2.34 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.05

$$\int \sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx =$$

$$-\frac{12b^2d^2x + 2a^2d^3x^{3/2} - 2a^2d^3e^{2c}x^{3/2} + 12b^2d\sqrt{x}\log\left(1-e^{-c-d\sqrt{x}}\right) - 12b^2de^{2c}\sqrt{x}\log\left(1-e^{-c-d\sqrt{x}}\right) + 12a}{}$$

```
[In] Integrate[Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] -1/3*(12*b^2*d^2*x + 2*a^2*d^3*x^(3/2) - 2*a^2*d^3*E^(2*c)*x^(3/2) + 12*b^2
*d*Sqrt[x]*Log[1 - E^(-c - d*Sqrt[x])] - 12*b^2*d*E^(2*c)*Sqrt[x]*Log[1 - E
^(-c - d*Sqrt[x])] + 12*a*b*d^2*x*Log[1 - E^(-c - d*Sqrt[x])] - 12*a*b*d^2*
E^(2*c)*x*Log[1 - E^(-c - d*Sqrt[x])] + 12*b^2*d*Sqrt[x]*Log[1 + E^(-c - d*
Sqrt[x])] - 12*b^2*d*E^(2*c)*Sqrt[x]*Log[1 + E^(-c - d*Sqrt[x])] - 12*a*b*d
^2*x*Log[1 + E^(-c - d*Sqrt[x])] + 12*a*b*d^2*E^(2*c)*x*Log[1 + E^(-c - d*S
qrt[x])] + 12*b*(-1 + E^(2*c))*(b - 2*a*d*Sqrt[x])*PolyLog[2, -E^(-c - d*Sq
rt[x])] + 12*b*(-1 + E^(2*c))*(b + 2*a*d*Sqrt[x])*PolyLog[2, E^(-c - d*Sqrt
[x])] + 24*a*b*PolyLog[3, -E^(-c - d*Sqrt[x])] - 24*a*b*E^(2*c)*PolyLog[3,
-E^(-c - d*Sqrt[x])] - 24*a*b*PolyLog[3, E^(-c - d*Sqrt[x])] + 24*a*b*E^(2*
c)*PolyLog[3, E^(-c - d*Sqrt[x])] + 3*b^2*d^2*x*Csch[c/2]*Csch[(c + d*Sqrt[
x])/2]*Sinh[(d*Sqrt[x])/2] - 3*b^2*d^2*E^(2*c)*x*Csch[c/2]*Csch[(c + d*Sqrt
[x])/2]*Sinh[(d*Sqrt[x])/2] - 3*b^2*d^2*x*Sech[c/2]*Sech[(c + d*Sqrt[x])/2]
*Sinh[(d*Sqrt[x])/2] + 3*b^2*d^2*E^(2*c)*x*Sech[c/2]*Sech[(c + d*Sqrt[x])/2
]*Sinh[(d*Sqrt[x])/2])/(d^3*(-1 + E^(2*c)))
```

Maple [F]

$$\int \left(a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}\sqrt{x}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))^2*x^(1/2),x)
```

```
[Out] int((a+b*csch(c+d*x^(1/2)))^2*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int \left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^{2}\sqrt{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(b^2*sqrt(x)*csch(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csch(d*sqrt(x) +
 c) + a^2*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int \sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.26

$$\int \sqrt{x}\big(a+b\mathrm{csch}\big(c+d\sqrt{x}\big)\big)^2\,dx$$
$$= \frac{2}{3}a^2x^{\frac{3}{2}} - \frac{4b^2x}{de^{(2d\sqrt{x}+2c)}-d} - \frac{4\left(d^2x\log\left(e^{(d\sqrt{x}+c)}+1\right)+2d\sqrt{x}\mathrm{Li}_2\left(-e^{(d\sqrt{x}+c)}\right)-2\,\mathrm{Li}_3(-e^{(d\sqrt{x}+c)})\right)ab}{d^3} + \frac{4\left(d^2x\log\left(-e^{(d\sqrt{x}+c)}+1\right)+2d\sqrt{x}\mathrm{Li}_2\left(e^{(d\sqrt{x}+c)}\right)-2\,\mathrm{Li}_3(e^{(d\sqrt{x}+c)})\right)ab}{d^3} + \frac{4\left(d\sqrt{x}\log\left(e^{(d\sqrt{x}+c)}+1\right)+\mathrm{Li}_2\left(-e^{(d\sqrt{x}+c)}\right)\right)b^2}{d^3} + \frac{4\left(d\sqrt{x}\log\left(-e^{(d\sqrt{x}+c)}+1\right)+\mathrm{Li}_2\left(e^{(d\sqrt{x}+c)}\right)\right)b^2}{d^3} - \frac{2\left(2\,abd^3x^{\frac{3}{2}}+3\,b^2d^2x\right)}{3\,d^3} + \frac{2\left(2\,abd^3x^{\frac{3}{2}}-3\,b^2d^2x\right)}{3\,d^3}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*a^2*x^(3/2) - 4*b^2*x/(d*e^(2*d*sqrt(x) + 2*c) - d) - 4*(d^2*x*log(e^(d
*sqrt(x) + c) + 1) + 2*d*sqrt(x)*dilog(-e^(d*sqrt(x) + c)) - 2*polylog(3, -
e^(d*sqrt(x) + c)))*a*b/d^3 + 4*(d^2*x*log(-e^(d*sqrt(x) + c) + 1) + 2*d*sq
rt(x)*dilog(e^(d*sqrt(x) + c)) - 2*polylog(3, e^(d*sqrt(x) + c)))*a*b/d^3 +
 4*(d*sqrt(x)*log(e^(d*sqrt(x) + c) + 1) + dilog(-e^(d*sqrt(x) + c)))*b^2/d
^3 + 4*(d*sqrt(x)*log(-e^(d*sqrt(x) + c) + 1) + dilog(e^(d*sqrt(x) + c)))*b
^2/d^3 - 2/3*(2*a*b*d^3*x^(3/2) + 3*b^2*d^2*x)/d^3 + 2/3*(2*a*b*d^3*x^(3/2)
 - 3*b^2*d^2*x)/d^3
```

Giac [F]

$$\int \sqrt{x}\big(a+b\mathrm{csch}\big(c+d\sqrt{x}\big)\big)^2\,dx = \int \left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\,\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(1/2)*(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)*(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.58 $\int \frac{\left(a+b\,\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$

Optimal result 394
Rubi [A] (verified) 394
Mathematica [A] (verified) 395
Maple [A] (verified) 396
Fricas [B] (verification not implemented) 396
Sympy [F] 397
Maxima [A] (verification not implemented) 397
Giac [A] (verification not implemented) 397
Mupad [B] (verification not implemented) 398

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2a^2\sqrt{x}-\frac{4ab\operatorname{arctanh}\left(\cosh\left(c+d\sqrt{x}\right)\right)}{d}-\frac{2b^2\coth\left(c+d\sqrt{x}\right)}{d}$$

```
[Out] -4*a*b*arctanh(cosh(c+d*x^(1/2)))/d-2*b^2*coth(c+d*x^(1/2))/d+2*a^2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5545, 3858, 3855, 3852, 8}

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2a^2\sqrt{x}-\frac{4ab\operatorname{arctanh}\left(\cosh\left(c+d\sqrt{x}\right)\right)}{d}-\frac{2b^2\coth\left(c+d\sqrt{x}\right)}{d}$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] - (4*a*b*ArcTanh[Cosh[c + d*Sqrt[x]]])/d - (2*b^2*Coth[c + d*
Sqrt[x]])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int (a+b\operatorname{csch}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}+(4ab)\text{Subst}\left(\int \operatorname{csch}(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int \operatorname{csch}^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}-\frac{4ab\operatorname{arctanh}\left(\cosh\left(c+d\sqrt{x}\right)\right)}{d}-\frac{\left(2ib^2\right)\text{Subst}\left(\int 1\,dx, x, -i\coth\left(c+d\sqrt{x}\right)\right)}{d}\\
&= 2a^2\sqrt{x}-\frac{4ab\operatorname{arctanh}\left(\cosh\left(c+d\sqrt{x}\right)\right)}{d}-\frac{2b^2\coth\left(c+d\sqrt{x}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = -\frac{b^2\coth\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)-2a\left(ac+ad\sqrt{x}-2b\log\left(\cosh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)+2b\log\left(\sinh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)\right)+}{d}$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/Sqrt[x],x]
[Out] -((b^2*Coth[(c + d*Sqrt[x])/2] - 2*a*(a*c + a*d*Sqrt[x] - 2*b*Log[Cosh[(c +
 d*Sqrt[x])/2]] + 2*b*Log[Sinh[(c + d*Sqrt[x])/2]]) + b^2*Tanh[(c + d*Sqrt[
x])/2])/d)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2a^2(c+d\sqrt{x})-8ab\,\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)-2b^2\coth\left(c+d\sqrt{x}\right)}{d}$	44
default	$\frac{2a^2(c+d\sqrt{x})-8ab\,\operatorname{arctanh}\left(e^{c+d\sqrt{x}}\right)-2b^2\coth\left(c+d\sqrt{x}\right)}{d}$	44
parts	$2a^2\sqrt{x}-\frac{2b^2\coth\left(c+d\sqrt{x}\right)}{d}+\frac{4ab\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{d}$	45

```
[In] int((a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(a^2*(c+d*x^(1/2))-4*a*b*arctanh(exp(c+d*x^(1/2)))-b^2*coth(c+d*x^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 5.77

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \frac{2\left(a^2d\sqrt{x}\cosh\left(d\sqrt{x}+c\right)^2+2\,a^2d\sqrt{x}\cosh\left(d\sqrt{x}+c\right)\sinh\left(d\sqrt{x}+c\right)+a^2d\sqrt{x}\sinh\left(d\sqrt{x}+c\right)^2-a^2d\sqrt{x}\right.}{}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
[Out] 2*(a^2*d*sqrt(x)*cosh(d*sqrt(x) + c)^2 + 2*a^2*d*sqrt(x)*cosh(d*sqrt(x) + c
)*sinh(d*sqrt(x) + c) + a^2*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 - a^2*d*sqrt(x)
 - 2*b^2 - 2*(a*b*cosh(d*sqrt(x) + c)^2 + 2*a*b*cosh(d*sqrt(x) + c)*sinh(d*
sqrt(x) + c) + a*b*sinh(d*sqrt(x) + c)^2 - a*b)*log(cosh(d*sqrt(x) + c) + s
inh(d*sqrt(x) + c) + 1) + 2*(a*b*cosh(d*sqrt(x) + c)^2 + 2*a*b*cosh(d*sqrt(
x) + c)*sinh(d*sqrt(x) + c) + a*b*sinh(d*sqrt(x) + c)^2 - a*b)*log(cosh(d*s
qrt(x) + c) + sinh(d*sqrt(x) + c) - 1))/(d*cosh(d*sqrt(x) + c)^2 + 2*d*cosh
(d*sqrt(x) + c)*sinh(d*sqrt(x) + c) + d*sinh(d*sqrt(x) + c)^2 - d)
```

Sympy [F]

$$\int \frac{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=\int \frac{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))**2/sqrt(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2\,a^2\sqrt{x}+\frac{4\,ab\log\left(\tanh\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)\right)}{d}+\frac{4\,b^2}{d\left(e^{\left(-2\,d\sqrt{x}-2\,c\right)}-1\right)}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
[Out] 2*a^2*sqrt(x) + 4*a*b*log(tanh(1/2*d*sqrt(x) + 1/2*c))/d + 4*b^2/(d*(e^(-2*
d*sqrt(x) - 2*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=\frac{2\left(d\sqrt{x}+c\right)a^2}{d}-\frac{4\,ab\log\left(e^{\left(d\sqrt{x}+c\right)}+1\right)}{d}+\frac{4\,ab\log\left(\left|e^{\left(d\sqrt{x}+c\right)}-1\right|\right)}{d}-\frac{4\,b^2}{d\left(e^{\left(2\,d\sqrt{x}+2\,c\right)}-1\right)}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
[Out] 2*(d*sqrt(x) + c)*a^2/d - 4*a*b*log(e^(d*sqrt(x) + c) + 1)/d + 4*a*b*log(ab
s(e^(d*sqrt(x) + c) - 1))/d - 4*b^2/(d*(e^(2*d*sqrt(x) + 2*c) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.72

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\,\sqrt{x}-\frac{4\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,\sqrt{x}}-1\right)}-\frac{8\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))^2/x^(1/2),x)
```

```
[Out] 2*a^2*x^(1/2) - (4*b^2)/(d*(exp(2*c + 2*d*x^(1/2)) - 1)) - (8*atan((a*b*exp
(d*x^(1/2))*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^
2)^(1/2)
```

3.59 $\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$

Optimal result 399
Rubi [N/A] 399
Mathematica [N/A] 400
Maple [N/A] (verified) 400
Fricas [N/A] 400
Sympy [N/A] 401
Maxima [N/A] 401
Giac [N/A] 401
Mupad [N/A] 402

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\operatorname{Int}\left(\frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])^2/x^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 126.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^(3/2),x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\,\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*csch(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csch(d*sqrt(x)
+ c) + a^2*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\,\text{csch}\,\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2/x**(3/2),x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))**2/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.27

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\,\text{csch}\,\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")
[Out] -2*(a^2*d*sqrt(x)*e^(2*d*sqrt(x) + 2*c) - a^2*d*sqrt(x) + 2*b^2)/(d*x*e^(2*
d*sqrt(x) + 2*c) - d*x) + integrate(2*(a*b*d*x + b^2*sqrt(x))/(d*x^(5/2)*e^
(d*sqrt(x) + c) + d*x^(5/2)), x) - integrate(-2*(a*b*d*x - b^2*sqrt(x))/(d*
x^(5/2)*e^(d*sqrt(x) + c) - d*x^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\,\text{csch}\,\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2}{x^{3/2}}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a + b/sinh(c + d*x^(1/2)))^2/x^(3/2), x)
```

3.60 $\int \frac{\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$

Optimal result 403
Rubi [N/A] 403
Mathematica [N/A] 404
Maple [N/A] (verified) 404
Fricas [N/A] 404
Sympy [N/A] 405
Maxima [F(-1)] 405
Giac [N/A] 405
Mupad [N/A] 405

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Int}\left(\frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Csch[c + d*Sqrt[x]])^2/x^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Csch[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 125.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^(5/2),x]
[Out] Integrate[(a + b*Csch[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*csch(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csch(d*sqrt(x)
+ c) + a^2*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x**(1/2)))**2/x**(5/2),x)
[Out] Integral((a + b*csch(c + d*sqrt(x)))**2/x**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csch(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")
[Out] integrate((b*csch(d*sqrt(x) + c) + a)^2/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2}{x^{5/2}}\,dx$$

```
[In] int((a + b/sinh(c + d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a + b/sinh(c + d*x^(1/2)))^2/x^(5/2), x)
```

3.61 $\int \frac{x^{3/2}}{a+b\operatorname{csch}(c+d\sqrt{x})} \, dx$

Optimal result . . . 406
Rubi [A] (verified) . . . 407
Mathematica [A] (verified) . . . 411
Maple [F] . . . 412
Fricas [F] . . . 412
Sympy [F] . . . 412
Maxima [F] . . . 413
Giac [F] . . . 413
Mupad [F(-1)] . . . 413

Optimal result

Integrand size = 22, antiderivative size = 561

$$\int \frac{x^{3/2}}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)} \, dx = \frac{2x^{5/2}}{5a} - \frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{24bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{24bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{48b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$

```
[Out] 2/5*x^(5/2)/a-2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d/(a^2
+b^2)^(1/2)+2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d/(a^2+b
^2)^(1/2)-8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/
d^2/(a^2+b^2)^(1/2)+8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^
(1/2)))/a/d^2/(a^2+b^2)^(1/2)+24*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+
b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-24*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+
(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)+48*b*polylog(5,-a*exp(c+d*x^(1/2))/
(b-(a^2+b^2)^(1/2)))/a/d^5/(a^2+b^2)^(1/2)-48*b*polylog(5,-a*exp(c+d*x^(1/2
))/(b+(a^2+b^2)^(1/2)))/a/d^5/(a^2+b^2)^(1/2)-48*b*polylog(4,-a*exp(c+d*x^(
```

```
1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(a^2+b^2)^(1/2)+48*b*polylog(4,-a*
exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5545, 4276, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^{3/2}}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{48b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} - \frac{48b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} - \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{24bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{24bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} - \frac{2bx^2\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}} + \frac{2bx^2\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{a^2+b^2}+b}+1\right)}{ad\sqrt{a^2+b^2}} + \frac{2x^{5/2}}{5a}$$

```
[In] Int[x^(3/2)/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(5/2))/(5*a) - (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
b^2])])/(a*Sqrt[a^2 + b^2]*d) + (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (8*b*x^(3/2)*PolyLog[2, -((a*E^
(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (8*b*x^
(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a
^2 + b^2]*d^2) + (24*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
 b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (24*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x
]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (48*b*Sqrt[x]*PolyLo
g[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^
4) + (48*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])
)])/(a*Sqrt[a^2 + b^2]*d^4) + (48*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b -
 Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^5) - (48*b*PolyLog[5, -((a*E^(c +
 d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
```

```
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a}-\frac{bx^4}{a(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{5/2}}{5a}-\frac{(2b)\text{Subst}\left(\int \frac{x^4}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}\\
&\quad+\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}\\
&= \frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}\\
&\quad+\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}\\
&\quad-\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}
$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{(24b)\operatorname{Subst}\left(\int x^2 \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2} - \frac{(24b)\operatorname{Subst}\left(\int x^2 \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{24bx \operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{24bx \operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{(48b)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3} + \frac{(48b)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{24bx \operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{24bx \operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{(48b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4} - \frac{(48b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}d^4}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(4,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(4,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^5}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx^2 \log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^4} + \frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5} - \frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^5}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.78

$$\int \frac{x^{3/2}}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{2\left(\sqrt{a^2+b^2}d^5x^{5/2} - 5bd^4x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) + 5bd^4x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) -$$

```
[In] Integrate[x^(3/2)/(a + b*Csch[c + d*Sqrt[x]]),x]
[Out] (2*(Sqrt[a^2 + b^2]*d^5*x^(5/2) - 5*b*d^4*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))
/(b - Sqrt[a^2 + b^2])] + 5*b*d^4*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sq
```

```
rt[a^2 + b^2])] - 20*b*d^3*x^(3/2)*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + S
qrt[a^2 + b^2])] + 20*b*d^3*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2]))] + 60*b*d^2*x*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqr
t[a^2 + b^2])] - 60*b*d^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^
2 + b^2]))] - 120*b*d*Sqrt[x]*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a
^2 + b^2])] + 120*b*d*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[
a^2 + b^2]))] + 120*b*PolyLog[5, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2 + b^2
])] - 120*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))]))/(5
*a*Sqrt[a^2 + b^2]*d^5)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(3/2)/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x^(3/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{a+b\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^(3/2)/(b*csch(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{a+b\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)/(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2*b*integrate(x^(3/2)*e^(d*sqrt(x) + c)/(a^2*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*e^(d*sqrt(x) + c) - a^2), x) + 2/5*x^(5/2)/a
```

Giac [F]

$$\int \frac{x^{3/2}}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\operatorname{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(b*csch(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a+b\operatorname{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{3/2}}{a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(3/2)/(a + b/sinh(c + d*x^(1/2))),x)
```

```
[Out] int(x^(3/2)/(a + b/sinh(c + d*x^(1/2))), x)
```

3.62 $\int \frac{\sqrt{x}}{a+b\,\mathrm{csch}(c+d\sqrt{x})}\,dx$

Optimal result 414
Rubi [A] (verified) 415
Mathematica [A] (verified) 418
Maple [F] 419
Fricas [F] 419
Sympy [F] 419
Maxima [F] 419
Giac [F] 420
Mupad [F(-1)] 420

Optimal result

Integrand size = 22, antiderivative size = 337

$$\begin{aligned}\int \frac{\sqrt{x}}{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx = {} & \frac{2x^{3/2}}{3a} - \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} \\ & - \frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^2} \\ & + \frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^2} \\ & + \frac{4b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^3} - \frac{4b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^3}\end{aligned}$$

```
[Out] 2/3*x^(3/2)/a-2*b*x*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d/(a^2+b
^2)^(1/2)+2*b*x*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^
(1/2)+4*b*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2
)^(1/2)-4*b*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b
^2)^(1/2)-4*b*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a/
d^2/(a^2+b^2)^(1/2)+4*b*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*
x^(1/2)/a/d^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5545, 4276, 3403, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{\sqrt{x}}{a+b\text{csch}\left(c+d\sqrt{x}\right)}\,dx = \frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{2bx\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{2bx\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{a^2+b^2}+b}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{2x^{3/2}}{3a}$$

```
[In] Int[Sqrt[x]/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(3/2))/(3*a) - (2*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^
2])])/(a*Sqrt[a^2 + b^2]*d) + (2*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqr
t[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (4*b*Sqrt[x]*PolyLog[2, -((a*E^(c +
 d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (4*b*Sqrt[x
]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 +
 b^2]*d^2) + (4*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))
])/(a*Sqrt[a^2 + b^2]*d^3) - (4*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^2}{a + b\text{csch}(c + dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^2}{a} - \frac{bx^2}{a(b + a\sinh(c + dx))}\right) dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{3/2}}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+b^2}}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{(4b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d} - \frac{(4b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2} - \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}d^2}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{4b\sqrt{x}\, \text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{4b\sqrt{x}\, \text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\, dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^3}$$
$$- \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2, -\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\, dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{a^2+b^2}d^3}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{2bx \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$
$$- \frac{4b\sqrt{x}\, \text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{4b\sqrt{x}\, \text{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}$$
$$+ \frac{4b\, \text{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3} - \frac{4b\, \text{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\, dx$$
$$= \frac{2\left(\sqrt{a^2+b^2}d^3x^{3/2} - 3bd^2x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) + 3bd^2x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) - 6bd\sqrt{x}\, \text{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{-b+\sqrt{a^2+b^2}}\right.\right.}{3a\sqrt{a^2+b^2}}$$

```
[In] Integrate[Sqrt[x]/(a + b*Csch[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[a^2 + b^2]*d^3*x^(3/2) - 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(
b - Sqrt[a^2 + b^2])] + 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[a
^2 + b^2])] - 6*b*d*Sqrt[x]*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2
 + b^2])] + 6*b*d*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2
+ b^2]))] + 6*b*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[a^2 + b^2])] -
6*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))]))/(3*a*Sqrt[
a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{\sqrt{x}}{a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(1/2)/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(x^(1/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{a+b\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*csch(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{a+b\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(sqrt(x)/(a + b*csch(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{a+b\mathrm{csch}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(sqrt(x)*e^(d*sqrt(x) + c)/(a^2*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*e^(d*sqrt(x) + c) - a^2), x) + 2/3*x^(3/2)/a
```

Giac [F]

$$\int \frac{\sqrt{x}}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\, dx = \int \frac{\sqrt{x}}{b\,\text{csch}\left(d\sqrt{x} + c\right) + a}\, dx$$

```
[In] integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*csch(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a + b\text{csch}\left(c + d\sqrt{x}\right)}\, dx = \int \frac{\sqrt{x}}{a + \frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}}\, dx$$

```
[In] int(x^(1/2)/(a + b/sinh(c + d*x^(1/2))),x)
[Out] int(x^(1/2)/(a + b/sinh(c + d*x^(1/2))), x)
```

3.63 $\int \frac{1}{\sqrt{x}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result 421
Rubi [A] (verified) 421
Mathematica [A] (verified) 423
Maple [A] (verified) 423
Fricas [B] (verification not implemented) 423
Sympy [F] 424
Maxima [A] (verification not implemented) 424
Giac [A] (verification not implemented) 424
Mupad [B] (verification not implemented) 425

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}+\frac{4b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

```
[Out] 4*b*arctanh((a-b*tanh(1/2*c+1/2*d*x^(1/2)))/(a^2+b^2)^(1/2))/a/d/(a^2+b^2)^
(1/2)+2*x^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5545, 3868, 2739, 632, 210}

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{4b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{2\sqrt{x}}{a}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])),x]
```

```
[Out] (2*Sqrt[x])/a + (4*b*ArcTanh[(a - b*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 + b^2
]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= \frac{2\sqrt{x}}{a} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2\sqrt{x}}{a} + \frac{(4i)\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}\\
&= \frac{2\sqrt{x}}{a} - \frac{(8i)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}\\
&= \frac{2\sqrt{x}}{a} + \frac{4b\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\left(\frac{c}{d}+\sqrt{x}-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}\right)}{a}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])),x]
[Out] (2*(c/d + Sqrt[x] - (2*b*ArcTan[(a - b*Tanh[(c + d*Sqrt[x])/2])/Sqrt[-a^2 -
 b^2]])/(Sqrt[-a^2 - b^2]*d)))/a
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a}+\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a}+\frac{4b\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	89
default	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a}+\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a}+\frac{4b\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	89

```
[In] int(1/(a+b*csch(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(-1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))-1)+1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))+
1)+2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*c+1/2*d*x^(1/2))+2*a)/(
a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$
$$=\frac{2\left((a^2+b^2)d\sqrt{x}+\sqrt{a^2+b^2}b\log\left(\frac{ab+\left(a^2+b^2+\sqrt{a^2+b^2}b\right)\cosh\left(d\sqrt{x}+c\right)-\left(b^2+\sqrt{a^2+b^2}b\right)\sinh\left(d\sqrt{x}+c\right)+\sqrt{a^2+b^2}a}{a\sinh\left(d\sqrt{x}+c\right)+b}\right)\right)}{(a^3+ab^2)d}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*((a^2 + b^2)*d*sqrt(x) + sqrt(a^2 + b^2)*b*log((a*b + (a^2 + b^2 + sqrt(a
^2 + b^2)*b)*cosh(d*sqrt(x) + c) - (b^2 + sqrt(a^2 + b^2)*b)*sinh(d*sqrt(x)
 + c) + sqrt(a^2 + b^2)*a)/(a*sinh(d*sqrt(x) + c) + b)))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/(a+b*csch(c+d*x**(1/2)))/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a + b*csch(c + d*sqrt(x)))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=-\frac{2\,b\log\left(\frac{ae^{\left(-d\sqrt{x}-c\right)}-b-\sqrt{a^2+b^2}}{ae^{\left(-d\sqrt{x}-c\right)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ad}+\frac{2\left(d\sqrt{x}+c\right)}{ad}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*b*log((a*e^(-d*sqrt(x) - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*sqrt(x) - c)
 - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + 2*(d*sqrt(x) + c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=-\frac{2\,b\log\left(\frac{\left|2\,ae^{\left(d\sqrt{x}+c\right)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{\left(d\sqrt{x}+c\right)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ad}+\frac{2\left(d\sqrt{x}+c\right)}{ad}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(abs(2*a*e^(d*sqrt(x) + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*
sqrt(x) + c) + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + 2*(d*sqrt(
x) + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}-\frac{2\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}}{a^2\,\sqrt{x}}-\frac{2\,b\,\left(a-b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\right)}{a^2\,\sqrt{x}\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}+\frac{2\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}}{a^2\,\sqrt{x}}+\frac{2\,b\,\left(a-b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\right)}{a^2\,\sqrt{x}\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

```
[In] int(1/(x^(1/2)*(a + b/sinh(c + d*x^(1/2)))),x)
```

```
[Out] (2*x^(1/2))/a - (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^2*x^(1/2)) - (2*b*(
a - b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a^2 + b^2)^(1/2))))/(a*d*(a^2 +
 b^2)^(1/2)) + (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^2*x^(1/2)) + (2*b*(a
 - b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a^2 + b^2)^(1/2))))/(a*d*(a^2 +
b^2)^(1/2))
```

3.64 $\int \frac{1}{x^{3/2}\left(a+b\,\mathbf{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result 426
Rubi [N/A] 426
Mathematica [N/A] 427
Maple [N/A] (verified) 427
Fricas [N/A] 427
Sympy [N/A] 427
Maxima [N/A] 428
Giac [N/A] 428
Mupad [N/A] 428

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])),x]
```

```
[Out] Defer[Int][1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\ \text{csch}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^2*csch(d*sqrt(x) + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(3/2)*(a + b*csch(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(3/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*x^(3/2)*e^(d*sqrt(x) + c) - a^2*x^(3/2)), x) - 2/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csch(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))), x)
```

3.65 $\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result 429
Rubi [N/A] 429
Mathematica [N/A] 430
Maple [N/A] (verified) 430
Fricas [N/A] 430
Sympy [N/A] 430
Maxima [N/A] 431
Giac [N/A] 431
Mupad [N/A] 431

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\mathrm{Int}\left(\frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])),x]
```

```
[Out] Defer[Int][1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{x^{5/2} \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} \, dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b \operatorname{csch}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x)
[Out] int(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{\left(b \operatorname{csch}\left(d\sqrt{x}+c\right)+a\right) x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^3*csch(d*sqrt(x) + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{x^{\frac{5}{2}} \left(a+b \operatorname{csch}\left(c+d\sqrt{x}\right)\right)} \, dx$$

```
[In] integrate(1/x**(5/2)/(a+b*csch(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(5/2)*(a + b*csch(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*x^(5/2)*e^(d*sqrt(x) + c) - a^2*x^(5/2)), x) - 2/3/(a*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(1/((b*csch(d*sqrt(x) + c) + a)*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/sinh(c + d*x^(1/2)))),x)
[Out] int(1/(x^(5/2)*(a + b/sinh(c + d*x^(1/2)))), x)
```

3.66 $\int \frac{x^{3/2}}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 433
Rubi [A] (verified) 434
Mathematica [A] (verified) 442
Maple [F] 443
Fricas [F] 443
Sympy [F] 443
Maxima [F] 444
Giac [F] 444
Mupad [F(-1)] 444

Optimal result

Integrand size = 22, antiderivative size = 1639

$$\int \frac{x^{3/2}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2b^2x^2}{a^2(a^2+b^2)d}+\frac{2x^{5/2}}{5a^2}$$
$$+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}+\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$-\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$-\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}+\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+\frac{24b^2x\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}+\frac{8b^3x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$-\frac{16bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}+\frac{24b^2x\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$-\frac{8b^3x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}+\frac{16bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$-\frac{48b^2\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^4}-\frac{24b^3x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$+\frac{48bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}-\frac{48b^2\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^4}$$
$$+\frac{24b^3x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}-\frac{48bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$+\frac{48b^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^5}+\frac{48b^3\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^4}$$
$$-\frac{96b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^4}+\frac{48b^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^5}$$
$$-\frac{48b^3\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^4}+\frac{96b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^4}$$
$$-\frac{48b^3\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^5}+\frac{96b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^5}$$
$$+\frac{48b^3\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^5}-\frac{96b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^5}$$
$$-\frac{2b^2x^2\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -2*b^2*x^2/a^2/(a^2+b^2)/d-48*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2
)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^5+48*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b+(
a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^5+96*b*polylog(5,-a*exp(c+d*x^(1/2))
/(b-(a^2+b^2)^(1/2)))/a^2/d^5/(a^2+b^2)^(1/2)-96*b*polylog(5,-a*exp(c+d*x^(
1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^5/(a^2+b^2)^(1/2)+48*b^2*polylog(4,-a*exp(
c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^5+48*b^2*polylog(4,-a*exp
(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^5-48*b^2*polylog(3,-a*ex
p(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^4+48*b^3*polylo
g(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^
4-48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^
2+b^2)^(3/2)/d^4-96*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^
(1/2)/a^2/d^4/(a^2+b^2)^(1/2)+96*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(a^2+b^
2)^(1/2)))*x^(1/2)/a^2/d^4/(a^2+b^2)^(1/2)+8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(
1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+2*b^3*x^2*ln(1+a*exp(c+d*x^(1/
2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+8*b^2*x^(3/2)*ln(1+a*exp(c+d
*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2-2*b^3*x^2*ln(1+a*exp(c+d*x
^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+24*b^2*x*polylog(2,-a*ex
p(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3+8*b^3*x^(3/2)*polylog
(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2+24*b^2*
x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-8*b^
3*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^
(3/2)/d^2-24*b^3*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/(
a^2+b^2)^(3/2)/d^3+24*b^3*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2
)))/a^2/(a^2+b^2)^(3/2)/d^3-4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1
/2)))/a^2/d/(a^2+b^2)^(1/2)+4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1
/2)))/a^2/d/(a^2+b^2)^(1/2)-16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(
a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+16*b*x^(3/2)*polylog(2,-a*exp(c+d*
x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)+48*b*x*polylog(3,-a*e
xp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)-48*b*x*polylog
(3,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)-48*b^2*
polylog(3,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^
4+2/5*x^(5/2)/a^2-2*b^2*x^2*cosh(c+d*x^(1/2))/a/(a^2+b^2)/d/(b+a*sinh(c+d*x
^(1/2)))
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 1639, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 6744, 2320, 6724, 5680}

$$\int \frac{x^{3/2}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d} + \frac{8x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2} - \frac{8x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2} - \frac{24x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3} + \frac{24x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3} + \frac{48\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4} - \frac{48\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^4} - \frac{48\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^5} + \frac{48\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^5} - \frac{2x^2b^2}{a^2\left(a^2+b^2\right)d} + \frac{8x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2} + \frac{8x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2} + \frac{24x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3} + \frac{24x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3} - \frac{48\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4} - \frac{48\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^4} + \frac{48\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^5} + \frac{48\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^5} - \frac{2x^2\cosh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} - \frac{4x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d} + \frac{4x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d} - \frac{16x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2} + \frac{16x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2} + \frac{48x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^3} - \frac{48x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^3} - \frac{96\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^4} + \frac{96\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^4} + \frac{96\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^5} - \frac{96\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^5} + \frac{2x^{5/2}}{5a^2}$$

```
[In] Int[x^(3/2)/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x^2)/(a^2*(a^2 + b^2)*d) + (2*x^(5/2))/(5*a^2) + (8*b^2*x^(3/2)*Log
[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) +
(2*b^3*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2
+ b^2)^(3/2)*d) - (4*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^
2])])/(a^2*Sqrt[a^2 + b^2]*d) + (8*b^2*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x])
)/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (2*b^3*x^2*Log[1 + (a*E^(
c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) + (4*b*x^
2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2
]*d) + (24*b^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))]
)/(a^2*(a^2 + b^2)*d^3) + (8*b^3*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))
/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (16*b*x^(3/2)*PolyL
og[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]
*d^2) + (24*b^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))
])/(a^2*(a^2 + b^2)*d^3) - (8*b^3*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x])
)/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (16*b*x^(3/2)*Poly
Log[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2
]*d^2) - (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 +
 b^2]))])/(a^2*(a^2 + b^2)*d^4) - (24*b^3*x*PolyLog[3, -((a*E^(c + d*Sqrt[x
]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (48*b*x*PolyLog[
3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^
3) - (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2
]))])/(a^2*(a^2 + b^2)*d^4) + (24*b^3*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/
(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (48*b*x*PolyLog[3, -
((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) +
 (48*b^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(
a^2 + b^2)*d^5) + (48*b^3*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - S
qrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^4) - (96*b*Sqrt[x]*PolyLog[4, -
((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) +
 (48*b^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(
a^2 + b^2)*d^5) - (48*b^3*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + S
qrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^4) + (96*b*Sqrt[x]*PolyLog[4, -
((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) -
 (48*b^3*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(
a^2 + b^2)^(3/2)*d^5) + (96*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[
a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^5) + (48*b^3*PolyLog[5, -((a*E^(c + d
*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^5) - (96*b*Pol
yLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^
2]*d^5) - (2*b^2*x^2*Cosh[c + d*Sqrt[x]])/(a*(a^2 + b^2)*d*(b + a*Sinh[c +
d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
```

```
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{(a+b\operatorname{csch}(c+dx))^2}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a^2}+\frac{b^2x^4}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^4}{a^2(b+a\sinh(c+dx))}\right)dx, x, \sqrt{x}\right) \\
&= \frac{2x^{5/2}}{5a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^4}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^4}{(b+a\sinh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}
\end{aligned}$$

$$= \frac{2x^{5/2}}{5a^2} - \frac{2b^2x^2\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{a^2} + \frac{\left(2b^3\right)\text{Subst}\left(\int\frac{x^4}{b+a\sinh(c+dx)}\,dx,x,\sqrt{x}\right)}{a^2\left(a^2+b^2\right)} + \frac{\left(8b^2\right)\text{Subst}\left(\int\frac{x^3\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx,x,\sqrt{x}\right)}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b^2x^2}{a^2\left(a^2+b^2\right)d} + \frac{2x^{5/2}}{5a^2} - \frac{2b^2x^2\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} + \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{a^2\left(a^2+b^2\right)} - \frac{(8b)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}} + \frac{(8b)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\sqrt{a^2+b^2}} + \frac{\left(8b^2\right)\text{Subst}\left(\int\frac{e^{c+dx}x^3}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(a^2+b^2\right)d} + \frac{\left(8b^2\right)\text{Subst}\left(\int\frac{e^{c+dx}x^3}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b^2x^2}{a^2(a^2+b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$

$$- \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$

$$+ \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{2b^2x^2\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$

$$+ \frac{(4b^3)\operatorname{Subst}\left(\int\frac{e^{c+dx}x^4}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$

$$- \frac{(4b^3)\operatorname{Subst}\left(\int\frac{e^{c+dx}x^4}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$

$$- \frac{(24b^2)\operatorname{Subst}\left(\int x^2\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$

$$- \frac{(24b^2)\operatorname{Subst}\left(\int x^2\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$

$$+ \frac{(16b)\operatorname{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$- \frac{(16b)\operatorname{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$= -\frac{2b^2x^2}{a^2(a^2+b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$+ \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{24b^2x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{16bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{24b^2x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{16bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2x^2\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(48b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$- \frac{(48b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{(48b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(48b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$- \frac{(8b^3)\,\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{(8b^3)\,\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}$$

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Mathematica [A] (verified)

Time = 6.35 (sec) , antiderivative size = 1696, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\text{csch}^2\left(c+d\sqrt{x}\right)\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)\left(x^{5/2}\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)-\frac{5be^{c}\cdots}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[x^(3/2)/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*Csch[c + d*Sqrt[x]]^2*(b + a*Sinh[c + d*Sqrt[x]])*(x^(5/2)*(b + a*Sinh[c
 + d*Sqrt[x]]) - (5*b*E^c*(2*b*E^c*x^2 - ((-1 + E^(2*c))*(4*b*d^3*Sqrt[(a^2
 + b^2)*E^(2*c)]*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)])] - 2*a^2*d^4*E^c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - b^2*d^4*E^c*x^2*Log[1 + (a*E^(2*c + d*Sq
rt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 4*b*d^3*Sqrt[(a^2 + b^2)*E^(
2*c)]*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)])] + 2*a^2*d^4*E^c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)])] + b^2*d^4*E^c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 4*d^2*(-3*b*Sqrt[(a^2 + b^2)*E^(2*c)] +
 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x*PolyLog[2, -((a*E^(2*c + d*Sqrt
[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*d^2*(3*b*Sqrt[(a^2 + b^2)*E
^(2*c)] + 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x*PolyLog[2, -((a*E^(2*c
 + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*b*d*Sqrt[(a^2 + b
^2)*E^(2*c)]*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))] + 24*a^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x])
)/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*b^2*d^2*E^c*x*PolyLog[3, -((a*
E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*b*d*Sqrt[(a
^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] - 24*a^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sq
rt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*b^2*d^2*E^c*x*PolyLog[3,
 -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*b*Sqr
t[(a^2 + b^2)*E^(2*c)]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))] - 48*a^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sq
rt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*b^2*d*E^c*Sqrt[x]*PolyLo
g[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*b
*Sqrt[(a^2 + b^2)*E^(2*c)]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] + 48*a^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c +
d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*b^2*d*E^c*Sqrt[x]*Po
```

```
lyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] +
48*a^2*E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] + 24*b^2*E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 48*a^2*E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(
b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*b^2*E^c*PolyLog[5, -((a*E^(2*c +
d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^4*E^c*Sqrt[(a^2 + b^
2)*E^(2*c)]))*(b + a*Sinh[c + d*Sqrt[x]]))/((a^2 + b^2)*d*(-1 + E^(2*c)) +
 (5*b^2*x^2*Csch[c]*(b*Cosh[c] + a*Sinh[d*Sqrt[x]]))/((a^2 + b^2)*d)))/(5*a
^2*(a + b*Csch[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{\left(a+b\ \mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

```
[In] int(x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\,\mathrm{csch}\left(d\sqrt{x}+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^(3/2)/(b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(a+b\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(a+b\,\mathrm{csch}\left(c+d\sqrt{x}\right)\right)^{2}}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(x**(3/2)/(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")

[Out] 2/5*(10*a*b^2*x^2 - (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^(5/2)*e^(2*d*sqrt(x
)) + (a^3*d + a*b^2*d)*x^(5/2) - 2*(5*b^3*x^2*e^c + (a^2*b*d*e^c + b^3*d*e^
c)*x^(5/2))*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*
e^(2*c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))) -
 integrate(-2*(4*a*b^2*x^2 - (4*b^3*x^2*e^c + (2*a^2*b*d*e^c + b^3*d*e^c)*x
^(5/2))*e^(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x
)) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*
x), x)

Giac [F]

$$\int \frac{x^{3/2}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x^(3/2)/(b*csch(d*sqrt(x) + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{3/2}}{\left(a+\frac{b}{\sinh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

[In] int(x^(3/2)/(a + b/sinh(c + d*x^(1/2)))^2,x)

[Out] int(x^(3/2)/(a + b/sinh(c + d*x^(1/2)))^2, x)

3.67 $\int \frac{\sqrt{x}}{\left(a+b\,\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 446
Rubi [A] (verified) 447
Mathematica [A] (verified) 456
Maple [F] 457
Fricas [F] 457
Sympy [F] 458
Maxima [F] 458
Giac [F] 458
Mupad [F(-1)] 459

Optimal result

Integrand size = 22, antiderivative size = 959

$$\int \frac{\sqrt{x}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2b^2x}{a^2(a^2+b^2)\,d}+\frac{2x^{3/2}}{3a^2}+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2}+\frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d}-\frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2}-\frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d}+\frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}+\frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3}+\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2}-\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2}+\frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3}-\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2}+\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2}-\frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3}+\frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3}+\frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3}-\frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3}-\frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)\,d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -2*b^2*x/a^2/(a^2+b^2)/d+2/3*x^(3/2)/a^2+2*b^3*x*ln(1+a*exp(c+d*x^(1/2))/(b
-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-2*b^3*x*ln(1+a*exp(c+d*x^(1/2))/(b
+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+4*b^2*polylog(2,-a*exp(c+d*x^(1/2)
)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3+4*b^2*polylog(2,-a*exp(c+d*x^(1/2)
)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-4*b^3*polylog(3,-a*exp(c+d*x^(1/2)
)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3+4*b^3*polylog(3,-a*exp(c+d*x
^(1/2))/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-2*b^2*x*cosh(c+d*x^(1/
2))/a/(a^2+b^2)/d/(b+a*sinh(c+d*x^(1/2)))-4*b*x*ln(1+a*exp(c+d*x^(1/2))/(b-
(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+4*b*x*ln(1+a*exp(c+d*x^(1/2))/(b+(a
^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)+8*b*polylog(3,-a*exp(c+d*x^(1/2))/(b-
(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)-8*b*polylog(3,-a*exp(c+d*x^(1/2))
/(b+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+4*b^2*ln(1+a*exp(c+d*x^(1/2))
/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^2+4*b^2*ln(1+a*exp(c+d*x^(1/2
))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)/d^2+4*b^3*polylog(2,-a*exp(c+
d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d^2-4*b^3*polyl
og(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2+b^2)^(3/2)/d
^2-8*b*polylog(2,-a*exp(c+d*x^(1/2))/(b-(a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(
a^2+b^2)^(1/2)+8*b*polylog(2,-a*exp(c+d*x^(1/2))/(b+(a^2+b^2)^(1/2)))*x^(1/
2)/a^2/d^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 959, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules

used = {5545, 4276, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\int \frac{\sqrt{x}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d}$$
$$+ \frac{4\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$
$$- \frac{4\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^2}$$
$$- \frac{4\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3}$$
$$+ \frac{4\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^3}{a^2\left(a^2+b^2\right)^{3/2}d^3} - \frac{2xb^2}{a^2\left(a^2+b^2\right)d}$$
$$+ \frac{4\sqrt{x}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2} + \frac{4\sqrt{x}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b^2}{a^2\left(a^2+b^2\right)d^2}$$
$$+ \frac{4\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$
$$+ \frac{4\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b^2}{a^2\left(a^2+b^2\right)d^3}$$
$$- \frac{2x\cosh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{4x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d} + \frac{4x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{a^2+b^2}}+1\right)b}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{8\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{8\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{8\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{8\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)b}{a^2\sqrt{a^2+b^2}d^3} + \frac{2x^{3/2}}{3a^2}$$

```
[In] Int[Sqrt[x]/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (-2*b^2*x)/(a^2*(a^2 + b^2)*d) + (2*x^(3/2))/(3*a^2) + (4*b^2*Sqrt[x]*Log[1
 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (2
*b^3*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^
2)^(3/2)*d) - (4*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2])])/
(a^2*Sqrt[a^2 + b^2]*d) + (4*b^2*Sqrt[x]*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (2*b^3*x*Log[1 + (a*E^(c + d*Sq
rt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) + (4*b*x*Log[1 +
(a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (4*
b^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
 b^2)*d^3) + (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^
2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (8*b*Sqrt[x]*PolyLog[2, -((a*E^(
c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (4*b^2
*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^
2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[a^2 +
 b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) + (8*b*Sqrt[x]*PolyLog[2, -((a*E^(c +
 d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (4*b^3*Po
lyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^
(3/2)*d^3) + (8*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[a^2 + b^2]))
])/(a^2*Sqrt[a^2 + b^2]*d^3) + (4*b^3*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (8*b*PolyLog[3, -((a*E
^(c + d*Sqrt[x]))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (2*b
^2*x*Cosh[c + d*Sqrt[x]])/(a*(a^2 + b^2)*d*(b + a*Sinh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
```

```
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{(a+b\text{csch}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sinh(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(b+a\sinh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{2x^{3/2}}{3a^2}-\frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad+\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}+\frac{(4b^2)\,\text{Subst}\left(\int \frac{x\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)d}
\end{aligned}$$

$$= -\frac{2b^2x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)}$$
$$- \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{a^2+b^2}}$$
$$+ \frac{(4b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)d}$$
$$+ \frac{(4b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)d}$$

$$= -\frac{2b^2x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2+b^2)^{3/2}}$$
$$- \frac{(4b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$- \frac{(4b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)d^2}$$
$$+ \frac{(8b)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$
$$- \frac{(8b)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d}$$

$$
\begin{aligned}
&= -\frac{2b^2 x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{2b^3 x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3 x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad + \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&\quad + \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2 x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} \\
&\quad - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ax}{b-\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)d^3} \\
&\quad - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)d^3} \\
&\quad + \frac{(8b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&\quad - \frac{(8b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&\quad - \frac{(4b^3)\,\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad + \frac{(4b^3)\,\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d}
\end{aligned}
$$

$$= -\frac{2b^2x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$- \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{4b^3\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{4b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{4b^3\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(8b)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{(8b)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2\sqrt{a^2+b^2}d^3}$$
$$- \frac{(4b^3)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{(4b^3)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$

$$
\begin{aligned}
&= -\frac{2b^2x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad + \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} \\
&\quad + \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} \\
&\quad + \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} \\
&\quad + \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} + \frac{8b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} \\
&\quad - \frac{8b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} - \frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)} \\
&\quad - \frac{(4b^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)^{3/2}d^3} \\
&\quad + \frac{(4b^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2+b^2)^{3/2}d^3}
\end{aligned}
$$

$$= -\frac{2b^2x}{a^2(a^2+b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$- \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} + \frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3}$$
$$+ \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}$$
$$+ \frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} - \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2} - \frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$+ \frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} + \frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$- \frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3} - \frac{2b^2x\cosh\left(c+d\sqrt{x}\right)}{a(a^2+b^2)d\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 948, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{x}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{\operatorname{csch}^2\left(c+d\sqrt{x}\right)\left(b+a\sinh\left(c+d\sqrt{x}\right)\right)\left(2x^{3/2}\left(b+a\sinh\left(c+d\sqrt{x}\right)\right) - \frac{6be^c\left(2be^cx - \frac{e^{-c}\left(-1+e^{2c}\right)\left(2bd\sqrt{\left(a^2+b^2\right)e^{2c}}\right.\cdots}{\cdots}\right)}{\cdots}\right.\cdots}{\cdots}$$

```
[In] Integrate[Sqrt[x]/(a + b*Csch[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Csch[c + d*Sqrt[x]]^2*(b + a*Sinh[c + d*Sqrt[x]])*(2*x^(3/2)*(b + a*Sinh[c
 + d*Sqrt[x]]) - (6*b*E^c*(2*b*E^c*x - ((-1 + E^(2*c))*(2*b*d*Sqrt[(a^2 + b
```

```
^2)*E^(2*c)]*Sqrt[x]*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])] - 2*a^2*d^2*E^c*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - S
qrt[(a^2 + b^2)*E^(2*c)])] - b^2*d^2*E^c*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/
(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*b*d*Sqrt[(a^2 + b^2)*E^(2*c)]*Sqrt
[x]*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] +
2*a^2*d^2*E^c*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E
^(2*c)])] + b^2*d^2*E^c*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)])] + 2*(b*Sqrt[(a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x]
- b^2*d*E^c*Sqrt[x])*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))] + 2*(b*Sqrt[(a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x]
 + b^2*d*E^c*Sqrt[x])*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))] + 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sq
rt[x]))/(b*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 4*a^2*E^c*PolyLog[3, -((a*E
^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^c*PolyL
og[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^
2*E^c*Sqrt[(a^2 + b^2)*E^(2*c)]))*(b + a*Sinh[c + d*Sqrt[x]]))/((a^2 + b^2)
*d*(-1 + E^(2*c))) + (6*b^2*x*Csch[c]*(b*Cosh[c] + a*Sinh[d*Sqrt[x]]))/((a^
2 + b^2)*d)))/(3*a^2*(a + b*Csch[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{\sqrt{x}}{\left(a + b\ \text{csch}\left(c + d\sqrt{x}\right)\right)^2} dx$$

[In] `int(x^(1/2)/(a+b*csch(c+d*x^(1/2)))^2,x)`

[Out] `int(x^(1/2)/(a+b*csch(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{\sqrt{x}}{\left(a + b\text{csch}\left(c + d\sqrt{x}\right)\right)^2} dx = \int \frac{\sqrt{x}}{\left(b\,\text{csch}\left(d\sqrt{x} + c\right) + a\right)^2} dx$$

[In] `integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")`

[Out] `integral(sqrt(x)/(b^2*csch(d*sqrt(x) + c)^2 + 2*a*b*csch(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(sqrt(x)/(a + b*csch(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 2/3*(6*a*b^2*x - (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x^(3/2)*e^(2*d*sqrt(x))
+ (a^3*d + a*b^2*d)*x^(3/2) - 2*(3*b^3*x*e^c + (a^2*b*d*e^c + b^3*d*e^c)*x^
(3/2))*e^(d*sqrt(x)))/(a^5*d + a^3*b^2*d - (a^5*d*e^(2*c) + a^3*b^2*d*e^(2*
c))*e^(2*d*sqrt(x)) - 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*e^(d*sqrt(x))) - inte
grate(-2*(2*a*b^2*x - (2*b^3*x*e^c + (2*a^2*b*d*e^c + b^3*d*e^c)*x^(3/2))*e
^(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x)) + 2*(a
^4*b*d*e^c + a^2*b^3*d*e^c)*x*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*csch(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(a+\frac{b}{\sinh\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(1/2)/(a + b/sinh(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)/(a + b/sinh(c + d*x^(1/2)))^2, x)
```

3.68 $\int \frac{1}{\sqrt{x}\left(a+b\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 460
Rubi [A] (verified) 460
Mathematica [A] (verified) 462
Maple [A] (verified) 463
Fricas [B] (verification not implemented) 463
Sympy [F] 464
Maxima [A] (verification not implemented) 464
Giac [A] (verification not implemented) 465
Mupad [B] (verification not implemented) 465

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{1}{\sqrt{x}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\sqrt{x}}{a^2}+\frac{4b(2a^2+b^2)\,\text{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] 4*b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*c+1/2*d*x^(1/2)))/(a^2+b^2)^(1/2))/a^
2/(a^2+b^2)^(3/2)/d-2*b^2*coth(c+d*x^(1/2))/a/(a^2+b^2)/d/(a+b*csch(c+d*x^(
1/2)))+2*x^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5545, 3870, 4004, 3916, 2739, 632, 210}

$$\int \frac{1}{\sqrt{x}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{4b(2a^2+b^2)\,\text{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{ad\left(a^2+b^2\right)\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)}+\frac{2\sqrt{x}}{a^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*Sqrt[x])/a^2 + (4*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*Sqrt[x])/2]
)/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - (2*b^2*Coth[c + d*Sqrt[x]])
/(a*(a^2 + b^2)*d*(a + b*Csch[c + d*Sqrt[x]]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
```

```
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} - \frac{2\text{Subst}\left(\int \frac{-a^2-b^2+ab\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2+b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2b(2a^2+b^2)\right)\text{Subst}\left(\int \frac{\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2+b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} + \frac{\left(4i(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)} - \frac{\left(8i(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{4b(2a^2+b^2)\,\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{2b^2\coth\left(c+d\sqrt{x}\right)}{a\left(a^2+b^2\right)d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{2\text{csch}\left(c+d\sqrt{x}\right)\left(-\frac{ab^2\coth\left(c+d\sqrt{x}\right)}{a^2+b^2} + \left(c+d\sqrt{x}\right)\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right) + \frac{2b(2a^2+b^2)\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{-a^2-b^2}}\right)\left(\cdots\right.}{\left(-a^2-b^2\right)^{3/2}}\right.}{a^2d\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] (2*Csch[c + d*Sqrt[x]]*(-((a*b^2*Coth[c + d*Sqrt[x]])/(a^2 + b^2)) + (c + d
*Sqrt[x])*(a + b*Csch[c + d*Sqrt[x]]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Ta
nh[(c + d*Sqrt[x])/2])/Sqrt[-a^2 - b^2]]*(a + b*Csch[c + d*Sqrt[x]]))/(-a^2
 - b^2)^(3/2))*(b + a*Sinh[c + d*Sqrt[x]]))/(a^2*d*(a + b*Csch[c + d*Sqrt[x
]])^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a^{2}}-\frac{4b\left(\frac{\frac{a^{2}\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^{2}+2b^{2}}+\frac{ab}{2a^{2}+2b^{2}}}{-\frac{\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^{2}b}{2}+a\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(2a^{2}+2b^{2}\right)\sqrt{a^{2}+b^{2}}}\right)}{a^{2}}-}{d}$
default	$\frac{\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a^{2}}-\frac{4b\left(\frac{\frac{a^{2}\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^{2}+2b^{2}}+\frac{ab}{2a^{2}+2b^{2}}}{-\frac{\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^{2}b}{2}+a\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(2a^{2}+2b^{2}\right)\sqrt{a^{2}+b^{2}}}\right)}{a^{2}}-}{d}$

```
[In] int(1/(a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(1/a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))+1)-2/a^2*b*((1/2*a^2/(a^2+b^2)*tanh
(1/2*c+1/2*d*x^(1/2))+1/2*b*a/(a^2+b^2))/(-1/2*tanh(1/2*c+1/2*d*x^(1/2))^2*
b+a*tanh(1/2*c+1/2*d*x^(1/2))+1/2*b)-2*(2*a^2+b^2)/(2*a^2+2*b^2)/(a^2+b^2)^
(1/2)*arctanh(1/2*(-2*b*tanh(1/2*c+1/2*d*x^(1/2))+2*a)/(a^2+b^2)^(1/2)))-1/
a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(107) = 214$.

Time = 0.32 (sec) , antiderivative size = 670, normalized size of antiderivative = 5.68

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^{2}}\,dx =$$

$$-\frac{2\left(2a^{3}b^{2}+2ab^{4}-(a^{5}+2a^{3}b^{2}+ab^{4})d\sqrt{x}\cosh\left(d\sqrt{x}+c\right)^{2}-(a^{5}+2a^{3}b^{2}+ab^{4})d\sqrt{x}\sinh\left(d\sqrt{x}+c\right)^{2}\right.}{}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sqrt(x
) + c)^2 - (a^5 + 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 + (a^5
 + 2*a^3*b^2 + a*b^4)*d*sqrt(x) - 2*(a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b
^5)*d*sqrt(x))*cosh(d*sqrt(x) + c) - ((2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*cos
h(d*sqrt(x) + c)^2 + (2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*sinh(d*sqrt(x) + c)^
2 + 2*(2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cosh(d*sqrt(x) + c) + 2*((2*a^3*b +
 a*b^3)*sqrt(a^2 + b^2)*cosh(d*sqrt(x) + c) + (2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2))*sinh(d*sqrt(x) + c) - (2*a^3*b + a*b^3)*sqrt(a^2 + b^2))*log((a*b + (
a^2 + b^2 + sqrt(a^2 + b^2)*b)*cosh(d*sqrt(x) + c) - (b^2 + sqrt(a^2 + b^2)
*b)*sinh(d*sqrt(x) + c) + sqrt(a^2 + b^2)*a)/(a*sinh(d*sqrt(x) + c) + b)) -
 2*(a^2*b^3 + b^5 + (a^5 + 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sqrt(x) + c)
 + (a^4*b + 2*a^2*b^3 + b^5)*d*sqrt(x))*sinh(d*sqrt(x) + c))/((a^7 + 2*a^5*
b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c)^2 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*sinh
(d*sqrt(x) + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*sqrt(x) + c) -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sq
rt(x) + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*sqrt(x) + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a + b\mathrm{csch}\left(c + d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\sqrt{x}\left(a + b\operatorname{csch}\left(c + d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*csch(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(a + b*csch(c + d*sqrt(x)))**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{x}\left(a + b\mathrm{csch}\left(c + d\sqrt{x}\right)\right)^2}\,dx = -\frac{2\,(2\,a^2b + b^3)\log\left(\frac{ae^{(-d\sqrt{x}-c)} - b - \sqrt{a^2+b^2}}{ae^{(-d\sqrt{x}-c)} - b + \sqrt{a^2+b^2}}\right)}{(a^4 + a^2b^2)\sqrt{a^2+b^2}d} - \frac{4\left(b^3e^{(-d\sqrt{x}-c)} + ab^2\right)}{\left(a^5 + a^3b^2 + 2\,(a^4b + a^2b^3)e^{(-d\sqrt{x}-c)} - (a^5 + a^3b^2)e^{(-2\,d\sqrt{x}-2\,c)}\right)d} + \frac{2\,\left(d\sqrt{x} + c\right)}{a^2d}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
[Out] -2*(2*a^2*b + b^3)*log((a*e^(-d*sqrt(x) - c) - b - sqrt(a^2 + b^2))/(a*e^(-
d*sqrt(x) - c) - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d)
```

```
- 4*(b^3*e^(-d*sqrt(x) - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*
e^(-d*sqrt(x) - c) - (a^5 + a^3*b^2)*e^(-2*d*sqrt(x) - 2*c))*d) + 2*(d*sqrt
(x) + c)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2\left(2a^2b+b^3\right)\log\left(\frac{\left|2ae^{\left(d\sqrt{x}+c\right)}+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^{\left(d\sqrt{x}+c\right)}+2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^4d+a^2b^2d\right)\sqrt{a^2+b^2}} + \frac{4\left(b^3e^{\left(d\sqrt{x}+c\right)}-ab^2\right)}{\left(a^4d+a^2b^2d\right)\left(ae^{\left(2d\sqrt{x}+2c\right)}+2be^{\left(d\sqrt{x}+c\right)}-a\right)} + \frac{2\left(d\sqrt{x}+c\right)}{a^2d}$$

```
[In] integrate(1/(a+b*csch(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*(2*a^2*b + b^3)*log(abs(2*a*e^(d*sqrt(x) + c) + 2*b - 2*sqrt(a^2 + b^2))
/abs(2*a*e^(d*sqrt(x) + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4*d + a^2*b^2*d)
*sqrt(a^2 + b^2)) + 4*(b^3*e^(d*sqrt(x) + c) - a*b^2)/((a^4*d + a^2*b^2*d)*
(a*e^(2*d*sqrt(x) + 2*c) + 2*b*e^(d*sqrt(x) + c) - a)) + 2*(d*sqrt(x) + c)/
(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.70

$$\int \frac{1}{\sqrt{x}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\sqrt{x}}{a^2} - \frac{\frac{4b^2\sqrt{x}}{d\left(a^3\sqrt{x}+ab^2\sqrt{x}\right)} - \frac{4b^3\sqrt{x}\,\mathrm{e}^{c+d\sqrt{x}}}{a\,d\left(a^3\sqrt{x}+ab^2\sqrt{x}\right)}}{2b\,\mathrm{e}^{c+d\sqrt{x}}-a+a\,\mathrm{e}^{2c+2d\sqrt{x}}} - \frac{2b\ln\left(\frac{2\mathrm{e}^{c+d\sqrt{x}}\left(2a^2b+b^3\right)}{a^3\sqrt{x}\left(a^2+b^2\right)} - \frac{2b\left(2a^2+b^2\right)\left(a-b\,\mathrm{e}^{c+d\sqrt{x}}\right)}{a^3\sqrt{x}\left(a^2+b^2\right)^{3/2}}\right)\left(2a^2+b^2\right)}{a^2d\left(a^2+b^2\right)^{3/2}} + \frac{2b\ln\left(\frac{2\mathrm{e}^{c+d\sqrt{x}}\left(2a^2b+b^3\right)}{a^3\sqrt{x}\left(a^2+b^2\right)} + \frac{2b\left(2a^2+b^2\right)\left(a-b\,\mathrm{e}^{c+d\sqrt{x}}\right)}{a^3\sqrt{x}\left(a^2+b^2\right)^{3/2}}\right)\left(2a^2+b^2\right)}{a^2d\left(a^2+b^2\right)^{3/2}}$$

```
[In] int(1/(x^(1/2)*(a + b/sinh(c + d*x^(1/2)))^2),x)
```

```
[Out] (2*x^(1/2))/a^2 - ((4*b^2*x^(1/2))/(d*(a^3*x^(1/2) + a*b^2*x^(1/2))) - (4*b
^3*x^(1/2)*exp(c + d*x^(1/2)))/(a*d*(a^3*x^(1/2) + a*b^2*x^(1/2))))/(2*b*ex
p(c + d*x^(1/2)) - a + a*exp(2*c + 2*d*x^(1/2))) - (2*b*log((2*exp(c + d*x^
(1/2))*(2*a^2*b + b^3))/(a^3*x^(1/2)*(a^2 + b^2)) - (2*b*(2*a^2 + b^2)*(a -
 b*exp(c + d*x^(1/2))))/(a^3*x^(1/2)*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^
2*d*(a^2 + b^2)^(3/2)) + (2*b*log((2*exp(c + d*x^(1/2))*(2*a^2*b + b^3))/(a
^3*x^(1/2)*(a^2 + b^2)) + (2*b*(2*a^2 + b^2)*(a - b*exp(c + d*x^(1/2))))/(a
^3*x^(1/2)*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b^2)^(3/2))
```

3.69 $\int \frac{1}{x^{3/2}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 467
Rubi [N/A] 467
Mathematica [N/A] 468
Maple [N/A] (verified) 468
Fricas [N/A] 468
Sympy [N/A] 469
Maxima [N/A] 469
Giac [N/A] 469
Mupad [N/A] 470

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 60.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^2*csch(d*sqrt(x) + c)^2 + 2*a*b*x^2*csch(d*sqrt(x)
+ c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(3/2)*(a + b*csch(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 14.14

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -2*(2*a*b^2 + (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*sqrt(x)*e^(2*d*sqrt(x)) - 2
*(b^3*e^c - (a^2*b*d*e^c + b^3*d*e^c)*sqrt(x))*e^(d*sqrt(x)) - (a^3*d + a*b
^2*d)*sqrt(x))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x)) + 2*(
a^4*b*d*e^c + a^2*b^3*d*e^c)*x*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*x) + int
egrate(-2*(2*a*b^2*sqrt(x) - (2*b^3*sqrt(x)*e^c - (2*a^2*b*d*e^c + b^3*d*e^
c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^(5/2)*e^(2*d*sq
rt(x)) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^(5/2)*e^(d*sqrt(x)) - (a^5*d + a
^3*b^2*d)*x^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csch(d*sqrt(x) + c) + a)^2*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a + b\operatorname{csch}\left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{x^{3/2} \left(a + \frac{b}{\sinh\left(c + d\sqrt{x}\right)}\right)^2} \, dx$$

```
[In] int(1/(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(3/2)*(a + b/sinh(c + d*x^(1/2)))^2), x)
```

3.70 $\int \frac{1}{x^{5/2}\left(a+b\mathbf{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result 471
Rubi [N/A] 471
Mathematica [N/A] 472
Maple [N/A] (verified) 472
Fricas [N/A] 472
Sympy [N/A] 473
Maxima [N/A] 473
Giac [N/A] 473
Mupad [N/A] 474

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 61.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Csch[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}}\left(a+b\,\text{csch}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^3*csch(d*sqrt(x) + c)^2 + 2*a*b*x^3*csch(d*sqrt(x)
+ c) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{\frac{5}{2}}\left(a+b\operatorname{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(5/2)/(a+b*csch(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(5/2)*(a + b*csch(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 318, normalized size of antiderivative = 14.45

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -2/3*(6*a*b^2 + (a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*sqrt(x)*e^(2*d*sqrt(x)) -
 2*(3*b^3*e^c - (a^2*b*d*e^c + b^3*d*e^c)*sqrt(x))*e^(d*sqrt(x)) - (a^3*d +
 a*b^2*d)*sqrt(x))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x))
 + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^2*e^(d*sqrt(x)) - (a^5*d + a^3*b^2*d)*
x^2) + integrate(-2*(4*a*b^2*sqrt(x) - (4*b^3*sqrt(x)*e^c - (2*a^2*b*d*e^c
+ b^3*d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) + a^3*b^2*d*e^(2*c))*x^(7/2)
*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c + a^2*b^3*d*e^c)*x^(7/2)*e^(d*sqrt(x)) -
(a^5*d + a^3*b^2*d)*x^(7/2)), x)
```

Giac [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^{5/2}\left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{csch}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csch(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} \left(a+b\text{csch}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{x^{5/2}\,{\left(a+\frac{b}{\mathrm{sinh}\left(c+d\,\sqrt{x}\right)}\right)}^2} \,d x$$

```
[In] int(1/(x^(5/2)*(a + b/sinh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(5/2)*(a + b/sinh(c + d*x^(1/2)))^2), x)
```

3.71 $\int (ex)^m (a+b\,\mathrm{csch}(c+dx^n))^p \, dx$

Optimal result 475
Rubi [N/A] 475
Mathematica [N/A] 476
Maple [N/A] (verified) 476
Fricas [N/A] 476
Sympy [N/A] 476
Maxima [N/A] 477
Giac [N/A] 477
Mupad [N/A] 477

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\,\mathrm{csch}(c+dx^n))^p \, dx = x^{-m}(ex)^m \mathrm{Int}(x^m(a+b\,\mathrm{csch}(c+dx^n))^p, x)$$

```
[Out] (e*x)^m*Unintegrable(x^m*(a+b*csch(c+d*x^n))^p,x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\,\mathrm{csch}(c+dx^n))^p \, dx = \int (ex)^m (a+b\,\mathrm{csch}(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Csch[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*(a + b*Csch[c + d*x^n])^p, x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m(a+b\,\mathrm{csch}(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 14.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\text{csch}(c+dx^n))^p \, dx = \int (ex)^m (a+b\text{csch}(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Csch[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Csch[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b \operatorname{csch}(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*csch(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*csch(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\text{csch}(c+dx^n))^p \, dx = \int (ex)^m (b \operatorname{csch}(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csch(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*csch(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\text{csch}(c+dx^n))^p \, dx = \int (ex)^m (a + b \operatorname{csch}(c+dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*csch(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*csch(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\text{csch}(c + dx^n))^p \, dx = \int (ex)^m (b \operatorname{csch}(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csch(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*csch(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\text{csch}(c + dx^n))^p \, dx = \int (ex)^m (b \operatorname{csch}(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csch(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*csch(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b\text{csch}(c + dx^n))^p \, dx = \int \left(a + \frac{b}{\sinh(c + d\,x^n)}\right)^p (e\,x)^m \, dx$$

```
[In] int((a + b/sinh(c + d*x^n))^p*(e*x)^m,x)
[Out] int((a + b/sinh(c + d*x^n))^p*(e*x)^m, x)
```

3.72 $\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n)) \, dx$

Optimal result 478
Rubi [A] (verified) 478
Mathematica [A] (verified) 479
Maple [C] (warning: unable to verify) 480
Fricas [B] (verification not implemented) 480
Sympy [F] 481
Maxima [A] (verification not implemented) 481
Giac [F] 481
Mupad [B] (verification not implemented) 481

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n)) \, dx = \frac{a(ex)^n}{en} - \frac{bx^{-n}(ex)^n \text{arctanh}(\cosh(c + dx^n))}{den}$$

```
[Out] a*(e*x)^n/e/n-b*(e*x)^n*arctanh(cosh(c+d*x^n))/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {14, 5549, 5545, 3855}

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n)) \, dx = \frac{a(ex)^n}{en} - \frac{bx^{-n}(ex)^n \text{arctanh}(\cosh(c + dx^n))}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^n)/(e*n) - (b*(e*x)^n*ArcTanh[Cosh[c + d*x^n]])/(d*e*n*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+n}+b(ex)^{-1+n}\text{csch}(c+dx^n)\right)\,dx\\
&= \frac{a(ex)^n}{en}+b\int (ex)^{-1+n}\text{csch}(c+dx^n)\,dx\\
&= \frac{a(ex)^n}{en}+\frac{\left(bx^{-n}(ex)^n\right)\int x^{-1+n}\text{csch}(c+dx^n)\,dx}{e}\\
&= \frac{a(ex)^n}{en}+\frac{\left(bx^{-n}(ex)^n\right)\text{Subst}\left(\int \text{csch}(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a(ex)^n}{en}-\frac{bx^{-n}(ex)^n\text{arctanh}(\cosh\left(c+dx^n\right))}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int (ex)^{-1+n}\left(a+b\text{csch}(c+dx^n)\right)\,dx = \frac{x^{-n}(ex)^n\left(a(c+dx^n)-b\log\left(\cosh\left(\frac{1}{2}(c+dx^n)\right)\right)+b\log\left(\sinh\left(\frac{1}{2}(c+dx^n)\right)\right)\right)}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Csch[c + d*x^n]),x]
[Out] ((e*x)^n*(a*(c + d*x^n) - b*Log[Cosh[(c + d*x^n)/2]] + b*Log[Sinh[(c + d*x^
n)/2]]))/(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.44

method	result
risch	$\frac{a x \,{\mathrm e}^{\frac{\left(-1+n \right) \left(-i \,\mathrm{csgn}\left(i e \right) \mathrm{csgn}\left(i x \right) \mathrm{csgn}\left(i e x \right) \pi +i \,\mathrm{csgn}\left(i e \right) \mathrm{csgn}\left(i e x \right)^{2} \pi +i \,\mathrm{csgn}\left(i x \right) \mathrm{csgn}\left(i e x \right)^{2} \pi -i \mathrm{csgn}\left(i e x \right)^{3} \pi +2 \ln \left(e \right)+2 \ln \left(x \right)\right)}{2}}}{n}-\frac{2 \,\mathrm{arctanh}\left({\mathrm e}^{c +d x}\right.}{}$

[In] int((e*x)^(-1+n)*(a+b*csch(c+d*x^n)),x,method=_RETURNVERBOSE)

[Out] a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csg
n(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(
x)))-2*arctanh(exp(c+d*x^n))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(c
sgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(45) = 90.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.00

$$\int (ex)^{-1+n} \left(a + b\text{csch}(c + dx^n)\right) \, dx$$
$$= \frac{ad\cosh\left((n-1)\log\left(e\right)\right)\cosh\left(n\log\left(x\right)\right) + ad\cosh\left(n\log\left(x\right)\right)\sinh\left((n-1)\log\left(e\right)\right) - \left(b\cosh\left((n-1)\log\left(e\right)\right.\right.}{}$$

[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n)),x, algorithm="fricas")

[Out] (a*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + a*d*cosh(n*log(x))*sinh((n - 1)*
log(e)) - (b*cosh((n - 1)*log(e)) + b*sinh((n - 1)*log(e)))*log(cosh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x
)) + c) + 1) + (b*cosh((n - 1)*log(e)) + b*sinh((n - 1)*log(e)))*log(cosh(d
*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c) - 1) + (a*d*cosh((n - 1)*log(e)) + a*d*sinh((n - 1)*log(e)))*s
inh(n*log(x)))/(d*n)

Sympy [F]

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))\, dx = \int (ex)^{n-1} (a + b\operatorname{csch}(c + dx^n))\, dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*csch(c+d*x**n)),x)
[Out] Integral((e*x)**(n - 1)*(a + b*csch(c + d*x**n)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.67

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))\, dx = -b\left(\frac{e^{n-1}\log\left(\left(e^{(dx^n+c)}+1\right)e^{(-c)}\right)}{dn} - \frac{e^{n-1}\log\left(\left(e^{(dx^n+c)}-1\right)e^{(-c)}\right)}{dn}\right) + \frac{(ex)^n a}{en}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n)),x, algorithm="maxima")
[Out] -b*(e^(n - 1)*log((e^(d*x^n + c) + 1)*e^(-c))/(d*n) - e^(n - 1)*log((e^(d*x
^n + c) - 1)*e^(-c))/(d*n)) + (e*x)^n*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))\, dx = \int (b\operatorname{csch}(dx^n + c) + a)(ex)^{n-1}\, dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^n + c) + a)*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))\, dx = \frac{a\,x\,(e\,x)^{n-1}}{n} - \frac{2\,\mathrm{atan}\left(\frac{b\,x\,\mathrm{e}^{d\,x^n}\,\mathrm{e}^c\,(e\,x)^{n-1}\,\sqrt{-d^2\,n^2\,x^{2\,n}}}{d\,n\,x^n\,\sqrt{b^2\,x^2\,(e\,x)^{2\,n-2}}}\right)\,\sqrt{b^2\,x^2\,(e\,x)^{2\,n-2}}}{\sqrt{-d^2\,n^2\,x^{2\,n}}}$$

```
[In] int((a + b/sinh(c + d*x^n))*(e*x)^(n - 1),x)

[Out] (a*x*(e*x)^(n - 1))/n - (2*atan((b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(-d^2*
n^2*x^(2*n))^(1/2))/(d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(b^2*x^2*(e*
x)^(2*n - 2))^(1/2))/(-d^2*n^2*x^(2*n))^(1/2)
```

3.73 $\int (ex)^{-1+2n}(a+b\,\mathrm{csch}(c+dx^n))\,dx$

Optimal result 483
Rubi [A] (verified) 483
Mathematica [A] (verified) 485
Maple [C] (warning: unable to verify) 486
Fricas [B] (verification not implemented) 486
Sympy [F] 487
Maxima [F] 487
Giac [F] 487
Mupad [F(-1)] 488

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int (ex)^{-1+2n}(a+b\,\mathrm{csch}(c+dx^n))\,dx = \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\mathrm{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en}$$

```
[Out] 1/2*a*(e*x)^(2*n)/e/n-2*b*(e*x)^(2*n)*arctanh(exp(c+d*x^n))/d/e/n/(x^n)-b*(
e*x)^(2*n)*polylog(2,-exp(c+d*x^n))/d^2/e/n/(x^(2*n))+b*(e*x)^(2*n)*polylog
(2,exp(c+d*x^n))/d^2/e/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 5549, 5545, 4267, 2317, 2438}

$$\int (ex)^{-1+2n}(a+b\,\mathrm{csch}(c+dx^n))\,dx = \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\mathrm{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,-e^{dx^n+c}\right)}{d^2en} + \frac{bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,e^{dx^n+c}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(2*n))/(2*e*n) - (2*b*(e*x)^(2*n)*ArcTanh[E^(c + d*x^n)])/(d*e*n*x
^n) - (b*(e*x)^(2*n)*PolyLog[2, -E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (b*(e*
x)^(2*n)*PolyLog[2, E^(c + d*x^n)])/(d^2*e*n*x^(2*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \left(a(ex)^{-1+2n} + b(ex)^{-1+2n}\text{csch}(c + dx^n)\right) dx$$

$$
\begin{aligned}
&= \frac{a(ex)^{2n}}{2en} + b\int (ex)^{-1+2n}\text{csch}(c+dx^n)\,dx \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n})\int x^{-1+2n}\text{csch}(c+dx^n)\,dx}{e} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int x\text{csch}(c+dx)\,dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1-e^{c+dx}\right)\,dx, x, x^n\right)}{den} \\
&\quad + \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1+e^{c+dx}\right)\,dx, x, x^n\right)}{den} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{c+dx^n}\right)}{d^2en} \\
&\quad + \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{c+dx^n}\right)}{d^2en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.41

$$
\int (ex)^{-1+2n}\left(a+b\text{csch}(c+dx^n)\right)\,dx
= \frac{x^{-2n}(ex)^{2n}\left(ad^2x^{2n}+2bc\log\left(1-e^{-c-dx^n}\right)+2bdx^n\log\left(1-e^{-c-dx^n}\right)-2bc\log\left(1+e^{-c-dx^n}\right)-2bdx^n\log\right.}{2d^2en}
$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Csch[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*(a*d^2*x^(2*n) + 2*b*c*Log[1 - E^(-c - d*x^n)] + 2*b*d*x^n*Log
[1 - E^(-c - d*x^n)] - 2*b*c*Log[1 + E^(-c - d*x^n)] - 2*b*d*x^n*Log[1 + E^
(-c - d*x^n)] - 2*b*c*Log[Tanh[(c + d*x^n)/2]] + 2*b*PolyLog[2, -E^(-c - d*
x^n)] - 2*b*PolyLog[2, E^(-c - d*x^n)]))/(2*d^2*e*n*x^(2*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.63

method	result
risch	$\frac{a x \,\mathrm{e}^{\frac{(2n-1)\left(-i \,\mathrm{csgn}(ie) \,\mathrm{csgn}(ix) \,\mathrm{csgn}(iex)\pi + i \,\mathrm{csgn}(ie) \,\mathrm{csgn}(iex)^{2}\pi + i \,\mathrm{csgn}(ix) \,\mathrm{csgn}(iex)^{2}\pi - i \,\mathrm{csgn}(iex)^{3}\pi + 2\ln(e) + 2\ln(x)\right)}{2}}}{2n} + \frac{2b \,\mathrm{e}^{-i\pi n \,\mathrm{csgn}(ie) \,\mathrm{c}}}{}$

[In] int((e*x)^(2*n-1)*(a+b*csch(c+d*x^n)),x,method=_RETURNVERBOSE)

[Out] 1/2*a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))+2*b*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(1/2*(ln(1-exp(c+d*x^n))-ln(exp(c+d*x^n)+1))*d*x^n*exp(-c)+1/2*(dilog(1-exp(c+d*x^n))-dilog(exp(c+d*x^n)+1))*exp(-c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(121) = 242.

Time = 0.30 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.48

$$\int (ex)^{-1+2n}(a+b\operatorname{csch}(c+dx^n))\,dx = \text{Too large to display}$$

[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n)),x, algorithm="fricas")

[Out] 1/2*(a*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + a*d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + (a*d^2*cosh((2*n - 1)*log(e)) + a*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + 2*(b*cosh((2*n - 1)*log(e)) + b*sinh((2*n - 1)*log(e)))*dilog(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(b*cosh((2*n - 1)*log(e)) + b*sinh((2*n - 1)*log(e)))*dilog(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + b*d*cosh(n*log(x))*sinh((2*n - 1)*log(e)) + (b*d*cosh((2*n - 1)*log(e)) + b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) - 2*(b*c*cosh((2*n - 1)*log(e)) + b*c*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 1) + 2*(b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + b*c*cosh((2*n - 1)*log(e)) + (b*d*cosh(n*log(x)) + b*c)*sinh((2*n -

```
 1)*log(e)) + (b*d*cosh((2*n - 1)*log(e)) + b*d*sinh((2*n - 1)*log(e)))*sin
h(n*log(x)))*log(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - sinh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + 2*(a*d^2*cosh((2*n - 1)*log(e))
*cosh(n*log(x)) + a*d^2*cosh(n*log(x))*sinh((2*n - 1)*log(e)))*sinh(n*log(x
)))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\text{csch}(c+dx^n))\, dx = \int (ex)^{2n-1}\left(a+b\operatorname{csch}\left(c+dx^n\right)\right)\, dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*csch(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*csch(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\text{csch}(c+dx^n))\, dx = \int (b\operatorname{csch}\left(dx^n+c\right)+a)(ex)^{2\,n-1}\, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*integrate((e*x)^(2*n - 1)/(e^(d*x^n + c) - e^(-d*x^n - c)), x) + 1/2*(e
*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}(a+b\text{csch}(c+dx^n))\, dx = \int (b\operatorname{csch}\left(dx^n+c\right)+a)(ex)^{2\,n-1}\, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^n + c) + a)*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\text{csch}(c+dx^n))\,dx=\int \left(a+\frac{b}{\sinh(c+d\,x^n)}\right)(e\,x)^{2\,n-1}\,dx$$

```
[In] int((a + b/sinh(c + d*x^n))*(e*x)^(2*n - 1),x)
[Out] int((a + b/sinh(c + d*x^n))*(e*x)^(2*n - 1), x)
```

3.74 $\int (ex)^{-1+3n}(a+b\operatorname{csch}(c+dx^n))\,dx$

Optimal result . . . 489
Rubi [A] (verified) . . . 489
Mathematica [F] . . . 492
Maple [F] . . . 493
Fricas [B] (verification not implemented) . . . 493
Sympy [F] . . . 494
Maxima [F] . . . 494
Giac [F] . . . 494
Mupad [F(-1)] . . . 494

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int (ex)^{-1+3n}(a+b\operatorname{csch}(c+dx^n))\,dx = \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-e^{c+dx^n}\right)}{d^3en} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,e^{c+dx^n}\right)}{d^3en}$$

```
[Out] 1/3*a*(e*x)^(3*n)/e/n-2*b*(e*x)^(3*n)*arctanh(exp(c+d*x^n))/d/e/n/(x^n)-2*b
*(e*x)^(3*n)*polylog(2,-exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*b*(e*x)^(3*n)*pol
ylog(2,exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*b*(e*x)^(3*n)*polylog(3,-exp(c+d*x
^n))/d^3/e/n/(x^(3*n))-2*b*(e*x)^(3*n)*polylog(3,exp(c+d*x^n))/d^3/e/n/(x^(
3*n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{14, 5549, 5545, 4267, 2611, 2320, 6724\}$

$$\int (ex)^{-1+3n}\left(a+b\,\mathrm{csch}(c+dx^n)\right)\,dx = \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\mathrm{arctanh}\left(e^{c+dx^n}\right)}{den} + \frac{2bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-e^{dx^n+c}\right)}{d^3en} - \frac{2bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,e^{dx^n+c}\right)}{d^3en} - \frac{2bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-e^{dx^n+c}\right)}{d^2en} + \frac{2bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,e^{dx^n+c}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(3*n))/(3*e*n) - (2*b*(e*x)^(3*n)*ArcTanh[E^(c + d*x^n)])/(d*e*n*x
^n) - (2*b*(e*x)^(3*n)*PolyLog[2, -E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (2*b
*(e*x)^(3*n)*PolyLog[2, E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (2*b*(e*x)^(3*n
)*PolyLog[3, -E^(c + d*x^n)])/(d^3*e*n*x^(3*n)) - (2*b*(e*x)^(3*n)*PolyLog[
3, E^(c + d*x^n)])/(d^3*e*n*x^(3*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+3n} + b(ex)^{-1+3n}\operatorname{csch}(c+dx^n)\right)\,dx \\
&= \frac{a(ex)^{3n}}{3en} + b\int (ex)^{-1+3n}\operatorname{csch}(c+dx^n)\,dx \\
&= \frac{a(ex)^{3n}}{3en} + \frac{\left(bx^{-3n}(ex)^{3n}\right)\int x^{-1+3n}\operatorname{csch}(c+dx^n)\,dx}{e} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{\left(bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2\operatorname{csch}(c+dx)\,dx,x,x^n\right)}{en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1-e^{c+dx}\right)\,dx,x,x^n\right)}{den} \\
&\quad + \frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+e^{c+dx}\right)\,dx,x,x^n\right)}{den}
\end{aligned}$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,x^n\right)}{d^2en} - \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,x^n\right)}{d^2en}$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^3en} - \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^3en}$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-e^{c+dx^n}\right)}{d^3en} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,e^{c+dx^n}\right)}{d^3en}$$

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b\operatorname{csch}(c+dx^n)\right)\,dx = \int (ex)^{-1+3n}\left(a+b\operatorname{csch}(c+dx^n)\right)\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n]), x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a + b\ \mathrm{csch}\left(c + d\,x^{n}\right)\right) dx$$

```
[In] int((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n)),x)
[Out] int((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(196) = 392$.

Time = 0.28 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.83

$$\int (ex)^{-1+3n} \left(a + b\mathrm{csch}(c + dx^n)\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n)),x, algorithm="fricas")
[Out] 1/3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + a*d^3*cosh(n*log(x))^3
*sinh((3*n - 1)*log(e)) + (a*d^3*cosh((3*n - 1)*log(e)) + a*d^3*sinh((3*n -
 1)*log(e)))*sinh(n*log(x))^3 + 3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(
x)) + a*d^3*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + 6*(b*
d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + b*d*cosh(n*log(x))*sinh((3*n - 1)
*log(e)) + (b*d*cosh((3*n - 1)*log(e)) + b*d*sinh((3*n - 1)*log(e)))*sinh(n
*log(x)))*dilog(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c)) - 6*(b*d*cosh((3*n - 1)*log(e))*cosh(n*
log(x)) + b*d*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (b*d*cosh((3*n - 1)*l
og(e)) + b*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*dilog(-cosh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c) - sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
 c)) - 3*(b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + b*d^2*cosh(n*log(
x))^2*sinh((3*n - 1)*log(e)) + (b*d^2*cosh((3*n - 1)*log(e)) + b*d^2*sinh((
3*n - 1)*log(e)))*sinh(n*log(x))^2 + 2*(b*d^2*cosh((3*n - 1)*log(e))*cosh(n
*log(x)) + b*d^2*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*log
(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c) + 1) + 3*(b*c^2*cosh((3*n - 1)*log(e)) + b*c^2*sinh((3*
n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*
cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 1) + 3*(b*d^2*cosh((3*n - 1)*log(e
))*cosh(n*log(x))^2 - b*c^2*cosh((3*n - 1)*log(e)) + (b*d^2*cosh((3*n - 1)*
log(e)) + b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (b*d^2*cosh(n*lo
g(x))^2 - b*c^2)*sinh((3*n - 1)*log(e)) + 2*(b*d^2*cosh((3*n - 1)*log(e))*c
osh(n*log(x)) + b*d^2*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))
)*log(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - sinh(d*cosh(n*log(x)
) + d*sinh(n*log(x)) + c) + 1) - 6*(b*cosh((3*n - 1)*log(e)) + b*sinh((3*n
- 1)*log(e)))*polylog(3, cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + si
nh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + 6*(b*cosh((3*n - 1)*log(e))
```

```
+ b*sinh((3*n - 1)*log(e)))*polylog(3, -cosh(d*cosh(n*log(x)) + d*sinh(n*lo
g(x)) + c) - sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + 3*(a*d^3*cosh
((3*n - 1)*log(e))*cosh(n*log(x))^2 + a*d^3*cosh(n*log(x))^2*sinh((3*n - 1)
*log(e)))*sinh(n*log(x)))/(d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))\,dx=\int (ex)^{3n-1}\left(a+b\,\text{csch}\left(c+dx^n\right)\right)\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*csch(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*csch(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))\,dx=\int (b\,\text{csch}\left(dx^n+c\right)+a)(ex)^{3\,n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*integrate((e*x)^(3*n - 1)/(e^(d*x^n + c) - e^(-d*x^n - c)), x) + 1/3*(e
*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))\,dx=\int (b\,\text{csch}\left(dx^n+c\right)+a)(ex)^{3\,n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*csch(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))\,dx=\int \left(a+\frac{b}{\sinh\left(c+d\,x^n\right)}\right)\,{\left(e\,x\right)}^{3\,n-1}\,dx$$

```
[In] int((a + b/sinh(c + d*x^n))*(e*x)^(3*n - 1),x)
[Out] int((a + b/sinh(c + d*x^n))*(e*x)^(3*n - 1), x)
```

3.75 $\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))^2 \, dx$

Optimal result 495
Rubi [A] (verified) 495
Mathematica [A] (verified) 497
Maple [C] (warning: unable to verify) 497
Fricas [B] (verification not implemented) 497
Sympy [F] 498
Maxima [A] (verification not implemented) 499
Giac [F] 499
Mupad [B] (verification not implemented) 499

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))^2 \, dx = \frac{a^2 (ex)^n}{en} - \frac{2abx^{-n}(ex)^n \text{arctanh}(\cosh(c + dx^n))}{den} - \frac{b^2 x^{-n}(ex)^n \coth(c + dx^n)}{den}$$

```
[Out] a^2*(e*x)^n/e/n-2*a*b*(e*x)^n*arctanh(cosh(c+d*x^n))/d/e/n/(x^n)-b^2*(e*x)^
n*coth(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5549, 5545, 3858, 3855, 3852, 8}

$$\int (ex)^{-1+n} (a + b\text{csch}(c + dx^n))^2 \, dx = \frac{a^2 (ex)^n}{en} - \frac{2abx^{-n}(ex)^n \text{arctanh}(\cosh(c + dx^n))}{den} - \frac{b^2 x^{-n}(ex)^n \coth(c + dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^n)/(e*n) - (2*a*b*(e*x)^n*ArcTanh[Cosh[c + d*x^n]])/(d*e*n*x^n)
- (b^2*(e*x)^n*Coth[c + d*x^n])/(d*e*n*x^n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\operatorname{csch}(c+dx^n))^2\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int (a+b\operatorname{csch}(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}+\frac{(2abx^{-n}(ex)^n)\operatorname{Subst}\left(\int \operatorname{csch}(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{(b^2x^{-n}(ex)^n)\operatorname{Subst}\left(\int \operatorname{csch}^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}-\frac{2abx^{-n}(ex)^n\operatorname{arctanh}(\cosh(c+dx^n))}{den}\\
&\quad-\frac{(ib^2x^{-n}(ex)^n)\operatorname{Subst}\left(\int 1\,dx,x,-i\coth(c+dx^n)\right)}{den}\\
&= \frac{a^2(ex)^n}{en}-\frac{2abx^{-n}(ex)^n\operatorname{arctanh}(\cosh(c+dx^n))}{den}-\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{den}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int (ex)^{-1+n} \left(a + b\operatorname{csch}(c + dx^n)\right)^2 \, dx$$
$$= \frac{x^{-n}(ex)^n \left(-b^2 \coth\left(\frac{1}{2}(c + dx^n)\right) + 2a\left(ac + adx^n - 2b\log\left(\cosh\left(\frac{1}{2}(c + dx^n)\right)\right) + 2b\log\left(\sinh\left(\frac{1}{2}(c + dx^n)\right.\right.\right.}{2den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^n*(-(b^2*Coth[(c + d*x^n)/2]) + 2*a*(a*c + a*d*x^n - 2*b*Log[Cosh[(c
 + d*x^n)/2]] + 2*b*Log[Sinh[(c + d*x^n)/2]]) - b^2*Tanh[(c + d*x^n)/2]))/(
2*d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.90 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.39

method	result
risch	$\frac{a^2 x\, e^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi + i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi + i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi - i\,\mathrm{csgn}(iex)^3\pi + 2\ln(e) + 2\ln(x)\right)}{2}}}{n} - 2x\,x^{-n}e^{(-1+\cdots}$

```
[In] int((e*x)^(-1+n)*(a+b*csch(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2/d/n*x/(x^n)*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+
I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*P
i+2*ln(e)+2*ln(x)))*b^2/(exp(2*c+2*d*x^n)-1)-4*arctanh(exp(c+d*x^n))/d/e*e^
n/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*
x)+csgn(I*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 854, normalized size of antiderivative = 10.68

$$\int (ex)^{-1+n} \left(a + b\operatorname{csch}(c + dx^n)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*cosh((n - 1)*log(e))*cosh(n*log(x)) - (a^2*d*cosh((n - 1)*log(e))*c
osh(n*log(x)) + a^2*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + (a^2*d*cosh((n
- 1)*log(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c)^2 + 2*b^2*cosh((n - 1)*log(e)) - 2*(a^2*d*cos
h((n - 1)*log(e))*cosh(n*log(x)) + a^2*d*cosh(n*log(x))*sinh((n - 1)*log(e)
) + (a^2*d*cosh((n - 1)*log(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x)
))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c) - (a^2*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + a^2*d*co
sh(n*log(x))*sinh((n - 1)*log(e)) + (a^2*d*cosh((n - 1)*log(e)) + a^2*d*sin
h((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
 + c)^2 + 2*((a*b*cosh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - a*b*cosh((n - 1)*log(e)) + 2*(a*b
*cosh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*c
osh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*s
inh(n*log(x)) + c)^2 - a*b*sinh((n - 1)*log(e)))*log(cosh(d*cosh(n*log(x))
+ d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1
) - 2*((a*b*cosh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*cosh(n*
log(x)) + d*sinh(n*log(x)) + c)^2 - a*b*cosh((n - 1)*log(e)) + 2*(a*b*cosh(
(n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*cosh((n
 - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c)^2 - a*b*sinh((n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*si
nh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 1) + (a
^2*d*cosh(n*log(x)) + 2*b^2)*sinh((n - 1)*log(e)) + (a^2*d*cosh((n - 1)*log
(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/(d*n*cosh(d*cosh(n*log(x
)) + d*sinh(n*log(x)) + c)^2 + 2*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x
)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + d*n*sinh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c)^2 - d*n)
```

Sympy [F]

$$\int (ex)^{-1+n}\,(a+b\text{csch}(c+dx^n))^2\,dx=\int (ex)^{n-1}\,(a+b\,\text{csch}\,(c+dx^n))^2\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*csch(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*csch(c + d*x**n))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int (ex)^{-1+n}\,(a+b\mathrm{csch}(c+dx^n))^2\,dx$$
$$= -2\,ab\left(\frac{e^{n-1}\log\left(\left(e^{(dx^n+c)}+1\right)e^{(-c)}\right)}{dn} - \frac{e^{n-1}\log\left(\left(e^{(dx^n+c)}-1\right)e^{(-c)}\right)}{dn}\right) - \frac{2\,b^2e^n}{dene^{(2\,dx^n+2\,c)}-den} + \frac{(ex)^n\,a^2}{en}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(e^(n - 1)*log((e^(d*x^n + c) + 1)*e^(-c))/(d*n) - e^(n - 1)*log((e^
(d*x^n + c) - 1)*e^(-c))/(d*n)) - 2*b^2*e^n/(d*e*n*e^(2*d*x^n + 2*c) - d*e*
n) + (e*x)^n*a^2/(e*n)
```

Giac [F]

$$\int (ex)^{-1+n}\,(a+b\mathrm{csch}(c+dx^n))^2\,dx = \int (b\,\mathrm{csch}\,(dx^n+c)+a)^2(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csch(c+d*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.00

$$\int (ex)^{-1+n}\,(a+b\mathrm{csch}(c+dx^n))^2\,dx$$
$$= \frac{a^2\,x\,(e\,x)^{n-1}}{n} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,x\,\mathrm{e}^{d\,x^n}\,\mathrm{e}^c\,(e\,x)^{n-1}\,\sqrt{-d^2\,n^2\,x^{2\,n}}}{d\,n\,x^n\,\sqrt{a^2\,b^2\,x^2\,(e\,x)^{2\,n-2}}}\right)\sqrt{a^2\,b^2\,x^2\,(e\,x)^{2\,n-2}}}{\sqrt{-d^2\,n^2\,x^{2\,n}}} - \frac{2\,b^2\,x\,(e\,x)^{n-1}}{d\,n\,x^n\,\left(\mathrm{e}^{2\,c+2\,d\,x^n}-1\right)}$$

```
[In] int((a + b/sinh(c + d*x^n))^2*(e*x)^(n - 1),x)
```

```
[Out] (a^2*x*(e*x)^(n - 1))/n - (4*atan((a*b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(-
d^2*n^2*x^(2*n))^(1/2))/(d*n*x^n*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(a^2
*b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(-d^2*n^2*x^(2*n))^(1/2) - (2*b^2*x*(e*x)^
(n - 1))/(d*n*x^n*(exp(2*c + 2*d*x^n) - 1))
```

3.76 $\int (ex)^{-1+2n} (a + b\text{csch}(c + dx^n))^2 \, dx$

Optimal result 500
Rubi [A] (verified) 500
Mathematica [B] (verified) 503
Maple [F] 504
Fricas [B] (verification not implemented) 504
Sympy [F] 506
Maxima [F] 506
Giac [F] 506
Mupad [F(-1)] 507

Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int (ex)^{-1+2n} (a + b\text{csch}(c + dx^n))^2 \, dx &= \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} \\ &- \frac{b^2x^{-n}(ex)^{2n}\coth(c+dx^n)}{den} \\ &+ \frac{b^2x^{-2n}(ex)^{2n}\log(\sinh(c+dx^n))}{d^2en} \\ &- \frac{2abx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} \\ &+ \frac{2abx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en}\end{aligned}$$

```
[Out] 1/2*a^2*(e*x)^(2*n)/e/n-4*a*b*(e*x)^(2*n)*arctanh(exp(c+d*x^n))/d/e/n/(x^n)
-b^2*(e*x)^(2*n)*coth(c+d*x^n)/d/e/n/(x^n)+b^2*(e*x)^(2*n)*ln(sinh(c+d*x^n)
)/d^2/e/n/(x^(2*n))-2*a*b*(e*x)^(2*n)*polylog(2,-exp(c+d*x^n))/d^2/e/n/(x^(
2*n))+2*a*b*(e*x)^(2*n)*polylog(2,exp(c+d*x^n))/d^2/e/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{5549, 5545, 4275, 4267, 2317, 2438, 4269, 3556\}$

$$\int (ex)^{-1+2n}\left(a+b\operatorname{csch}(c+dx^n)\right)^2\,dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{2abx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-e^{dx^n+c}\right)}{d^2en} + \frac{2abx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,e^{dx^n+c}\right)}{d^2en} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(\sinh\left(c+dx^n\right)\right)}{d^2en} - \frac{b^2x^{-n}(ex)^{2n}\coth\left(c+dx^n\right)}{den}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(2*n))/(2*e*n) - (4*a*b*(e*x)^(2*n)*ArcTanh[E^(c + d*x^n)])/(d*e
*n*x^n) - (b^2*(e*x)^(2*n)*Coth[c + d*x^n])/(d*e*n*x^n) + (b^2*(e*x)^(2*n)*
Log[Sinh[c + d*x^n]])/(d^2*e*n*x^(2*n)) - (2*a*b*(e*x)^(2*n)*PolyLog[2, -E^
(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (2*a*b*(e*x)^(2*n)*PolyLog[2, E^(c + d*x^
n)])/(d^2*e*n*x^(2*n))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a+b\text{csch}(c+dx^n))^2\,dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int x(a+b\text{csch}(c+dx))^2\,dx,x,x^n\right)}{en} \\
&= \frac{(x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \left(a^2x+2abx\text{csch}(c+dx)+b^2x\text{csch}^2(c+dx)\right)\,dx,x,x^n\right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en}+\frac{(2abx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int x\text{csch}(c+dx)\,dx,x,x^n\right)}{en} \\
&\quad+\frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int x\text{csch}^2(c+dx)\,dx,x,x^n\right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en}-\frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den}-\frac{b^2x^{-n}(ex)^{2n}\coth\left(c+dx^n\right)}{den} \\
&\quad-\frac{(2abx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1-e^{c+dx}\right)\,dx,x,x^n\right)}{den} \\
&\quad+\frac{(2abx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1+e^{c+dx}\right)\,dx,x,x^n\right)}{den} \\
&\quad+\frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \coth(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{2n}\coth\left(c+dx^n\right)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(\sinh\left(c+dx^n\right)\right)}{d^2en} - \frac{(2abx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^2en} + \frac{(2abx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^2en}$$

$$= \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{2n}\coth\left(c+dx^n\right)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(\sinh\left(c+dx^n\right)\right)}{d^2en} - \frac{2abx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{2abx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. 2(198) = 396.

Time = 2.75 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.46

$$\int (ex)^{-1+2n}\left(a+b\text{csch}(c+dx^n)\right)^2\,dx$$
$$= \frac{x^{-2n}(ex)^{2n}\left(-4b^2dx^n - a^2d^2x^{2n} + a^2d^2e^{2c}x^{2n} - 2b^2\log\left(1-e^{-c-dx^n}\right) + 2b^2e^{2c}\log\left(1-e^{-c-dx^n}\right) - 4abdx^n\right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*(-4*b^2*d*x^n - a^2*d^2*x^(2*n) + a^2*d^2*E^(2*c)*x^(2*n) - 2*
b^2*Log[1 - E^(-c - d*x^n)] + 2*b^2*E^(2*c)*Log[1 - E^(-c - d*x^n)] - 4*a*b
*d*x^n*Log[1 - E^(-c - d*x^n)] + 4*a*b*d*E^(2*c)*x^n*Log[1 - E^(-c - d*x^n)
] - 2*b^2*Log[1 + E^(-c - d*x^n)] + 2*b^2*E^(2*c)*Log[1 + E^(-c - d*x^n)] +
 4*a*b*d*x^n*Log[1 + E^(-c - d*x^n)] - 4*a*b*d*E^(2*c)*x^n*Log[1 + E^(-c -
d*x^n)] + 4*a*b*(-1 + E^(2*c))*PolyLog[2, -E^(-c - d*x^n)] - 4*a*b*(-1 + E^
(2*c))*PolyLog[2, E^(-c - d*x^n)] - b^2*d*x^n*Csch[c/2]*Csch[(c + d*x^n)/2]
*Sinh[(d*x^n)/2] + b^2*d*E^(2*c)*x^n*Csch[c/2]*Csch[(c + d*x^n)/2]*Sinh[(d*
x^n)/2] + b^2*d*x^n*Sech[c/2]*Sech[(c + d*x^n)/2]*Sinh[(d*x^n)/2] - 4*b^2*d
*E^(2*c)*x^n*Csch[c]*Csch[c + d*x^n]*Sinh[c/2]*Sinh[(d*x^n)/2]*Sinh[(c + d*
x^n)/2]))/(2*d^2*e*(-1 + E^(2*c))*n*x^(2*n))
```

Maple [F]

$$\int (ex)^{2n-1} (a + b\,\mathrm{csch}\,(c + d\,x^n))^2 dx$$

```
[In] int((e*x)^(2*n-1)*(a+b*csch(c+d*x^n))^2,x)
```

```
[Out] int((e*x)^(2*n-1)*(a+b*csch(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2678 vs. $2(197) = 394$.

Time = 0.32 (sec) , antiderivative size = 2678, normalized size of antiderivative = 13.53

$$\int (ex)^{-1+2n} (a + b\mathrm{csch}(c + dx^n))^2 \, dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 - 4*b^2*c*cosh((2*n -
 1)*log(e)) - (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 - 4*b^2*d*co
sh((2*n - 1)*log(e))*cosh(n*log(x)) - 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2
*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x)
)^2 + (a^2*d^2*cosh(n*log(x))^2 - 4*b^2*d*cosh(n*log(x)) - 4*b^2*c)*sinh((2
*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) - 2*b^2*
d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) - 2*b^2*d)*sinh((2*n - 1
)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2
- 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 - 4*b^2*d*cosh((2*n -
1)*log(e))*cosh(n*log(x)) - 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(
(2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2
*d^2*cosh(n*log(x))^2 - 4*b^2*d*cosh(n*log(x)) - 4*b^2*c)*sinh((2*n - 1)*lo
g(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) - 2*b^2*d*cosh((2*
n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) - 2*b^2*d)*sinh((2*n - 1)*log(e)))
*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) - (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*
log(x))^2 - 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) - 4*b^2*c*cosh((2
*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*
log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 - 4*b^2*d*cosh(n*log(
x)) - 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*c
osh(n*log(x)) - 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) -
2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c)^2 + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n
 - 1)*log(e)))*sinh(n*log(x))^2 - 4*((a*b*cosh((2*n - 1)*log(e)) + a*b*sinh
((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - a*b*c
osh((2*n - 1)*log(e)) + 2*(a*b*cosh((2*n - 1)*log(e)) + a*b*sinh((2*n - 1)*
log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)
```

```
) + d*sinh(n*log(x)) + c) + (a*b*cosh((2*n - 1)*log(e)) + a*b*sinh((2*n - 1
)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - a*b*sinh((2*n
- 1)*log(e)))*dilog(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*
cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + 4*((a*b*cosh((2*n - 1)*log(e)) +
a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2
 - a*b*cosh((2*n - 1)*log(e)) + 2*(a*b*cosh((2*n - 1)*log(e)) + a*b*sinh((2
*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*cosh((2*n - 1)*log(e)) + a*b*sinh(
(2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - a*b*si
nh((2*n - 1)*log(e)))*dilog(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
- sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(2*a*b*d*cosh((2*n - 1
)*log(e))*cosh(n*log(x)) - (2*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) -
 b^2*cosh((2*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) - b^2)*sinh((2*n - 1)
*log(e)) + 2*(a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2*n - 1)*log(e)))*
sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - b^2*cosh(
(2*n - 1)*log(e)) - 2*(2*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) - b^2*
cosh((2*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) - b^2)*sinh((2*n - 1)*log(
e)) + 2*(a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2*n - 1)*log(e)))*sinh(
n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c) - (2*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)
) - b^2*cosh((2*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) - b^2)*sinh((2*n -
 1)*log(e)) + 2*(a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2*n - 1)*log(e)
))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2*a*b
*d*cosh(n*log(x)) - b^2)*sinh((2*n - 1)*log(e)) + 2*(a*b*d*cosh((2*n - 1)*l
og(e)) + a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(cosh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
c) + 1) + 2*(((2*a*b*c - b^2)*cosh((2*n - 1)*log(e)) + (2*a*b*c - b^2)*sinh
((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*((2
*a*b*c - b^2)*cosh((2*n - 1)*log(e)) + (2*a*b*c - b^2)*sinh((2*n - 1)*log(e
)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c) + ((2*a*b*c - b^2)*cosh((2*n - 1)*log(e)) + (2*a*b*c -
 b^2)*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
^2 - (2*a*b*c - b^2)*cosh((2*n - 1)*log(e)) - (2*a*b*c - b^2)*sinh((2*n - 1
)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) - 1) + 4*(a*b*d*cosh((2*n - 1)*log(e))*co
sh(n*log(x)) + a*b*c*cosh((2*n - 1)*log(e)) - (a*b*d*cosh((2*n - 1)*log(e))
*cosh(n*log(x)) + a*b*c*cosh((2*n - 1)*log(e)) + (a*b*d*cosh(n*log(x)) + a*
b*c)*sinh((2*n - 1)*log(e)) + (a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2
*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
 c)^2 - 2*(a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + a*b*c*cosh((2*n -
1)*log(e)) + (a*b*d*cosh(n*log(x)) + a*b*c)*sinh((2*n - 1)*log(e)) + (a*b*d
*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*cos
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) - (a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + a*b*c*cosh((
2*n - 1)*log(e)) + (a*b*d*cosh(n*log(x)) + a*b*c)*sinh((2*n - 1)*log(e)) +
```

```
(a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)
))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (a*b*d*cosh(n*log(x))
+ a*b*c)*sinh((2*n - 1)*log(e)) + (a*b*d*cosh((2*n - 1)*log(e)) + a*b*d*sin
h((2*n - 1)*log(e)))*sinh(n*log(x)))*log(-cosh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c) - sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + (a^2*d^
2*cosh(n*log(x))^2 - 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n
 - 1)*log(e))*cosh(n*log(x)) + a^2*d^2*cosh(n*log(x))*sinh((2*n - 1)*log(e)
))*sinh(n*log(x)))/(d^2*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 +
 2*d^2*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)
) + d*sinh(n*log(x)) + c) + d^2*n*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c)^2 - d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}\left(a+b\text{csch}(c+dx^n)\right)^2\,dx=\int (ex)^{2n-1}\left(a+b\operatorname{csch}\left(c+dx^n\right)\right)^2\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*csch(c+d*x**n))**2,x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*csch(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+b\text{csch}(c+dx^n)\right)^2\,dx=\int \left(b\operatorname{csch}\left(dx^n+c\right)+a\right)^2(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")
[Out] 4*(e^(2*n)*integrate(1/2*x^(2*n)/(e*x*e^(d*x^n + c) + e*x), x) + e^(2*n)*in
tegrate(1/2*x^(2*n)/(e*x*e^(d*x^n + c) - e*x), x))*a*b - b^2*(2*e^(2*n)*e^(
2*d*x^n + n*log(x) + 2*c)/(d*e*n*e^(2*d*x^n + 2*c) - d*e*n) - e^(2*n - 1)*l
og((e^(d*x^n + c) + 1)*e^(-c))/(d^2*n) - e^(2*n - 1)*log((e^(d*x^n + c) - 1
)*e^(-c))/(d^2*n)) + 1/2*(e*x)^(2*n)*a^2/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+b\text{csch}(c+dx^n)\right)^2\,dx=\int \left(b\operatorname{csch}\left(dx^n+c\right)+a\right)^2(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\operatorname{csch}(c+dx^n))^2\,dx=\int \left(a+\frac{b}{\sinh\left(c+d\,x^n\right)}\right)^2\,{\left(e\,x\right)}^{2\,n-1}\,dx$$

```
[In] int((a + b/sinh(c + d*x^n))^2*(e*x)^(2*n - 1),x)
[Out] int((a + b/sinh(c + d*x^n))^2*(e*x)^(2*n - 1), x)
```

3.77 $\int (ex)^{-1+3n} (a+b\text{csch}(c+dx^n))^2\,dx$

Optimal result . . . 508
Rubi [A] (verified) . . . 509
Mathematica [F] . . . 513
Maple [F] . . . 513
Fricas [B] (verification not implemented) . . . 513
Sympy [F] . . . 517
Maxima [F] . . . 517
Giac [F] . . . 517
Mupad [F(-1)] . . . 518

Optimal result

Integrand size = 24, antiderivative size = 344

$$\begin{aligned}\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))^2\,dx &= \frac{a^2(ex)^{3n}}{3en}-\frac{b^2x^{-n}(ex)^{3n}}{den} \\ &-\frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} \\ &-\frac{b^2x^{-n}(ex)^{3n}\coth(c+dx^n)}{den} \\ &+\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2(c+dx^n)}\right)}{d^2en} \\ &-\frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} \\ &+\frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} \\ &+\frac{b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{2(c+dx^n)}\right)}{d^3en} \\ &+\frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{c+dx^n}\right)}{d^3en} \\ &-\frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{c+dx^n}\right)}{d^3en}\end{aligned}$$

```
[Out] 1/3*a^2*(e*x)^(3*n)/e/n-b^2*(e*x)^(3*n)/d/e/n/(x^n)-4*a*b*(e*x)^(3*n)*arcta
nh(exp(c+d*x^n))/d/e/n/(x^n)-b^2*(e*x)^(3*n)*coth(c+d*x^n)/d/e/n/(x^n)+2*b^
2*(e*x)^(3*n)*ln(1-exp(2*c+2*d*x^n))/d^2/e/n/(x^(2*n))-4*a*b*(e*x)^(3*n)*po
lylog(2,-exp(c+d*x^n))/d^2/e/n/(x^(2*n))+4*a*b*(e*x)^(3*n)*polylog(2,exp(c+
d*x^n))/d^2/e/n/(x^(2*n))+b^2*(e*x)^(3*n)*polylog(2,exp(2*c+2*d*x^n))/d^3/e
/n/(x^(3*n))+4*a*b*(e*x)^(3*n)*polylog(3,-exp(c+d*x^n))/d^3/e/n/(x^(3*n))-4
*a*b*(e*x)^(3*n)*polylog(3,exp(c+d*x^n))/d^3/e/n/(x^(3*n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5549, 5545, 4275, 4267, 2611, 2320, 6724, 4269, 3797, 2221, 2317, 2438}

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))^2\,dx = \frac{a^2(ex)^{3n}}{3en} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} + \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{dx^n+c}\right)}{d^3en} - \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{dx^n+c}\right)}{d^3en} - \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{dx^n+c}\right)}{d^2en} + \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{dx^n+c}\right)}{d^2en} + \frac{b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{2(dx^n+c)}\right)}{d^3en} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2(c+dx^n)}\right)}{d^2en} - \frac{b^2x^{-n}(ex)^{3n}\coth(c+dx^n)}{den} - \frac{b^2x^{-n}(ex)^{3n}}{den}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(3*n))/(3*e*n) - (b^2*(e*x)^(3*n))/(d*e*n*x^n) - (4*a*b*(e*x)^(3
*n)*ArcTanh[E^(c + d*x^n)])/(d*e*n*x^n) - (b^2*(e*x)^(3*n)*Coth[c + d*x^n])
/(d*e*n*x^n) + (2*b^2*(e*x)^(3*n)*Log[1 - E^(2*(c + d*x^n))])/(d^2*e*n*x^(2
*n)) - (4*a*b*(e*x)^(3*n)*PolyLog[2, -E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (
4*a*b*(e*x)^(3*n)*PolyLog[2, E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (b^2*(e*x)
^(3*n)*PolyLog[2, E^(2*(c + d*x^n))])/(d^3*e*n*x^(3*n)) + (4*a*b*(e*x)^(3*n
)*PolyLog[3, -E^(c + d*x^n)])/(d^3*e*n*x^(3*n)) - (4*a*b*(e*x)^(3*n)*PolyLo
g[3, E^(c + d*x^n)])/(d^3*e*n*x^(3*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-3n}(ex)^{3n}) \int x^{-1+3n}(a+b\operatorname{csch}(c+dx^n))^2 \, dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2(a+b\operatorname{csch}(c+dx))^2 \, dx, x, x^n\right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \left(a^2x^2+2abx^2\operatorname{csch}(c+dx)+b^2x^2\operatorname{csch}^2(c+dx)\right) \, dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^{3n}}{3en} + \frac{(2abx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2\operatorname{csch}(c+dx) \, dx, x, x^n\right)}{en} \\
&\quad + \frac{(b^2x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2\operatorname{csch}^2(c+dx) \, dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^{3n}}{3en} - \frac{4abx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\coth\left(c+dx^n\right)}{den} \\
&\quad - \frac{(4abx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x\log\left(1-e^{c+dx}\right) \, dx, x, x^n\right)}{den} \\
&\quad + \frac{(4abx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x\log\left(1+e^{c+dx}\right) \, dx, x, x^n\right)}{den} \\
&\quad + \frac{(2b^2x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x\coth(c+dx) \, dx, x, x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{b^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\coth(c+dx^n)}{den} - \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{c+dx}\right)\,dx,x,x^n\right)}{d^2en} - \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{c+dx}\right)\,dx,x,x^n\right)}{d^2en} - \frac{(4b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x}{1-e^{2(c+dx)}}\,dx,x,x^n\right)}{den}$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{b^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\coth(c+dx^n)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2(c+dx^n)}\right)}{d^2en} - \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^3en} - \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{c+dx^n}\right)}{d^3en} - \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \log\left(1-e^{2(c+dx)}\right)\,dx,x,x^n\right)}{d^2en}$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{b^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\coth(c+dx^n)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2(c+dx^n)}\right)}{d^2en} - \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{4abx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{c+dx^n}\right)}{d^3en} - \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{c+dx^n}\right)}{d^3en} - \frac{(b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2(c+dx^n)}\right)}{d^3en}$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{b^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\coth\left(c+dx^n\right)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2(c+dx^n)}\right)}{d^2en} - \frac{4abx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{c+dx^n}\right)}{d^2en} + \frac{4abx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{c+dx^n}\right)}{d^2en} + \frac{b^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{2(c+dx^n)}\right)}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-e^{c+dx^n}\right)}{d^3en} - \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,e^{c+dx^n}\right)}{d^3en}$$

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b\operatorname{csch}(c+dx^n)\right)^2\,dx = \int (ex)^{-1+3n}\left(a+b\operatorname{csch}(c+dx^n)\right)^2\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csch[c + d*x^n])^2, x]
```

Maple [F]

$$\int (ex)^{-1+3n}\left(a+b\operatorname{csch}\left(c+d\,x^n\right)\right)^2dx$$

```
[In] int((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n))^2,x)
[Out] int((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4967 vs. $2(346) = 692$.

Time = 0.34 (sec) , antiderivative size = 4967, normalized size of antiderivative = 14.44

$$\int (ex)^{-1+3n}\left(a+b\operatorname{csch}(c+dx^n)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")
[Out] -1/3*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + 6*b^2*c^2*cosh((3*n
 - 1)*log(e)) + (a^2*d^3*cosh((3*n - 1)*log(e)) + a^2*d^3*sinh((3*n - 1)*lo
g(e)))*sinh(n*log(x))^3 - (a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3
- 6*b^2*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + 6*b^2*c^2*cosh((3*n -
 1)*log(e)) + (a^2*d^3*cosh((3*n - 1)*log(e)) + a^2*d^3*sinh((3*n - 1)*log(
```

```
e)))*sinh(n*log(x))^3 + 3*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) -
2*b^2*d^2*cosh((3*n - 1)*log(e)) + (a^2*d^3*cosh(n*log(x)) - 2*b^2*d^2)*sin
h((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^3*cosh(n*log(x))^3 - 6*b^2*d
^2*cosh(n*log(x))^2 + 6*b^2*c^2)*sinh((3*n - 1)*log(e)) + 3*(a^2*d^3*cosh((
3*n - 1)*log(e))*cosh(n*log(x))^2 - 4*b^2*d^2*cosh((3*n - 1)*log(e))*cosh(n
*log(x)) + (a^2*d^3*cosh(n*log(x))^2 - 4*b^2*d^2*cosh(n*log(x)))*sinh((3*n
- 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
^2 - 2*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 - 6*b^2*d^2*cosh((3
*n - 1)*log(e))*cosh(n*log(x))^2 + 6*b^2*c^2*cosh((3*n - 1)*log(e)) + (a^2*
d^3*cosh((3*n - 1)*log(e)) + a^2*d^3*sinh((3*n - 1)*log(e)))*sinh(n*log(x))
^3 + 3*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - 2*b^2*d^2*cosh((3*n
 - 1)*log(e)) + (a^2*d^3*cosh(n*log(x)) - 2*b^2*d^2)*sinh((3*n - 1)*log(e))
)*sinh(n*log(x))^2 + (a^2*d^3*cosh(n*log(x))^3 - 6*b^2*d^2*cosh(n*log(x))^2
 + 6*b^2*c^2)*sinh((3*n - 1)*log(e)) + 3*(a^2*d^3*cosh((3*n - 1)*log(e))*co
sh(n*log(x))^2 - 4*b^2*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a^2*d^3
*cosh(n*log(x))^2 - 4*b^2*d^2*cosh(n*log(x)))*sinh((3*n - 1)*log(e)))*sinh(
n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c) - (a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)
)^3 - 6*b^2*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + 6*b^2*c^2*cosh((3
*n - 1)*log(e)) + (a^2*d^3*cosh((3*n - 1)*log(e)) + a^2*d^3*sinh((3*n - 1)*
log(e)))*sinh(n*log(x))^3 + 3*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)
) - 2*b^2*d^2*cosh((3*n - 1)*log(e)) + (a^2*d^3*cosh(n*log(x)) - 2*b^2*d^2)
*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^3*cosh(n*log(x))^3 - 6*b
^2*d^2*cosh(n*log(x))^2 + 6*b^2*c^2)*sinh((3*n - 1)*log(e)) + 3*(a^2*d^3*co
sh((3*n - 1)*log(e))*cosh(n*log(x))^2 - 4*b^2*d^2*cosh((3*n - 1)*log(e))*co
sh(n*log(x)) + (a^2*d^3*cosh(n*log(x))^2 - 4*b^2*d^2*cosh(n*log(x)))*sinh((
3*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c)^2 + 3*(a^2*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a^2*d^3*cosh(n*
log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + 6*(2*a*b*d*cosh((3*n - 1
)*log(e))*cosh(n*log(x)) - (2*a*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) +
 b^2*cosh((3*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) + b^2)*sinh((3*n - 1)
*log(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e)) + a*b*d*sinh((3*n - 1)*log(e)))*
sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + b^2*cosh(
(3*n - 1)*log(e)) - 2*(2*a*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + b^2*
cosh((3*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) + b^2)*sinh((3*n - 1)*log(
e)) + 2*(a*b*d*cosh((3*n - 1)*log(e)) + a*b*d*sinh((3*n - 1)*log(e)))*sinh(
n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c) - (2*a*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)
) + b^2*cosh((3*n - 1)*log(e)) + (2*a*b*d*cosh(n*log(x)) + b^2)*sinh((3*n -
 1)*log(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e)) + a*b*d*sinh((3*n - 1)*log(e)
))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2*a*b
*d*cosh(n*log(x)) + b^2)*sinh((3*n - 1)*log(e)) + 2*(a*b*d*cosh((3*n - 1)*l
og(e)) + a*b*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*dilog(cosh(d*cosh(n*
log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c)) - 6*(2*a*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - (2*a*b*d*cosh((3
```

```
*n - 1)*log(e))*cosh(n*log(x)) - b^2*cosh((3*n - 1)*log(e)) + (2*a*b*d*cosh
(n*log(x)) - b^2)*sinh((3*n - 1)*log(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e))
+ a*b*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*s
inh(n*log(x)) + c)^2 - b^2*cosh((3*n - 1)*log(e)) - 2*(2*a*b*d*cosh((3*n -
1)*log(e))*cosh(n*log(x)) - b^2*cosh((3*n - 1)*log(e)) + (2*a*b*d*cosh(n*lo
g(x)) - b^2)*sinh((3*n - 1)*log(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e)) + a*b
*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (2*a*b*d*cosh
((3*n - 1)*log(e))*cosh(n*log(x)) - b^2*cosh((3*n - 1)*log(e)) + (2*a*b*d*c
osh(n*log(x)) - b^2)*sinh((3*n - 1)*log(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e
)) + a*b*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) +
d*sinh(n*log(x)) + c)^2 + (2*a*b*d*cosh(n*log(x)) - b^2)*sinh((3*n - 1)*log
(e)) + 2*(a*b*d*cosh((3*n - 1)*log(e)) + a*b*d*sinh((3*n - 1)*log(e)))*sinh
(n*log(x)))*dilog(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - sinh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c)) - 6*(a*b*d^2*cosh((3*n - 1)*log(e))*
cosh(n*log(x))^2 - b^2*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - (a*b*d^2*c
osh((3*n - 1)*log(e))*cosh(n*log(x))^2 - b^2*d*cosh((3*n - 1)*log(e))*cosh(
n*log(x)) + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*log(e)
))*sinh(n*log(x))^2 + (a*b*d^2*cosh(n*log(x))^2 - b^2*d*cosh(n*log(x)))*sin
h((3*n - 1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - b^
2*d*cosh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) - b^2*d)*sinh((3*n -
 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^
2 - 2*(a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 - b^2*d*cosh((3*n -
1)*log(e))*cosh(n*log(x)) + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh(
(3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a*b*d^2*cosh(n*log(x))^2 - b^2*d*cos
h(n*log(x)))*sinh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cos
h(n*log(x)) - b^2*d*cosh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) - b^
2*d)*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh
(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (a*b*d^2*co
sh((3*n - 1)*log(e))*cosh(n*log(x))^2 - b^2*d*cosh((3*n - 1)*log(e))*cosh(n
*log(x)) + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*log(e))
)*sinh(n*log(x))^2 + (a*b*d^2*cosh(n*log(x))^2 - b^2*d*cosh(n*log(x)))*sinh
((3*n - 1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - b^2
*d*cosh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) - b^2*d)*sinh((3*n -
1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2
 + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*log(e)))*sinh(n
*log(x))^2 + (a*b*d^2*cosh(n*log(x))^2 - b^2*d*cosh(n*log(x)))*sinh((3*n -
1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - b^2*d*cosh(
(3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) - b^2*d)*sinh((3*n - 1)*log(e
)))*sinh(n*log(x)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sin
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) - 6*(((a*b*c^2 - b^2*c)*cos
h((3*n - 1)*log(e)) + (a*b*c^2 - b^2*c)*sinh((3*n - 1)*log(e)))*cosh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*((a*b*c^2 - b^2*c)*cosh((3*n - 1)*
log(e)) + (a*b*c^2 - b^2*c)*sinh((3*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) +
 d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + ((a*
```

```
b*c^2 - b^2*c)*cosh((3*n - 1)*log(e)) + (a*b*c^2 - b^2*c)*sinh((3*n - 1)*lo
g(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - (a*b*c^2 - b^2*c)*
cosh((3*n - 1)*log(e)) - (a*b*c^2 - b^2*c)*sinh((3*n - 1)*log(e)))*log(cosh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) - 1) + 6*(a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 +
b^2*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - (a*b*d^2*cosh((3*n - 1)*log(e
))*cosh(n*log(x))^2 + b^2*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a*b*d^
2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2
 - (a*b*c^2 - b^2*c)*cosh((3*n - 1)*log(e)) + (a*b*d^2*cosh(n*log(x))^2 - a
*b*c^2 + b^2*d*cosh(n*log(x)) + b^2*c)*sinh((3*n - 1)*log(e)) + (2*a*b*d^2*
cosh((3*n - 1)*log(e))*cosh(n*log(x)) + b^2*d*cosh((3*n - 1)*log(e)) + (2*a
*b*d^2*cosh(n*log(x)) + b^2*d)*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*cosh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - 2*(a*b*d^2*cosh((3*n - 1)*log
(e))*cosh(n*log(x))^2 + b^2*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a*b*
d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))
^2 - (a*b*c^2 - b^2*c)*cosh((3*n - 1)*log(e)) + (a*b*d^2*cosh(n*log(x))^2 -
 a*b*c^2 + b^2*d*cosh(n*log(x)) + b^2*c)*sinh((3*n - 1)*log(e)) + (2*a*b*d^
2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + b^2*d*cosh((3*n - 1)*log(e)) + (2
*a*b*d^2*cosh(n*log(x)) + b^2*d)*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*co
sh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) - (a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + b^2*d*c
osh((3*n - 1)*log(e))*cosh(n*log(x)) + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*
b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 - (a*b*c^2 - b^2*c)*cosh((3*
n - 1)*log(e)) + (a*b*d^2*cosh(n*log(x))^2 - a*b*c^2 + b^2*d*cosh(n*log(x))
 + b^2*c)*sinh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n
*log(x)) + b^2*d*cosh((3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) + b^2*d
)*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c)^2 + (a*b*d^2*cosh((3*n - 1)*log(e)) + a*b*d^2*sinh((3*n - 1)*l
og(e)))*sinh(n*log(x))^2 - (a*b*c^2 - b^2*c)*cosh((3*n - 1)*log(e)) + (a*b*
d^2*cosh(n*log(x))^2 - a*b*c^2 + b^2*d*cosh(n*log(x)) + b^2*c)*sinh((3*n -
1)*log(e)) + (2*a*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + b^2*d*cosh(
(3*n - 1)*log(e)) + (2*a*b*d^2*cosh(n*log(x)) + b^2*d)*sinh((3*n - 1)*log(e
)))*sinh(n*log(x)))*log(-cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - si
nh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + 12*((a*b*cosh((3*n - 1)*
log(e)) + a*b*sinh((3*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(
x)) + c)^2 - a*b*cosh((3*n - 1)*log(e)) + 2*(a*b*cosh((3*n - 1)*log(e)) + a
*b*sinh((3*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*si
nh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*cosh((3*n - 1)*log(e)) +
 a*b*sinh((3*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^
2 - a*b*sinh((3*n - 1)*log(e)))*polylog(3, cosh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 12*((a*b*c
osh((3*n - 1)*log(e)) + a*b*sinh((3*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) +
 d*sinh(n*log(x)) + c)^2 - a*b*cosh((3*n - 1)*log(e)) + 2*(a*b*cosh((3*n -
1)*log(e)) + a*b*sinh((3*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*l
og(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*cosh((3*n
```

```
- 1)*log(e)) + a*b*sinh((3*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c)^2 - a*b*sinh((3*n - 1)*log(e)))*polylog(3, -cosh(d*cosh(n*log
(x)) + d*sinh(n*log(x)) + c) - sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c
)) + (a^2*d^3*cosh(n*log(x))^3 + 6*b^2*c^2)*sinh((3*n - 1)*log(e)) + 3*(a^2
*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + a^2*d^3*cosh(n*log(x))^2*sin
h((3*n - 1)*log(e)))*sinh(n*log(x)))/(d^3*n*cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c)^2 + 2*d^3*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*si
nh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + d^3*n*sinh(d*cosh(n*log(x)) +
 d*sinh(n*log(x)) + c)^2 - d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))^2\,dx=\int (ex)^{3n-1}\left(a+b\operatorname{csch}\left(c+dx^n\right)\right)^2\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*csch(c+d*x**n))**2,x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*csch(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))^2\,dx=\int \left(b\operatorname{csch}\left(dx^n+c\right)+a\right)^2(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")
[Out] -2*b^2*e^(3*n)*x^(2*n)/(d*e*n*e^(2*d*x^n + 2*c) - d*e*n) + 1/3*(e*x)^(3*n)*
a^2/(e*n) + integrate(2*(a*b*d*e^(3*n)*x^(3*n) - b^2*e^(3*n)*x^(2*n))/(d*e*
x*e^(d*x^n + c) + d*e*x), x) + integrate(2*(a*b*d*e^(3*n)*x^(3*n) + b^2*e^(
3*n)*x^(2*n))/(d*e*x*e^(d*x^n + c) - d*e*x), x)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b\text{csch}(c+dx^n))^2\,dx=\int \left(b\operatorname{csch}\left(dx^n+c\right)+a\right)^2(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csch(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*csch(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b\text{csch}(c + dx^n))^2 \, dx = \int \left(a + \frac{b}{\sinh(c + d\,x^n)}\right)^2 (e\,x)^{3\,n-1} \, dx$$

```
[In] int((a + b/sinh(c + d*x^n))^2*(e*x)^(3*n - 1),x)
[Out] int((a + b/sinh(c + d*x^n))^2*(e*x)^(3*n - 1), x)
```

3.78 $\int \frac{(ex)^{-1+n}}{a+b\text{csch}(c+dx^n)}\,dx$

Optimal result 519
Rubi [A] (verified) 519
Mathematica [A] (verified) 521
Maple [C] (warning: unable to verify) 521
Fricas [B] (verification not implemented) 522
Sympy [F] 522
Maxima [F] 522
Giac [F] 523
Mupad [B] (verification not implemented) 523

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}\,(c+dx^n)}\,dx=\frac{(ex)^n}{aen}+\frac{2bx^{-n}(ex)^n\text{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}$$

```
[Out] (e*x)^n/a/e/n+2*b*(e*x)^n*arctanh((a-b*tanh(1/2*c+1/2*d*x^n))/(a^2+b^2)^(1/
2))/a/d/e/n/(x^n)/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5549, 5545, 3868, 2739, 632, 210}

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}\,(c+dx^n)}\,dx=\frac{2bx^{-n}(ex)^n\text{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2+b^2}}\right)}{aden\sqrt{a^2+b^2}}+\frac{(ex)^n}{aen}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (e*x)^n/(a*e*n) + (2*b*(e*x)^n*ArcTanh[(a - b*Tanh[(c + d*x^n)/2])/Sqrt[a^2
 + b^2]])/(a*Sqrt[a^2 + b^2]*d*e*n*x^n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{a+b\operatorname{csch}(c+dx^n)}\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx, x, x^n\right)}{en}\\
&= \frac{(ex)^n}{aen} - \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx, x, x^n\right)}{aen}\\
&= \frac{(ex)^n}{aen} + \frac{(2ix^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}
\end{aligned}$$

$$= \frac{(ex)^n}{aen} - \frac{(4ix^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b} + 2i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}$$

$$= \frac{(ex)^n}{aen} + \frac{2bx^{-n}(ex)^n \text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \frac{(ex)^n \left(d + cx^{-n} - \frac{2bx^{-n}\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{aden}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Csch[c + d*x^n]),x]
```

```
[Out] ((e*x)^n*(d + c/x^n - (2*b*ArcTan[(a - b*Tanh[(c + d*x^n)/2])/Sqrt[-a^2 - b
^2]])/(Sqrt[-a^2 - b^2]*x^n)))/(a*d*e*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.89

method	result
risch	$\frac{x\,e^{\frac{(-1+n)\left(-i\,\text{csgn}(ie)\,\text{csgn}(ix)\,\text{csgn}(iex)\pi+i\,\text{csgn}(ie)\,\text{csgn}(iex)^2\pi+i\,\text{csgn}(ix)\,\text{csgn}(iex)^2\pi-i\,\text{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{an} - 2b\,e^{-\frac{i\pi n\,\text{csgn}(ie)}{}}$

```
[In] int((e*x)^(-1+n)/(a+b*csch(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2*b/a/n*exp(-1/2*I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*P
i*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-1
/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(
-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*e
xp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*exp(c)/d/(-a^2*exp(2*c)-exp(2*c)*b^2)^(1/2
)*arctan(1/2*(2*a*exp(2*c+d*x^n)+2*exp(c)*b)/(-a^2*exp(2*c)-exp(2*c)*b^2)^(
1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(79) = 158$.

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.02

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}(c+dx^n)}\,dx$$
$$= \frac{(a^2+b^2)d\cosh((n-1)\log(e))\cosh(n\log(x)) + (a^2+b^2)d\cosh(n\log(x))\sinh((n-1)\log(e)) + \left(\sqrt{a^2+\right.}{}$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2 + b^2)*d*cosh(n*l
og(x))*sinh((n - 1)*log(e)) + (sqrt(a^2 + b^2)*b*cosh((n - 1)*log(e)) + sqr
t(a^2 + b^2)*b*sinh((n - 1)*log(e)))*log((a*b + (a^2 + b^2 + sqrt(a^2 + b^2
)*b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (b^2 + sqrt(a^2 + b^2)
*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2 + b^2)*a)/(a*s
inh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + b)) + ((a^2 + b^2)*d*cosh((n
 - 1)*log(e)) + (a^2 + b^2)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/((a^3 +
 a*b^2)*d*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{a+b\operatorname{csch}(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*csch(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*csch(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\operatorname{csch}(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -2*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*
a*b*e*x*e^(d*x^n + c) - a^2*e*x), x) + e^(n - 1)*x^n/(a*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\,\text{csch}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)/(b*csch(d*x^n + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 410, normalized size of antiderivative = 5.00

$$\int \frac{(ex)^{-1+n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \frac{x\,(e\,x)^{n-1}}{a\,n} - \frac{\left(2\,\text{atan}\left(\frac{x\,(e\,x)^{n-1}\,\sqrt{-a^2\,d^2\,n^2\,x^{2\,n}\,(a^2+b^2)}}{a\,d\,n\,x^n\,\sqrt{b^2\,x^2\,(e\,x)^{2\,n-2}}}\right) - 2\,\text{atan}\left(\frac{a^2\,e^{d\,x^n}\,e^c\left(\frac{2\,b\,x\,(e\,x)^{n-1}}{a^4\,d\,n\,x^n\,\sqrt{b^2\,x^2\,(e\,x)^{2\,n-2}}}+\frac{2\,b\,d\,n\,x^n\,(e\,x)^{1-n}\,\sqrt{\cdots}}{a^2\,x\,\sqrt{-a^4\,d^2\,n^2\,x^{2\,n}-a^2\,b^2\,d^2\,n^2\cdots}}\cdots\right.}{2}\cdots\right.\right.}{\sqrt{-a^4\,d^2\,n^2\,x^{2\,n}-\cdots}}$$

```
[In] int((e*x)^(n - 1)/(a + b/sinh(c + d*x^n)),x)
[Out] (x*(e*x)^(n - 1))/(a*n) - ((2*atan((x*(e*x)^(n - 1)*(-a^2*d^2*n^2*x^(2*n)*(
a^2 + b^2))^(1/2))/(a*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))) - 2*atan((a
^2*exp(d*x^n)*exp(c)*((2*b*x*(e*x)^(n - 1))/(a^4*d*n*x^n*(b^2*x^2*(e*x)^(2*
n - 2))^(1/2)) + (2*b*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)
)/(a^2*x*(- a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)*(-a^2*d^2*
n^2*x^(2*n)*(a^2 + b^2))^(1/2)))*(- a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x
^(2*n))^(1/2))/2 - (a*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)
)/(x*(-a^2*d^2*n^2*x^(2*n)*(a^2 + b^2))^(1/2))))*(b^2*x^2*(e*x)^(2*n - 2))^
(1/2))/(- a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)
```

3.79 $\int \frac{(ex)^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx$

Optimal result 524
Rubi [A] (verified) 525
Mathematica [C] (verified) 528
Maple [C] (warning: unable to verify) 529
Fricas [B] (verification not implemented) 529
Sympy [F] 530
Maxima [F] 531
Giac [F] 531
Mupad [F(-1)] 531

Optimal result

Integrand size = 24, antiderivative size = 291

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx = \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} - \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en} + \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}$$

```
[Out] 1/2*(e*x)^(2*n)/a/e/n-b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2))
)/a/d/e/n/(x^n)/(a^2+b^2)^(1/2)+b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b+(a^2+b
^2)^(1/2)))/a/d/e/n/(x^n)/(a^2+b^2)^(1/2)-b*(e*x)^(2*n)*polylog(2,-a*exp(c+
d*x^n)/(b-(a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(a^2+b^2)^(1/2)+b*(e*x)^(2*
n)*polylog(2,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(a^2+
b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5549, 5545, 4276, 3403, 2296, 2221, 2317, 2438}

$$\int \frac{(ex)^{-1+2n}}{a+b\text{csch}\left(c+dx^n\right)}\,dx = -\frac{bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)}{ad^2en\sqrt{a^2+b^2}} + \frac{bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)}{ad^2en\sqrt{a^2+b^2}} - \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}+1\right)}{aden\sqrt{a^2+b^2}} + \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{a^2+b^2}+b}+1\right)}{aden\sqrt{a^2+b^2}} + \frac{(ex)^{2n}}{2aen}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (e*x)^(2*n)/(2*a*e*n) - (b*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[
a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^(2*n)*Log[1 + (a*E^(
c + d*x^n))/(b + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d*e*n*x^n) - (b*(e*x
)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
 + b^2]*d^2*e*n*x^(2*n)) + (b*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b
 + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2*e*n*x^(2*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx}{e} \\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \frac{x}{a+b\operatorname{csch}(c+dx)}\,dx, x, x^n\right)}{en} \\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\sinh(c+dx))}\right)\,dx, x, x^n\right)}{en}
\end{aligned}
$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{\left(2bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{\left(2bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{a^2+b^2}en} + \frac{\left(2bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{a^2+b^2}en}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{a^2+b^2}den} - \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^n}\right)}{a\sqrt{a^2+b^2}d^2en} - \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^n}\right)}{a\sqrt{a^2+b^2}d^2en}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} - \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en} + \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.65 (sec) , antiderivative size = 1181, normalized size of antiderivative = 4.06

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx$$

$$= \frac{(ex)^{2n}\operatorname{csch}(c+dx^n)\left(1-\frac{2bx^{-2n}\left(-\frac{i\pi\operatorname{arctanh}\left(\frac{-a+b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{2\left(c+i\arccos\left(-\frac{ib}{a}\right)\right)\arctan\left(\frac{(a-ib)\cot\left(\frac{1}{4}(2ic+\pi+2idx^n)\right)}{\sqrt{-a^2-b^2}}\right)+}{}\right.}{}\right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Csch[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*Csch[c + d*x^n]*(1 - (2*b*(((-I)*Pi*ArcTanh[(-a + b*Tanh[(c +
d*x^n)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - (2*(c + I*ArcCos[((-I)*b)/a]
)*ArcTan[((a - I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]]
+ ((-2*I)*c + Pi - (2*I)*d*x^n)*ArcTanh[(((-I)*a + b)*Tan[((2*I)*c + Pi + (
2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*b)/a] - 2*ArcTan[((a - I*
b)*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]])*Log[((a + I*b)*(
a - I*b + Sqrt[-a^2 - b^2])*(1 + I*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]))/(a
*(a + I*b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]))] - (Ar
cCos[((-I)*b)/a] + 2*ArcTan[((a - I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4])
/Sqrt[-a^2 - b^2]])*Log[(I*(a + I*b)*(-a + I*b + Sqrt[-a^2 - b^2])*(I + Cot
[((2*I)*c + Pi + (2*I)*d*x^n)/4]))/(a*(a + I*b + I*Sqrt[-a^2 - b^2]*Cot[((2
*I)*c + Pi + (2*I)*d*x^n)/4]))] + (ArcCos[((-I)*b)/a] + 2*ArcTan[((a - I*b)
*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[(((
-I)*a + b)*Tan[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]])*Log[-(((
-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x^n)/2))/(Sqrt[2]*Sqrt[(-I)*a]*Sq
rt[b + a*Sinh[c + d*x^n]]))] + (ArcCos[((-I)*b)/a] - 2*ArcTan[((a - I*b)*Co
t[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]] + (2*I)*ArcTanh[(((-I)
*a + b)*Tan[((2*I)*c + Pi + (2*I)*d*x^n)/4])/Sqrt[-a^2 - b^2]])*Log[((-1)^(
1/4)*Sqrt[-a^2 - b^2]*E^((c + d*x^n)/2))/(Sqrt[2]*Sqrt[(-I)*a]*Sqrt[b + a*S
inh[c + d*x^n]])] + I*(PolyLog[2, ((I*b + Sqrt[-a^2 - b^2])*(a + I*b - I*Sq
rt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]))/(a*(a + I*b + I*Sqrt[-
a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]))] - PolyLog[2, ((b + I*Sqrt
[-a^2 - b^2])*(I*a - b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/
4]))/(a*(a + I*b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]))
]))/Sqrt[-a^2 - b^2]))/(d^2*x^(2*n)))*(b + a*Sinh[c + d*x^n]))/(2*a*e*n*(a
+ b*Csch[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.98

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(2n-1)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^{2}\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^{2}\pi-i\,\mathrm{csgn}(iex)^{3}\pi+2\ln(e)+2\ln(x)\right)}{2}}}{2an} - \frac{2b\,\mathrm{e}^{-i\pi n\ \mathrm{csgn}(ie)\ \cdots}}{\cdots}$

```
[In] int((e*x)^(2*n-1)/(a+b*csch(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e
)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+
2*ln(x)))-2*b/a*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csg
n(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(
I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(
I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csg
n(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(1/2*x^n*d*(ln((a*exp(2*c+d*x^n)+exp(c)*
b-(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2))/(exp(c)*b-(a^2*exp(2*c)+exp(2*c)*b^2)^
(1/2)))-ln((a*exp(2*c+d*x^n)+exp(c)*b+(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2))/(e
xp(c)*b+(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2))))/(a^2*exp(2*c)+exp(2*c)*b^2)^(1
/2)+1/2*(dilog((a*exp(2*c+d*x^n)+exp(c)*b-(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2)
)/(exp(c)*b-(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2)))-dilog((a*exp(2*c+d*x^n)+exp
(c)*b+(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2))/(exp(c)*b+(a^2*exp(2*c)+exp(2*c)*b
^2)^(1/2))))/(a^2*exp(2*c)+exp(2*c)*b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(271) = 542.

Time = 0.30 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.07

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + (a^2 + b^2)*
d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + ((a^2 + b^2)*d^2*cosh((2*n -
1)*log(e)) + (a^2 + b^2)*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 - 2*(
a*b*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt((a^2 + b^2)/a^2
)*sinh((2*n - 1)*log(e)))*dilog(((a*sqrt((a^2 + b^2)/a^2) + b)*cosh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) + b)*sinh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) - a)/a + 1) + 2*(a*b*sqrt((a^2 + b^2)/
```

```
a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt((a^2 + b^2)/a^2)*sinh((2*n - 1)*log(
e)))*dilog(-((a*sqrt((a^2 + b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) - b)*sinh(d*cosh(n*log(x)) + d*sin
h(n*log(x)) + c) + a)/a + 1) - 2*(a*b*c*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1
)*log(e)) + a*b*c*sqrt((a^2 + b^2)/a^2)*sinh((2*n - 1)*log(e)))*log(2*a*cos
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sinh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) + 2*(a*b*c*sqrt((a^2
 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*c*sqrt((a^2 + b^2)/a^2)*sinh((2*n
 - 1)*log(e)))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*
sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*a*sqrt((a^2 + b^2)/a^2) +
 2*b) - 2*(a*b*d*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e))*cosh(n*log(x)
) + a*b*c*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + (a*b*d*sqrt((a^2 +
 b^2)/a^2)*cosh(n*log(x)) + a*b*c*sqrt((a^2 + b^2)/a^2))*sinh((2*n - 1)*log
(e)) + (a*b*d*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*d*sqrt((a^
2 + b^2)/a^2)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(-((a*sqrt((a^2 +
b^2)/a^2) + b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt((a^2
 + b^2)/a^2) + b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - a)/a) + 2
*(a*b*d*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + a*b*c
*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + (a*b*d*sqrt((a^2 + b^2)/a^2
)*cosh(n*log(x)) + a*b*c*sqrt((a^2 + b^2)/a^2))*sinh((2*n - 1)*log(e)) + (a
*b*d*sqrt((a^2 + b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*d*sqrt((a^2 + b^2)/
a^2)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(((a*sqrt((a^2 + b^2)/a^2)
- b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^
2) - b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a) + 2*((a^2 + b
^2)*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + (a^2 + b^2)*d^2*cosh(n*log(
x))*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))/((a^3 + a*b^2)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{csch}(c+dx^n)}\,dx=\int \frac{(ex)^{2n-1}}{a+b\operatorname{csch}(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*csch(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(2*n - 1)/(a + b*csch(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \int \frac{(ex)^{2\,n-1}}{b\,\text{csch}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -2*b*e^(2*n)*integrate(e^(d*x^n + 2*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c
) + 2*a*b*e*x*e^(d*x^n + c) - a^2*e*x), x) + 1/2*e^(2*n - 1)*x^(2*n)/(a*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \int \frac{(ex)^{2\,n-1}}{b\,\text{csch}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)/(b*csch(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a+b\text{csch}\,(c+dx^n)}\,dx = \int \frac{(e\,x)^{2\,n-1}}{a+\frac{b}{\sinh(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/sinh(c + d*x^n)),x)
```

```
[Out] int((e*x)^(2*n - 1)/(a + b/sinh(c + d*x^n)), x)
```

3.80 $\int \frac{(ex)^{-1+3n}}{a+b\operatorname{csch}(c+dx^n)}\,dx$

Optimal result 532
Rubi [A] (verified) 533
Mathematica [F] 536
Maple [F] 537
Fricas [B] (verification not implemented) 537
Sympy [F] 538
Maxima [F] 539
Giac [F] 539
Mupad [F(-1)] 539

Optimal result

Integrand size = 24, antiderivative size = 428

$$\begin{aligned}\int \frac{(ex)^{-1+3n}}{a+b\operatorname{csch}(c+dx^n)}\,dx &= \frac{(ex)^{3n}}{3aen}-\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}\\&+\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}\\&-\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}\\&+\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}\\&+\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3en}\\&-\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3en}\end{aligned}$$

```
[Out] 1/3*(e*x)^(3*n)/a/e/n-b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2))
)/a/d/e/n/(x^n)/(a^2+b^2)^(1/2)+b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(a^2+b
^2)^(1/2)))/a/d/e/n/(x^n)/(a^2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(2,-a*exp(
c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(a^2+b^2)^(1/2)+2*b*(e*x)
^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(
a^2+b^2)^(1/2)+2*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)
))/a/d^3/e/n/(x^(3*n))/(a^2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d
*x^n)/(b+(a^2+b^2)^(1/2)))/a/d^3/e/n/(x^(3*n))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5549, 5545, 4276, 3403, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{(ex)^{-1+3n}}{a+b\text{csch}(c+dx^n)}dx = \frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)}{ad^3en\sqrt{a^2+b^2}} - \frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)}{ad^3en\sqrt{a^2+b^2}} - \frac{2bx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)}{ad^2en\sqrt{a^2+b^2}} + \frac{2bx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)}{ad^2en\sqrt{a^2+b^2}} - \frac{bx^{-n}(ex)^{3n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}+1\right)}{aden\sqrt{a^2+b^2}} + \frac{bx^{-n}(ex)^{3n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{a^2+b^2}+b}+1\right)}{aden\sqrt{a^2+b^2}} + \frac{(ex)^{3n}}{3aen}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n]),x]
```

```
[Out] (e*x)^(3*n)/(3*a*e*n) - (b*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[
a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^(3*n)*Log[1 + (a*E^(
c + d*x^n))/(b + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d*e*n*x^n) - (2*b*(e
*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a
^2 + b^2]*d^2*e*n*x^(2*n)) + (2*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n)
)/(b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2*e*n*x^(2*n)) + (2*b*(e*x)
^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
+ b^2]*d^3*e*n*x^(3*n)) - (2*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(
b + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3*e*n*x^(3*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{a+b\operatorname{csch}(c+dx^n)}\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^2}{a+b\operatorname{csch}(c+dx)}\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sinh(c+dx))}\right)\,dx,x,x^n\right)}{en}\\
&= \frac{(ex)^{3n}}{3aen}-\frac{\left(bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx,x,x^n\right)}{aen}\\
&= \frac{(ex)^{3n}}{3aen}-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,x^n\right)}{aen}\\
&= \frac{(ex)^{3n}}{3aen}-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{\sqrt{a^2+b^2}en}\\
&\quad+\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{\sqrt{a^2+b^2}en}\\
&= \frac{(ex)^{3n}}{3aen}-\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}+\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}\\
&\quad+\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{a^2+b^2}den}\\
&\quad-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{a^2+b^2}den}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}$$
$$+ \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{a^2+b^2}d^2en}$$
$$- \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{a^2+b^2}d^2en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}$$
$$+ \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a\sqrt{a^2+b^2}d^3en}$$
$$- \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a\sqrt{a^2+b^2}d^3en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2en}$$
$$+ \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3en} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3en}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\operatorname{csch}(c+dx^n)}\,dx = \int \frac{(ex)^{-1+3n}}{a+b\operatorname{csch}(c+dx^n)}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n]), x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\,\operatorname{csch}\left(c+d\,x^{n}\right)}dx$$

```
[In] int((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n)),x)
[Out] int((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1850 vs. $2(402) = 804$.

Time = 0.31 (sec) , antiderivative size = 1850, normalized size of antiderivative = 4.32

$$\int \frac{(ex)^{-1+3n}}{a+b\operatorname{csch}\left(c+dx^{n}\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n)),x, algorithm="fricas")
[Out] 1/3*((a^2 + b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + (a^2 + b^2)*
d^3*cosh(n*log(x))^3*sinh((3*n - 1)*log(e)) + ((a^2 + b^2)*d^3*cosh((3*n -
1)*log(e)) + (a^2 + b^2)*d^3*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^3 + 3*(
(a^2 + b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a^2 + b^2)*d^3*cos
h(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 - 6*(a*b*d*sqrt((a^2 +
 b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt((a^2 + b^2)/a
^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt((a^2 + b^2)/a^2)*co
sh((3*n - 1)*log(e)) + a*b*d*sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e)))*
sinh(n*log(x)))*dilog(((a*sqrt((a^2 + b^2)/a^2) + b)*cosh(d*cosh(n*log(x))
+ d*sinh(n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) + b)*sinh(d*cosh(n*log(x
)) + d*sinh(n*log(x)) + c) - a)/a + 1) + 6*(a*b*d*sqrt((a^2 + b^2)/a^2)*cos
h((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt((a^2 + b^2)/a^2)*cosh(n*log
(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*l
og(e)) + a*b*d*sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e)))*sinh(n*log(x))
)*dilog(-((a*sqrt((a^2 + b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d*sinh(n*lo
g(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) - b)*sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) + a)/a + 1) + 3*(a*b*c^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)
*log(e)) + a*b*c^2*sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e)))*log(2*a*co
sh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sinh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c) + 2*a*sqrt((a^2 + b^2)/a^2) + 2*b) - 3*(a*b*c^2*sqrt((
a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*c^2*sqrt((a^2 + b^2)/a^2)*sinh
((3*n - 1)*log(e)))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
 2*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*a*sqrt((a^2 + b^2)/a
^2) + 2*b) - 3*(a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n
*log(x))^2 - a*b*c^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e)) + (a*b*d^
2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*d^2*sqrt((a^2 + b^2)/a
```

```
^2)*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a*b*d^2*sqrt((a^2 + b^2)/a^
2)*cosh(n*log(x))^2 - a*b*c^2*sqrt((a^2 + b^2)/a^2))*sinh((3*n - 1)*log(e))
 + 2*(a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) +
 a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(
n*log(x)))*log(-((a*sqrt((a^2 + b^2)/a^2) + b)*cosh(d*cosh(n*log(x)) + d*si
nh(n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) + b)*sinh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c) - a)/a) + 3*(a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n -
 1)*log(e))*cosh(n*log(x))^2 - a*b*c^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)
*log(e)) + (a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*d^2*
sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a*b*d^2*s
qrt((a^2 + b^2)/a^2)*cosh(n*log(x))^2 - a*b*c^2*sqrt((a^2 + b^2)/a^2))*sinh
((3*n - 1)*log(e)) + 2*(a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e)
)*cosh(n*log(x)) + a*b*d^2*sqrt((a^2 + b^2)/a^2)*cosh(n*log(x))*sinh((3*n -
 1)*log(e)))*sinh(n*log(x)))*log(((a*sqrt((a^2 + b^2)/a^2) - b)*cosh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) - b)*sinh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a) + 6*(a*b*sqrt((a^2 + b^2)/a^2
)*cosh((3*n - 1)*log(e)) + a*b*sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e))
)*polylog(3, ((a*sqrt((a^2 + b^2)/a^2) + b)*cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) + (a*sqrt((a^2 + b^2)/a^2) + b)*sinh(d*cosh(n*log(x)) + d*si
nh(n*log(x)) + c))/a) - 6*(a*b*sqrt((a^2 + b^2)/a^2)*cosh((3*n - 1)*log(e))
 + a*b*sqrt((a^2 + b^2)/a^2)*sinh((3*n - 1)*log(e)))*polylog(3, -((a*sqrt((
a^2 + b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqr
t((a^2 + b^2)/a^2) - b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))/a) +
 3*((a^2 + b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + (a^2 + b^2)*d
^3*cosh(n*log(x))^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))/((a^3 + a*b^2)*
d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a + b\operatorname{csch}(c + dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{a + b\operatorname{csch}(c + dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*csch(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)/(a + b*csch(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{csch}\,(c+dx^n)}\,dx=\int \frac{(ex)^{3\,n-1}}{b\,\text{csch}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -2*b*e^(3*n)*integrate(e^(d*x^n + 3*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c
) + 2*a*b*e*x*e^(d*x^n + c) - a^2*e*x), x) + 1/3*e^(3*n - 1)*x^(3*n)/(a*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{csch}\,(c+dx^n)}\,dx=\int \frac{(ex)^{3\,n-1}}{b\,\text{csch}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3*n - 1)/(b*csch(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a+b\text{csch}\,(c+dx^n)}\,dx=\int \frac{(e\,x)^{3\,n-1}}{a+\frac{b}{\sinh(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/sinh(c + d*x^n)),x)
```

```
[Out] int((e*x)^(3*n - 1)/(a + b/sinh(c + d*x^n)), x)
```

3.81 $\int \frac{(ex)^{-1+n}}{\left(a+b\,\mathrm{csch}(c+dx^n)\right)^2}\,dx$

Optimal result 540
Rubi [A] (verified) 540
Mathematica [A] (verified) 543
Maple [C] (warning: unable to verify) 543
Fricas [B] (verification not implemented) 544
Sympy [F] 545
Maxima [F] 545
Giac [F] 546
Mupad [B] (verification not implemented) 546

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{(ex)^{-1+n}}{(a+b\,\mathrm{csch}\,(c+dx^n))^2}\,dx = \frac{(ex)^n}{a^2en} + \frac{2b(2a^2+b^2)\,x^{-n}(ex)^n\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\,(a^2+b^2)^{3/2}\,den} - \frac{b^2x^{-n}(ex)^n\coth\,(c+dx^n)}{a\,(a^2+b^2)\,den\,(a+b\,\mathrm{csch}\,(c+dx^n))}$$

```
[Out] (e*x)^n/a^2/e/n+2*b*(2*a^2+b^2)*(e*x)^n*arctanh((a-b*tanh(1/2*c+1/2*d*x^n))
/(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(3/2)/d/e/n/(x^n)-b^2*(e*x)^n*coth(c+d*x^n)
/a/(a^2+b^2)/d/e/n/(x^n)/(a+b*csch(c+d*x^n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5549, 5545, 3870, 4004, 3916, 2739, 632, 210}

$$\int \frac{(ex)^{-1+n}}{(a+b\,\mathrm{csch}\,(c+dx^n))^2}\,dx = \frac{2b(2a^2+b^2)\,x^{-n}(ex)^n\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2+b^2}}\right)}{a^2den\,(a^2+b^2)^{3/2}} - \frac{b^2x^{-n}(ex)^n\coth\,(c+dx^n)}{aden\,(a^2+b^2)\,(a+b\,\mathrm{csch}\,(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (e*x)^n/(a^2*e*n) + (2*b*(2*a^2 + b^2)*(e*x)^n*ArcTanh[(a - b*Tanh[(c + d*x
^n)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d*e*n*x^n) - (b^2*(e*x)^n*
Coth[c + d*x^n])/(a*(a^2 + b^2)*d*e*n*x^n*(a + b*Csch[c + d*x^n]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
```

```
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{\left(a+b\text{csch}(c+dx^n)\right)^2}\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx,x,x^n\right)}{en}\\
&= -\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{a(a^2+b^2)\,den\,(a+b\text{csch}(c+dx^n))}-\frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{-a^2-b^2+ab\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx,x,x^n\right)}{a(a^2+b^2)\,en}\\
&= \frac{(ex)^n}{a^2en}-\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{a(a^2+b^2)\,den\,(a+b\text{csch}(c+dx^n))}\\
&\quad-\frac{(i(-ia^2b+ib(-a^2-b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx,x,x^n\right)}{a^2(a^2+b^2)\,en}\\
&= \frac{(ex)^n}{a^2en}-\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{a(a^2+b^2)\,den\,(a+b\text{csch}(c+dx^n))}\\
&\quad-\frac{(i(-ia^2b+ib(-a^2-b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx,x,x^n\right)}{a^2b(a^2+b^2)\,en}\\
&= \frac{(ex)^n}{a^2en}-\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{a(a^2+b^2)\,den\,(a+b\text{csch}(c+dx^n))}\\
&\quad-\frac{(2(-ia^2b+ib(-a^2-b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx,x,i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2+b^2)\,den}\\
&= \frac{(ex)^n}{a^2en}-\frac{b^2x^{-n}(ex)^n\coth(c+dx^n)}{a(a^2+b^2)\,den\,(a+b\text{csch}(c+dx^n))}\\
&\quad+\frac{(4(-ia^2b+ib(-a^2-b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,-\frac{2ia}{b}+2i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2+b^2)\,den}
\end{aligned}
$$

$$= \frac{(ex)^n}{a^2en} + \frac{2b(2a^2+b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\,(a^2+b^2)^{3/2}\,den} - \frac{b^2x^{-n}(ex)^n\coth\left(c+dx^n\right)}{a\,(a^2+b^2)\,den\,(a+b\text{csch}\left(c+dx^n\right))}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.12

$$\int \frac{(ex)^{-1+n}}{(a+b\text{csch}\left(c+dx^n\right))^2}\,dx = -\frac{x^{-n}(ex)^n\left(-ab^2\sqrt{-a^2-b^2}\coth\left(c+dx^n\right)+\left(-(-a^2-b^2)^{3/2}\left(c+dx^n\right)-2b(2a^2+b^2)\arctan\left(\frac{a-b\tanh}{\sqrt{-}}\right.\right.\right.}{a^2\left(-a^2-b^2\right)^{3/2}den\left(a+b\text{csch}\left(c+dx^n\right)\right)}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] -(((e*x)^n*(-(a*b^2*Sqrt[-a^2 - b^2]*Coth[c + d*x^n]) + (-((-a^2 - b^2)^(3/
2)*(c + d*x^n)) - 2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x^n)/2])/Sqrt
[-a^2 - b^2]])*(a + b*Csch[c + d*x^n])))/(a^2*(-a^2 - b^2)^(3/2)*d*e*n*x^n*
(a + b*Csch[c + d*x^n])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.43 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.29

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{a^2n} - \frac{2\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csg}\right.}{}}}{}$

```
[In] int((e*x)^(-1+n)/(a+b*csch(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)
*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2
*ln(x)))-2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e
)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+
2*ln(x)))*b^2*x*(-b*exp(c+d*x^n)+a)/a^2/(a^2+b^2)/d/n/(x^n)/(a*exp(2*c+2*d*
x^n)+2*b*exp(c+d*x^n)-a)-2*b/a^2*(2*a^2+b^2)/(a^2+b^2)/n*exp(-1/2*I*Pi*n*cs
gn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(
1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*
I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2
)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*
```

```
exp(c)/d/(-a^2*exp(2*c)-exp(2*c)*b^2)^(1/2)*arctan(1/2*(2*a*exp(2*c+d*x^n)+
2*exp(c)*b)/(-a^2*exp(2*c)-exp(2*c)*b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. 2(146) = 292.

Time = 0.29 (sec) , antiderivative size = 1729, normalized size of antiderivative = 11.60

$$\int \frac{(ex)^{-1+n}}{(a+b\text{csch}(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] -((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) - ((a^5 +
 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 + 2*a^3*b^
2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 + 2*a^3*b^2 + a*b^
4)*d*cosh((n - 1)*log(e)) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)
))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - ((a^5
+ 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 + 2*a^3*b
^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 + 2*a^3*b^2 + a*b
^4)*d*cosh((n - 1)*log(e)) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e
)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - 2*((a
^4*b + 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2*b^3 +
b^5)*cosh((n - 1)*log(e)) + (a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*d*co
sh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b + 2*a^2*b^3 + b^5)*d*cosh((n -
 1)*log(e)) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(
x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*(a^3*b^2 + a*b^4)*co
sh((n - 1)*log(e)) - (((2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*cosh((n - 1)*log(e
)) + (2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*
log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*cosh
((n - 1)*log(e)) + (2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*sinh((n - 1)*log(e)))*
sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - (2*a^3*b + a*b^3)*sqrt(a^
2 + b^2)*cosh((n - 1)*log(e)) - (2*a^3*b + a*b^3)*sqrt(a^2 + b^2)*sinh((n -
 1)*log(e)) + 2*((2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cosh((n - 1)*log(e)) + (
2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)
) + d*sinh(n*log(x)) + c) + 2*((2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cosh((n -
1)*log(e)) + (2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*sinh((n - 1)*log(e)) + ((2*a
^3*b + a*b^3)*sqrt(a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b + a*b^3)*sqrt
(a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))*log((a*b + (a^2 + b^2
+ sqrt(a^2 + b^2)*b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (b^2 +
 sqrt(a^2 + b^2)*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^
2 + b^2)*a)/(a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + b)) - 2*((a^
4*b + 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 + 2*a^
```

```
3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 + 2*a^3*b^2 + a
*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d*
cosh((n - 1)*log(e)) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*si
nh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2*b^3 + b^
5)*cosh((n - 1)*log(e)) + (a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*d*cosh
(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b + 2*a^2*b^3 + b^5)*d*cosh((n - 1
)*log(e)) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)
))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (2*a^3*b^2 + 2*a*b^4 + (
a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^5 + 2
*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh
((n - 1)*log(e)))*sinh(n*log(x)))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*n*s
inh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2
*b^5)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (a^7 + 2*a^5*b^2
+ a^3*b^4)*d*n + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) +
 d*sinh(n*log(x)) + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d*n)*sinh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c))
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\text{csch}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(a+b\,\text{csch}\,(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*csch(c+d*x**n))**2,x)
[Out] Integral((e*x)**(n - 1)/(a + b*csch(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\text{csch}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\,\text{csch}\,(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")
[Out] -2*(2*a^2*b*e^n*e^c + b^3*e^n*e^c)*integrate(e^(d*x^n + n*log(x))/((a^5*e*e
^(2*c) + a^3*b^2*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*e*e^c + a^2*b^3*e*e^c)
*x*e^(d*x^n) - (a^5*e + a^3*b^2*e)*x), x) + (2*a*b^2*e^n + (a^3*d*e^n + a*b
^2*d*e^n)*x^n - (a^3*d*e^n*e^(2*c) + a*b^2*d*e^n*e^(2*c))*e^(2*d*x^n + n*lo
g(x)) - 2*(b^3*e^n*e^c + (a^2*b*d*e^n*e^c + b^3*d*e^n*e^c)*x^n)*e^(d*x^n))/
(a^5*d*e*n + a^3*b^2*d*e*n - (a^5*d*e*n*e^(2*c) + a^3*b^2*d*e*n*e^(2*c))*e^
(2*d*x^n) - 2*(a^4*b*d*e*n*e^c + a^2*b^3*d*e*n*e^c)*e^(d*x^n))
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\operatorname{csch}(dx^n+c)+a)^2}\,dx$$

[In] integrate((e*x)^(-1+n)/(a+b*csch(c+d*x^n))^2,x, algorithm="giac")

[Out] integrate((e*x)^(n - 1)/(b*csch(d*x^n + c) + a)^2, x)

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.72

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \text{Too large to display}$$

[In] int((e*x)^(n - 1)/(a + b/sinh(c + d*x^n))^2,x)

[Out] ((2*atan(((a^5*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2))/2 + (a^3*b^2*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2))/2)*(exp(d*x^n)*exp(c)*((2*(e*x)^(1 - n)*(a^4*b*d*n*x^n*(b^6*x^2*(e*x)^(2*n - 2) + 4*a^2*b^4*x^2*(e*x)^(2*n - 2) + 4*a^4*b^2*x^2*(e*x)^(2*n - 2))^(1/2) + a^2*b^3*d*n*x^n*(b^6*x^2*(e*x)^(2*n - 2) + 4*a^2*b^4*x^2*(e*x)^(2*n - 2) + 4*a^4*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))/(a^2*x*(a^4 + a^2*b^2)*(2*a^2 + b^2)*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2)*(-a^4*d^2*n^2*x^(2*n)*(a^2 + b^2)^3)^(1/2)) + (2*(b^3*x*(e*x)^(n - 1)*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2) + 2*a^2*b*x*(e*x)^(n - 1)*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2)))/(a^4*d*n*x^n*(a^4 + a^2*b^2)*(a^2 + b^2)*(b^2*x^2*(e*x)^(2*n - 2)*(2*a^2 + b^2)^2)^(1/2)*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2))) - (2*(e*x)^(1 - n)*(a^5*d*n*x^n*(b^6*x^2*(e*x)^(2*n - 2) + 4*a^2*b^4*x^2*(e*x)^(2*n - 2) + 4*a^4*b^2*x^2*(e*x)^(2*n - 2))^(1/2) + a^3*b^2*d*n*x^n*(b^6*x^2*(e*x)^(2*n - 2) + 4*a^2*b^4*x^2*(e*x)^(2*n - 2) + 4*a^4*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))/(a^2*x*(a^4 + a^2*b^2)*(2*a^2 + b^2)*(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) - 3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2)*(-a^4*d^2*n^2*x^(2*n)*(a^2 + b^2)^3)^(1/2)))) - 2*atan((x*(e*x)^(n - 1)*(2*a^2 + b^2)*(-a^4*d^2*n^2*x^(2*n)*(a^2 + b^2)^3)^(1/2))/(a^2*d*n*x^n*(a^2 + b^2)*(b^2*x^2*(e*x)^(2*n - 2)*(2*a^2 + b^2)^2)^(1/2))))*(b^6*x^2*(e*x)^(2*n - 2) + 4*a^2*b^4*x^2*(e*x)^(2*n - 2) + 4*a^4*b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(- a^10*d^2*n^2*x^(2*n) - a^4*b^6*d^2*n^2*x^(2*n) -

```
3*a^6*b^4*d^2*n^2*x^(2*n) - 3*a^8*b^2*d^2*n^2*x^(2*n))^(1/2) - ((2*b^2*x^2*
(e*x)^(n - 1))/(d*n*x^n*(a^3*x + a*b^2*x)) - (2*b^3*x^2*exp(c + d*x^n)*(e*x
)^(n - 1))/(a*d*n*x^n*(a^3*x + a*b^2*x)))/(a*exp(2*c + 2*d*x^n) - a + 2*b*e
xp(c + d*x^n)) + (x*(e*x)^(n - 1))/(a^2*n)
```

3.82 $\int \frac{(ex)^{-1+2n}}{\left(a+b\,\mathbf{csch}(c+dx^n)\right)^2}\,dx$

Optimal result 549
Rubi [A] (verified) 550
Mathematica [C] (warning: unable to verify) 556
Maple [F] 557
Fricas [B] (verification not implemented) 558
Sympy [F] 558
Maxima [F] 558
Giac [F] 559
Mupad [F(-1)] 559

Optimal result

Integrand size = 24, antiderivative size = 681

$$\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx=\frac{(ex)^{2n}}{2a^2en}+\frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den}-\frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}-\frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den}+\frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}+\frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sinh(c+dx^n))}{a^2(a^2+b^2)d^2en}+\frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}-\frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en}-\frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}+\frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en}-\frac{b^2x^{-n}(ex)^{2n}\cosh(c+dx^n)}{a(a^2+b^2)den(b+a\sinh(c+dx^n))}$$

```
[Out] 1/2*(e*x)^(2*n)/a^2/e/n+b^2*(e*x)^(2*n)*ln(b+a*sinh(c+d*x^n))/a^2/(a^2+b^2)
/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))
/a^2/(a^2+b^2)^(3/2)/d/e/n/(x^n)-b^3*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b+(a^
2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d/e/n/(x^n)+b^3*(e*x)^(2*n)*polylog(2,-a
*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-b^
3*(e*x)^(2*n)*polylog(2,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^
(3/2)/d^2/e/n/(x^(2*n))-b^2*(e*x)^(2*n)*cosh(c+d*x^n)/a/(a^2+b^2)/d/e/n/(x^
n)/(b+a*sinh(c+d*x^n))-2*b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/
2)))/a^2/d/e/n/(x^n)/(a^2+b^2)^(1/2)+2*b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b
+(a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(a^2+b^2)^(1/2)-2*b*(e*x)^(2*n)*polylog(
2,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(a^2+b^2)^(1/2
)+2*b*(e*x)^(2*n)*polylog(2,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a^2/d^2/e/
n/(x^(2*n))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5549, 5545, 4276, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2}dx = -\frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d^2en\sqrt{a^2+b^2}} + \frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d^2en\sqrt{a^2+b^2}} + \frac{b^2x^{-2n}(ex)^{2n}\log(a\sinh(c+dx^n)+b)}{a^2d^2en(a^2+b^2)} - \frac{2bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}+1\right)}{a^2den\sqrt{a^2+b^2}} + \frac{2bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{a^2+b^2}+b}+1\right)}{a^2den\sqrt{a^2+b^2}} - \frac{b^2x^{-n}(ex)^{2n}\cosh(c+dx^n)}{aden(a^2+b^2)(a\sinh(c+dx^n)+b)} + \frac{b^3x^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d^2en(a^2+b^2)^{3/2}} - \frac{b^3x^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d^2en(a^2+b^2)^{3/2}} + \frac{b^3x^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}+1\right)}{a^2den(a^2+b^2)^{3/2}} - \frac{b^3x^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{a^2+b^2}+b}+1\right)}{a^2den(a^2+b^2)^{3/2}} + \frac{(ex)^{2n}}{2a^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(2*n)/(2*a^2*e*n) + (b^3*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b - S
qrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d*e*n*x^n) - (2*b*(e*x)^(2*n)*Log[
1 + (a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d*e*n*x^
n) - (b^3*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2])])/(a^
2*(a^2 + b^2)^(3/2)*d*e*n*x^n) + (2*b*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))
/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d*e*n*x^n) + (b^2*(e*x)^(2*n)
*Log[b + a*Sinh[c + d*x^n]])/(a^2*(a^2 + b^2)*d^2*e*n*x^(2*n)) + (b^3*(e*x)
^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
b^2)^(3/2)*d^2*e*n*x^(2*n)) - (2*b*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n
```

```
))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2*e*n*x^(2*n)) - (b^3*(e
*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2
 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (2*b*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*
x^n))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2*e*n*x^(2*n)) - (b^2
*(e*x)^(2*n)*Cosh[c + d*x^n])/(a*(a^2 + b^2)*d*e*n*x^n*(b + a*Sinh[c + d*x^
n]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{\left(a+b\operatorname{csch}(c+dx^n)\right)^2}\,dx}{e}\\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \frac{x}{(a+b\operatorname{csch}(c+dx))^2}\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx}{a^2(b+a\sinh(c+dx))}\right)dx,x,x^n\right)}{en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ex)^{2n}}{2a^2en} - \frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx,x,x^n\right)}{a^2en} \\
&\quad + \frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{(b+a\sinh(c+dx))^2}\,dx,x,x^n\right)}{a^2en} \\
&= \frac{(ex)^{2n}}{2a^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh\left(c+dx^n\right)}{a\left(a^2+b^2\right)den\left(b+a\sinh\left(c+dx^n\right)\right)} \\
&\quad - \frac{\left(4bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,x^n\right)}{a^2en} \\
&\quad + \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{b+a\sinh(c+dx)}\,dx,x,x^n\right)}{a^2\left(a^2+b^2\right)en} \\
&\quad + \frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx,x,x^n\right)}{a\left(a^2+b^2\right)den} \\
&= \frac{(ex)^{2n}}{2a^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh\left(c+dx^n\right)}{a\left(a^2+b^2\right)den\left(b+a\sinh\left(c+dx^n\right)\right)} \\
&\quad + \frac{\left(2b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,x^n\right)}{a^2\left(a^2+b^2\right)en} \\
&\quad - \frac{\left(4bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a\sqrt{a^2+b^2}en} \\
&\quad + \frac{\left(4bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a\sqrt{a^2+b^2}en} \\
&\quad + \frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{1}{b+x}\,dx,x,a\sinh\left(c+dx^n\right)\right)}{a^2\left(a^2+b^2\right)d^2en}
\end{aligned}
$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sinh(c+dx^n))}{a^2(a^2+b^2)d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh(c+dx^n)}{a(a^2+b^2)den(b+a\sinh(c+dx^n))} + \frac{(2b^3x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{e^{c+dx}x}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}dx,x,x^n\right)}{a(a^2+b^2)^{3/2}en} - \frac{(2b^3x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{e^{c+dx}x}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}dx,x,x^n\right)}{a(a^2+b^2)^{3/2}en} + \frac{(2bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a^2\sqrt{a^2+b^2}den} - \frac{(2bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a^2\sqrt{a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} - \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sinh(c+dx^n))}{a^2(a^2+b^2)d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh(c+dx^n)}{a(a^2+b^2)den(b+a\sinh(c+dx^n))} + \frac{(2bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{(2bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{(b^3x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2+b^2)^{3/2}den} + \frac{(b^3x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2+b^2)^{3/2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$

$$- \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$

$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\sinh\left(c+dx^n\right)\right)}{a^2\left(a^2+b^2\right)d^2en} - \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en}$$

$$+ \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh\left(c+dx^n\right)}{a\left(a^2+b^2\right)den\left(b+a\sinh\left(c+dx^n\right)\right)}$$

$$- \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}$$

$$+ \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$

$$- \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$

$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\sinh\left(c+dx^n\right)\right)}{a^2\left(a^2+b^2\right)d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}$$

$$- \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d^2en}$$

$$+ \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cosh\left(c+dx^n\right)}{a\left(a^2+b^2\right)den\left(b+a\sinh\left(c+dx^n\right)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.68 (sec) , antiderivative size = 3219, normalized size of antiderivative = 4.73

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Csch[c/2]*Csch[c + d*x^n]^2*Sech[c/2]*(b*Co
sh[c] + a*Sinh[d*x^n])*(b + a*Sinh[c + d*x^n]))/(2*a^2*(a^2 + b^2)*d*n*(a +
 b*Csch[c + d*x^n])^2) + (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Coth[c]*Csch[c + d
*x^n]^2*(b + a*Sinh[c + d*x^n])^2)/(a^2*(a^2 + b^2)*d*n*(a + b*Csch[c + d*x
^n])^2) + (2*b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*ArcTanh[(b + a*E^(c + d*x^n))
/Sqrt[a^2 + b^2]]*Coth[c]*Csch[c + d*x^n]^2*(b + a*Sinh[c + d*x^n])^2)/(a^2
*(a^2 + b^2)^(3/2)*d^2*n*(a + b*Csch[c + d*x^n])^2) - (b^2*E^c*x^(1 - 2*n)*
(e*x)^(-1 + 2*n)*Csch[c]*Csch[c + d*x^n]^2*((d*x^n)/a + (b*(1 + E^(2*c))*Ar
cTanh[(b*E^c + a*E^(2*c + d*x^n))/(Sqrt[a^2 + b^2]*E^c)])/(a*Sqrt[a^2 + b^2
]*E^(2*c)) - ((1 - E^(-2*c))*Log[a - 2*b*E^(c + d*x^n) - a*E^(2*c + 2*d*x^n
)])/(2*a))*(b + a*Sinh[c + d*x^n])^2)/(a*(a^2 + b^2)*d^2*n*(a + b*Csch[c +
d*x^n])^2) - (2*b*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csch[c + d*x^n]^2*(((-I)*Pi*
ArcTanh[(-a + b*Tanh[(c + d*x^n)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - (2
*((-I)*c + Pi/2 - I*d*x^n)*ArcTanh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x
^n)/2])/Sqrt[-a^2 - b^2]] - 2*((-I)*c + ArcCos[((-I)*b)/a])*ArcTanh[(((-I)*
a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]] + (ArcCos[((-I)*
b)/a] - (2*I)*(ArcTanh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt
[-a^2 - b^2]] - ArcTanh[(((-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqr
t[-a^2 - b^2]]))*Log[Sqrt[-a^2 - b^2]/(Sqrt[2]*Sqrt[(-I)*a]*E^((I/2)*((-I)*
c + Pi/2 - I*d*x^n))*Sqrt[b + a*Sinh[c + d*x^n]])] + (ArcCos[((-I)*b)/a] +
(2*I)*(ArcTanh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 -
b^2]] - ArcTanh[(((-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 -
 b^2]]))*Log[(Sqrt[-a^2 - b^2]*E^((I/2)*((-I)*c + Pi/2 - I*d*x^n)))/(Sqrt[2
]*Sqrt[(-I)*a]*Sqrt[b + a*Sinh[c + d*x^n]])] - (ArcCos[((-I)*b)/a] + (2*I)*
ArcTanh[(((-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]])*
Log[1 - (I*(b - I*Sqrt[-a^2 - b^2])*((-I)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I
)*c + Pi/2 - I*d*x^n)/2]))/(a*((-I)*a + b + Sqrt[-a^2 - b^2]*Tan[((-I)*c +
Pi/2 - I*d*x^n)/2]))] + (-ArcCos[((-I)*b)/a] + (2*I)*ArcTanh[(((-I)*a - b)*
Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*(b + I*Sqrt
[-a^2 - b^2])*((-I)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/
2]))/(a*((-I)*a + b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))]
+ I*(PolyLog[2, (I*(b - I*Sqrt[-a^2 - b^2])*((-I)*a + b - Sqrt[-a^2 - b^2]*
Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))/(a*((-I)*a + b + Sqrt[-a^2 - b^2]*Tan[((
-I)*c + Pi/2 - I*d*x^n)/2]))] - PolyLog[2, (I*(b + I*Sqrt[-a^2 - b^2])*((-I
)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))/(a*((-I)*a +
```

```
b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))]]))/Sqrt[-a^2 - b^2]
)*(b + a*Sinh[c + d*x^n])^2)/((a^2 + b^2)*d^2*n*(a + b*Csch[c + d*x^n])^2)
- (b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csch[c + d*x^n]^2*(((-I)*Pi*ArcTanh[(-a
 + b*Tanh[(c + d*x^n)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - (2*((-I)*c +
Pi/2 - I*d*x^n)*ArcTanh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqr
t[-a^2 - b^2]] - 2*((-I)*c + ArcCos[((-I)*b)/a])*ArcTanh[(((-I)*a - b)*Tan[
((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]] + (ArcCos[((-I)*b)/a] - (2*
I)*(ArcTanh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2
]] - ArcTanh[(((-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^
2]]))*Log[Sqrt[-a^2 - b^2]/(Sqrt[2]*Sqrt[(-I)*a]*E^((I/2)*((-I)*c + Pi/2 -
I*d*x^n))*Sqrt[b + a*Sinh[c + d*x^n]])] + (ArcCos[((-I)*b)/a] + (2*I)*(ArcT
anh[(((-I)*a + b)*Cot[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]] - Arc
Tanh[(((-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]]))*Lo
g[(Sqrt[-a^2 - b^2]*E^((I/2)*((-I)*c + Pi/2 - I*d*x^n)))/(Sqrt[2]*Sqrt[(-I)
*a]*Sqrt[b + a*Sinh[c + d*x^n]])] - (ArcCos[((-I)*b)/a] + (2*I)*ArcTanh[(((
-I)*a - b)*Tan[((-I)*c + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*
(b - I*Sqrt[-a^2 - b^2])*((-I)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2
- I*d*x^n)/2]))/(a*((-I)*a + b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*
x^n)/2]))] + (-ArcCos[((-I)*b)/a] + (2*I)*ArcTanh[(((-I)*a - b)*Tan[((-I)*c
 + Pi/2 - I*d*x^n)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*(b + I*Sqrt[-a^2 - b^2
])*((-I)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))/(a*((-
I)*a + b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))] + I*(PolyLo
g[2, (I*(b - I*Sqrt[-a^2 - b^2])*((-I)*a + b - Sqrt[-a^2 - b^2]*Tan[((-I)*c
 + Pi/2 - I*d*x^n)/2]))/(a*((-I)*a + b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/
2 - I*d*x^n)/2]))] - PolyLog[2, (I*(b + I*Sqrt[-a^2 - b^2])*((-I)*a + b - S
qrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))/(a*((-I)*a + b + Sqrt[-a
^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x^n)/2]))]))/Sqrt[-a^2 - b^2])*(b + a*Si
nh[c + d*x^n])^2)/(a^2*(a^2 + b^2)*d^2*n*(a + b*Csch[c + d*x^n])^2) + (x^(1
 - n)*(e*x)^(-1 + 2*n)*Csch[c/2]*Csch[c + d*x^n]^2*Sech[c/2]*(-2*b^2*Cosh[c
] + a^2*d*x^n*Sinh[c] + b^2*d*x^n*Sinh[c])*(b + a*Sinh[c + d*x^n])^2)/(4*a^
2*(a^2 + b^2)*d*n*(a + b*Csch[c + d*x^n])^2)
```

Maple [F]

$$\int \frac{(ex)^{2n-1}}{(a+b\,\mathrm{csch}\,(c+d\,x^n))^2}dx$$

[In] `int((e*x)^(2*n-1)/(a+b*csch(c+d*x^n))^2,x)`

[Out] `int((e*x)^(2*n-1)/(a+b*csch(c+d*x^n))^2,x)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8453 vs. $2(645) = 1290$.

Time = 0.41 (sec) , antiderivative size = 8453, normalized size of antiderivative = 12.41

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*csch(c+d*x**n))**2,x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*csch(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(b\operatorname{csch}(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")
[Out] 1/2*(4*a*b^2*e^(2*n)*x^n + (a^3*d*e^(2*n) + a*b^2*d*e^(2*n))*x^(2*n) - (a^3
*d*e^(2*n)*e^(2*c) + a*b^2*d*e^(2*n)*e^(2*c))*e^(2*d*x^n + 2*n*log(x)) - 2*
(2*b^3*e^(2*n)*e^(n*log(x) + c) + (a^2*b*d*e^(2*n)*e^c + b^3*d*e^(2*n)*e^c)
*x^(2*n))*e^(d*x^n))/(a^5*d*e*n + a^3*b^2*d*e*n - (a^5*d*e*n*e^(2*c) + a^3*
b^2*d*e*n*e^(2*c))*e^(2*d*x^n) - 2*(a^4*b*d*e*n*e^c + a^2*b^3*d*e*n*e^c)*e^
(d*x^n)) - integrate(-2*(a*b^2*e^(2*n)*x^n - (b^3*e^(2*n)*e^(n*log(x) + c)
+ (2*a^2*b*d*e^(2*n)*e^c + b^3*d*e^(2*n)*e^c)*x^(2*n))*e^(d*x^n))/((a^5*d*e
*e^(2*c) + a^3*b^2*d*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d*e*e^c + a^2*b^3*
d*e*e^c)*x*e^(d*x^n) - (a^5*d*e + a^3*b^2*d*e)*x), x)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2} \, dx = \int \frac{(ex)^{2n-1}}{(b\,\text{csch}(dx^n+c)+a)^2} \, dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*csch(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{csch}(c+dx^n))^2} \, dx = \int \frac{(e\,x)^{2\,n-1}}{\left(a+\frac{b}{\sinh(c+d\,x^n)}\right)^2} \, dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/sinh(c + d*x^n))^2,x)
[Out] int((e*x)^(2*n - 1)/(a + b/sinh(c + d*x^n))^2, x)
```

3.83 $\int \frac{(ex)^{-1+3n}}{\left(a+b\,\text{csch}(c+dx^n)\right)^2}\,dx$

Optimal result 561
Rubi [A] (verified) 562
Mathematica [F] 572
Maple [F] 573
Fricas [F(-1)] 573
Sympy [F] 573
Maxima [F] 573
Giac [F] 574
Mupad [F(-1)] 574

Optimal result

Integrand size = 24, antiderivative size = 1218

$$
\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx &= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} \\
&+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} \\
&- \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} \\
&- \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} \\
&+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} \\
&+ \frac{2b^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} \\
&+ \frac{2b^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} \\
&- \frac{2b^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en} \\
&+ \frac{4bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en} \\
&+ \frac{2b^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en} \\
&- \frac{4bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en}
\end{aligned}
$$

$b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)$

```
[Out] 1/3*(e*x)^(3*n)/a^2/e/n-b^2*(e*x)^(3*n)/a^2/(a^2+b^2)/d/e/n/(x^n)+2*b^2*(e*
x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2/e/n/(x^
(2*n))+b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^
2)^(3/2)/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/
2)))/a^2/(a^2+b^2)/d^2/e/n/(x^(2*n))-b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b
+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*polylo
g(2,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3/e/n/(x^(3*n))+2*
b^3*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2
)^(3/2)/d^2/e/n/(x^(2*n))+2*b^2*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(a
^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3/e/n/(x^(3*n))-2*b^3*(e*x)^(3*n)*polylog(2
,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))
-2*b^3*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/(a^2+
b^2)^(3/2)/d^3/e/n/(x^(3*n))+2*b^3*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b
+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))-b^2*(e*x)^(3*n)*co
sh(c+d*x^n)/a/(a^2+b^2)/d/e/n/(x^n)/(b+a*sinh(c+d*x^n))-2*b*(e*x)^(3*n)*ln(
1+a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(a^2+b^2)^(1/2)+2*b*(
e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(a^2+b^
2)^(1/2)-4*b*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2
/d^2/e/n/(x^(2*n))/(a^2+b^2)^(1/2)+4*b*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n
)/(b+(a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(a^2+b^2)^(1/2)+4*b*(e*x)^(3*n
)*polylog(3,-a*exp(c+d*x^n)/(b-(a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(a^2
+b^2)^(1/2)-4*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b+(a^2+b^2)^(1/2)))/
a^2/d^3/e/n/(x^(3*n))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 1218, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules

used = {5549, 5545, 4276, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)\,d^3en} + \frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)\,d^3en} + \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2\sqrt{a^2+b^2}d^3en} - \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)^{3/2}\,d^3en} - \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2\sqrt{a^2+b^2}d^3en} + \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)x^{-3n}}{a^2(a^2+b^2)^{3/2}\,d^3en} + \frac{2b^2(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^{-2n}}{a^2(a^2+b^2)\,d^2en} + \frac{2b^2(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^{-2n}}{a^2(a^2+b^2)\,d^2en} - \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2\sqrt{a^2+b^2}d^2en} + \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)^{3/2}\,d^2en} + \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2\sqrt{a^2+b^2}d^2en} - \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{a^2+b^2}}\right)x^{-2n}}{a^2(a^2+b^2)^{3/2}\,d^2en} - \frac{b^2(ex)^{3n}x^{-n}}{a^2(a^2+b^2)\,den} - \frac{2b(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^{-n}}{a^2\sqrt{a^2+b^2}den} + \frac{b^3(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{a^2+b^2}}+1\right)x^{-n}}{a^2(a^2+b^2)^{3/2}\,den} + \frac{2b(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^{-n}}{a^2\sqrt{a^2+b^2}den} - \frac{b^3(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{a^2+b^2}}+1\right)x^{-n}}{a^2(a^2+b^2)^{3/2}\,den} - \frac{b^2(ex)^{3n}\cosh(dx^n+c)\,x^{-n}}{a(a^2+b^2)\,den\,(b+a\sinh(dx^n+c))} + \frac{(ex)^{3n}}{3a^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(3*n)/(3*a^2*e*n) - (b^2*(e*x)^(3*n))/(a^2*(a^2 + b^2)*d*e*n*x^n) + (
2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2])])/(a^2*(a
^2 + b^2)*d^2*e*n*x^(2*n)) + (b^3*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b
- Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d*e*n*x^n) - (2*b*(e*x)^(3*n)*L
og[1 + (a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d*e*n
*x^n) + (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2])]
)/(a^2*(a^2 + b^2)*d^2*e*n*x^(2*n)) - (b^3*(e*x)^(3*n)*Log[1 + (a*E^(c + d*
x^n))/(b + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d*e*n*x^n) + (2*b*(e*x
)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b
^2]*d*e*n*x^n) + (2*b^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqr
t[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3*e*n*x^(3*n)) + (2*b^3*(e*x)^(3*n)*Pol
yLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)
*d^2*e*n*x^(2*n)) - (4*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sq
rt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2*e*n*x^(2*n)) + (2*b^2*(e*x)^(3*n
)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*
d^3*e*n*x^(3*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + S
qrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (4*b*(e*x)^(3*
n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 +
b^2]*d^2*e*n*x^(2*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(
b - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3*e*n*x^(3*n)) + (4*b*(e*x
)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a
^2 + b^2]*d^3*e*n*x^(3*n)) + (2*b^3*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^
n))/(b + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3*e*n*x^(3*n)) - (4*b
*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b + Sqrt[a^2 + b^2]))])/(a^2*S
qrt[a^2 + b^2]*d^3*e*n*x^(3*n)) - (b^2*(e*x)^(3*n)*Cosh[c + d*x^n])/(a*(a^2
 + b^2)*d*e*n*x^n*(b + a*Sinh[c + d*x^n]))
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2296</u>

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
```

```
Q[m, 0]
```

Rule 5545

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csch[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5549

```
Int[((a_.) + Csch[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Csch[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{\left(a+b\,\mathrm{csch}(c+dx^n)\right)^2}\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\mathrm{Subst}\left(\int \frac{x^2}{(a+b\,\mathrm{csch}(c+dx))^2}\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\mathrm{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sinh(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sinh(c+dx))}\right)dx,x,x^n\right)}{en}\\
&= \frac{(ex)^{3n}}{3a^2en}-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\mathrm{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx,x,x^n\right)}{a^2en}\\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\mathrm{Subst}\left(\int \frac{x^2}{(b+a\sinh(c+dx))^2}\,dx,x,x^n\right)}{a^2en}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))}$$
$$- \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{a^2en}$$
$$+ \frac{(b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{x^2}{b+a\sinh(c+dx)}\,dx, x, x^n\right)}{a^2(a^2+b^2)\,en}$$
$$+ \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{x\cosh(c+dx)}{b+a\sinh(c+dx)}\,dx, x, x^n\right)}{a(a^2+b^2)\,den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))}$$
$$+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{-a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{a^2(a^2+b^2)\,en}$$
$$- \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a\sqrt{a^2+b^2}en}$$
$$+ \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a\sqrt{a^2+b^2}en}$$
$$+ \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, x^n\right)}{a(a^2+b^2)\,den}$$
$$+ \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{a^2+b^2}+ae^{c+dx}}\,dx, x, x^n\right)}{a(a^2+b^2)\,den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en}$$

$$- \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en}$$

$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))}$$

$$+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a(a^2+b^2)^{3/2}\,en}$$

$$- \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a(a^2+b^2)^{3/2}\,en}$$

$$- \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b-\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a^2(a^2+b^2)\,d^2en}$$

$$- \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b+\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a^2(a^2+b^2)\,d^2en}$$

$$+ \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a^2\sqrt{a^2+b^2}den}$$

$$- \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx, x, x^n\right)}{a^2\sqrt{a^2+b^2}den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2\left(a^2+b^2\right)den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2en}$$
$$+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$
$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}den}$$
$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} - \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en}$$
$$+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{3n}\cosh\left(c+dx^n\right)}{a\left(a^2+b^2\right)den\left(b+a\sinh\left(c+dx^n\right)\right)}$$
$$- \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b-\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^n}\right)}{a^2\left(a^2+b^2\right)d^3en}$$
$$- \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^n}\right)}{a^2\left(a^2+b^2\right)d^3en}$$
$$+ \frac{\left(4bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{a^2+b^2}d^2en}$$
$$- \frac{\left(4bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{a^2+b^2}d^2en}$$
$$- \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2+b^2\right)^{3/2}den}$$
$$+ \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2+b^2\right)^{3/2}den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en}$$

$$+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den}$$

$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den}$$

$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en}$$

$$+ \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} - \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en}$$

$$+ \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en}$$

$$- \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en}$$

$$+ \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))}$$

$$+ \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+dx^n}\right)}{a^2\sqrt{a^2+b^2}d^3en}$$

$$- \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx, x, e^{c+dx^n}\right)}{a^2\sqrt{a^2+b^2}d^3en}$$

$$- \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2+b^2)^{3/2}\,d^2en}$$

$$+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2+b^2)^{3/2}\,d^2en}$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} \\
&+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} \\
&+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} - \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} \\
&+ \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} + \frac{4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en} \\
&- \frac{4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))} \\
&- \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en} \\
&+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2+b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} \\
&+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,den} \\
&+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}den} + \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} - \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} \\
&+ \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2en} - \frac{2b^3x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en} \\
&+ \frac{4bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en} + \frac{2b^3x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}\,d^3en} \\
&- \frac{4bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^3en} - \frac{b^2x^{-n}(ex)^{3n}\cosh(c+dx^n)}{a(a^2+b^2)\,den\,(b+a\sinh(c+dx^n))}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{(ex)^{-1+3n}}{(a+b\,\text{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{-1+3n}}{(a+b\,\text{csch}(c+dx^n))^2}\,dx
$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csch[c + d*x^n])^2, x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\,\operatorname{csch}(c+d\,x^n))^2}dx$$

[In] int((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n))^2,x)

[Out] int((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n))^2,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \text{Timed out}$$

[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx$$

[In] integrate((e*x)**(-1+3*n)/(a+b*csch(c+d*x**n))**2,x)

[Out] Integral((e*x)**(3*n - 1)/(a + b*csch(c + d*x**n))**2, x)

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\operatorname{csch}(dx^n+c)+a)^2}\,dx$$

[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="maxima")

[Out] 1/3*(6*a*b^2*e^(3*n)*x^(2*n) + (a^3*d*e^(3*n) + a*b^2*d*e^(3*n))*x^(3*n) - (a^3*d*e^(3*n)*e^(2*c) + a*b^2*d*e^(3*n)*e^(2*c))*e^(2*d*x^n + 3*n*log(x)) - 2*(3*b^3*e^(3*n)*e^(2*n*log(x) + c) + (a^2*b*d*e^(3*n)*e^c + b^3*d*e^(3*n)*e^c)*x^(3*n))*e^(d*x^n))/(a^5*d*e*n + a^3*b^2*d*e*n - (a^5*d*e*n*e^(2*c) + a^3*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) - 2*(a^4*b*d*e*n*e^c + a^2*b^3*d*e*n*e^c)*e^(d*x^n)) - integrate(-2*(2*a*b^2*e^(3*n)*x^(2*n) - (2*b^3*e^(3*n)*e^(2*n*log(x) + c) + (2*a^2*b*d*e^(3*n)*e^c + b^3*d*e^(3*n)*e^c)*x^(3*n))*e^(d*x^n))/((a^5*d*e*e^(2*c) + a^3*b^2*d*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d*e*e^c + a^2*b^3*d*e*e^c)*x*e^(d*x^n) - (a^5*d*e + a^3*b^2*d*e)*x), x)

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\operatorname{csch}(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csch(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*csch(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{csch}(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{3\,n-1}}{\left(a+\frac{b}{\sinh(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/sinh(c + d*x^n))^2,x)
[Out] int((e*x)^(3*n - 1)/(a + b/sinh(c + d*x^n))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 575

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```